Probabilistic Completion Time in Project Scheduling

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Abstract— *There are two common used methods to find the minimum completion time for a project scheduling. These methods are Critical Path Method (CPM) and Program Evaluation Review Technique (PERT). In CPM, a network diagram, which is Activity on Node (AON), is drawn and the slack time for every activity is calculated such that the project's critical path could be found. It is important that the critical path can suggest the shortest possible completion time. On the other hand, PERT concerns on uncertainty and risk in a project. It has three time estimates, which are optimistic, pessimistic and most likely, and all the time estimates mentioned follows the beta distribution. Besides, the probability in completing the project within certain duration is calculated by using the standard normal distribution. As the risk cannot be avoided in a project, it is important to keep track on any changes and to minimize the completion time for a project. Both of the methods are used to calculate the shortest possible completion time, slack and critical path. The difference between these methods is CPM has only one determined time estimate, while PERT has three time estimates, which shows the uncertainty in the duration of an activity in a project. For illustration, the data used for the construction of a three-room house was studied. The results show that the minimum completion time for the project is 44 days with a success probability 0.91. In conclusion, CPM and PERT are practical tool in the project scheduling.*

Keywords—Activity on Node, Critical Path Method, Program Evaluation Review Technique, Probabilistic Completion Time, Slack Time.

I. INTRODUCTION

Project scheduling is a complicated process of planning, scheduling and controlling a progressing event. The problem that deals with a project is always related to risk, which its occurrence will affect the project objective, scope, time, budget and quality [1]. Since variability is unavoidable, a project manager shall determine the risk and uncertainty before executing a project. A good estimation of project completion time is one of the solutions in diminishing uncertainty. On this basis, the project manager will be able to gauge the uncertainty and quickly adapt to any problem that might occur. The project manager should also figure out the shortest completion time in order to minimize the cost incurred. By virtue of this, two popular methods, which are critical path method (CPM) and program evaluation review technique (PERT), are applied to find the shortest completion time with consideration of risk factors and uncertainty [2]. Duration on a critical path is the possible shortest time to complete critical activities, and any delay of a critical activity will cause the impact on the project completion. Thus, critical path, which is having the least amount of scheduling flexibility, is identified by using the network diagram, that is, Activity on Node (AON) [3]. The probability of completing a project by certain time is calculated by using the standard normal distribution. Here, SAS University Edition is employed for the calculation.

The rest of the paper is organized as follows. In Section 2, the problem of scheduling for a project is described, where the activities and the corresponding time estimates are presented. In Section 3, the methods used, which are CPM and PERT, are clearly discussed. The forward and backward passes are considered in calculating the slack time such that the critical path of the activities could be determined. In Section 4, Gantt chart and network diagram AON are shown. The results show the critical path and the probabilities of the certain duration are given. Finally, some concluding remarks are made.

II. PROBLEM STATEMENT

Uncertainty is inevitable in most of the project scheduling and even an excellence project manager has difficulty to deal with the uncertainty. Every uncertainty is bringing a risk of money loss and time wastage. This situation will eventually overrun the project budget and make a project fail if it is not managed properly [4], [5]. Therefore, the project minimum completion time at a high probability to be completed should be chosen so as a target for completing a project could be set. In practice, the time estimates for construction of a three-bed room house at Nkronsa, Off the Santasi-Obuasi Road, Kumasi [6], which are provided by the project manager from Angel Estates and Construction Ltd, are shown in TABLE 1. In order to ensure there is no budget overrun and time wastage, the project completion time must within the estimated duration.

TABLE 1
ACTIVITIES DESCRIPTION WITH THEIR PREDECESSOR AND TIME ESTIMATES

Activity	Activity Description	Immediate Predecessor	Estimated Duration (x)	Optimistic Estimate (a)	Most Likely Estimate (m)	Pessimistic Estimate (b)
A	Site clearing	-	2	1	2	3
B	Foundation	*A*	4	2	3.5	8
C	Block laying	*B*	10	6	9	18
D	Roofing	*C*	6	4	5.5	10
E	Plumbing	*C*	4	1	4.5	5
F	Electrical work	*E*	5	4	4	10
G	Plastering	*D*	7	5	6.5	11
H	Fixing up the doors and windows	*E*, *G*	9	5	8	17
I	Ceiling	*C*	7	3	7.5	9
J	Flooring	*F*, *I*	8	3	9	9
K	Interior Fixtures	*J*	4	4	4	4
L	Exterior fixtures	*J*	5	1	5.5	7
M	Painting	*H*	2	1	2	3
N	Landscaping	*K*, *L*	6	5	5.5	9

The data consists of 14 activities with their immediate predecessors and estimated durations. This project is having only an estimation of completion time including risks. Hence, the project manager should always keep tracking and setting a minimum time as a target of the completion time.

III. CPM / PERT APPROACH

In finding a project's minimum completion time, Gantt chart is drawn to show all activities that are displayed over time. The list of activities is transferred in network diagram AON for a better understanding in terms of their duration in an order. Thus, CPM is used to calculate earliest start time (ES), late start time (LS), earliest finish time (EF) and late finish time (LF) for each activity. During the calculation, the critical path, which is the path with the shortest possible completion time, is determined. In computing ES, EF, LS, and LF, two nodes, which form a pass, need to be considered. For a forward pass,

$$\text{ES} = \max\{\text{EF}_i\}, \text{ for } i > 1 \quad (1)$$

is considered if there are more EF values from predecessors. Similarly, for a backward pass,

$$\text{LF} = \min\{\text{ES}_i\}, \textit{ for } i > 1 \quad (2)$$

is considered when there are more than one of ES values from the successor. After this, slack time for each activity is computed using the following formula:

$$\text{Slack} = \text{LF} - \text{EF} = \text{LS} - \text{ES}. \quad (3)$$

Every activity's slack time is listed and those with zero slack are a critical activity. The succession path that is formed by critical activity is regarded as the critical path. The duration on the critical path is calculated and identified as minimum completion time of the project.

PERT is more advanced with three time estimations, which are optimistic (a), most possible (m) and pessimistic (b), are considered in completion time of scheduling the project. PERT is more suitable for large project with more uncertainties if compared with CPM. In PERT, the expected time TE and the variance σ are calculated by:

$$\text{TE} = \mu = \frac{a + 4m + b}{6} \quad \text{and} \quad \sigma^2 = \left(\frac{b - a}{6}\right)^2 \quad (4)$$

Furthermore, the probability of the completion time is then calculated by using the standard *z*-score with considering the expected time and the variance calculated. The *z*-score formula, where *d* value is any duration that the project manager desires to complete a project, is given by:

$$Z = \frac{d - \mu}{\sigma} \tag{5}$$

On the other hand, SAS University Edition is used in calculating the completion time and the corresponding probability as well as drawing Gantt chart, while RF Flow is used for drawing network diagram as it is a very user-friendly software and free for all users.

IV. RESULT AND DISCUSSION

Gantt chart is shown in Fig. 1, where a clear display of activities with the respective time used in managing the project, is presented.

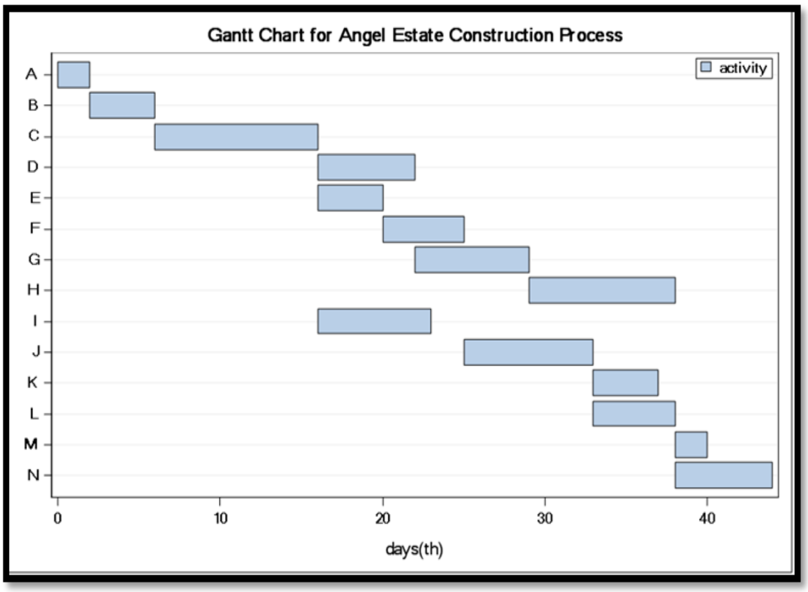

FIG. 1 GANTT CHART FOR ANGEL ESTATE CONSTRUCTION PROCESS

The network diagram AON is simpler and easy to understand. As such the duration of each activity is expressed in Fig. 2.

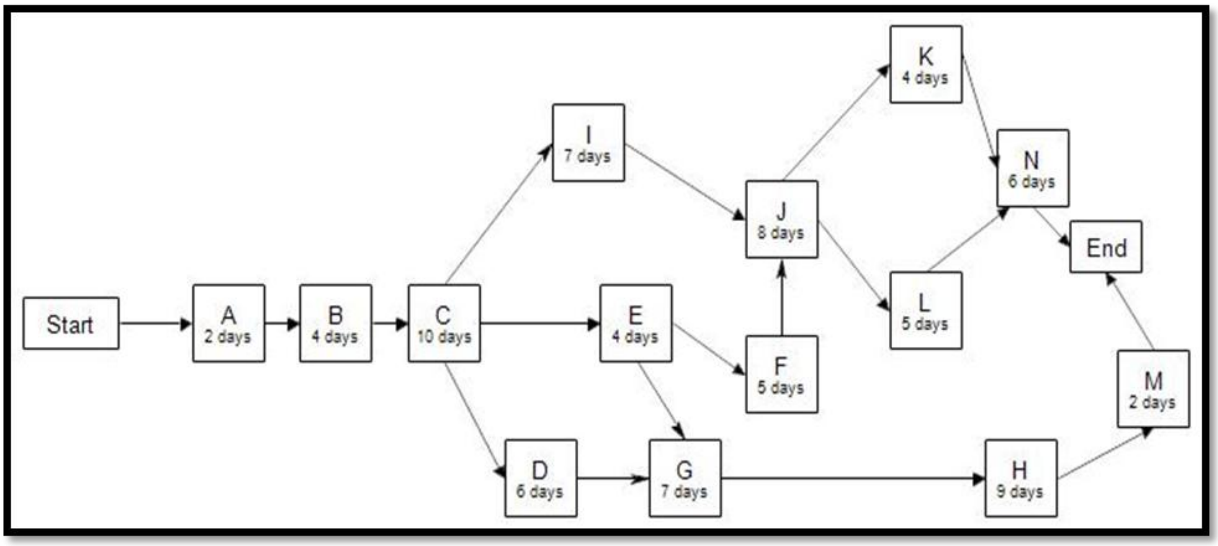

FIG. 2 AON FOR ANGEL ESTATE CONSTRUCTION PROCESS

Next, CPM is used to calculate all the ES, LS, EF and LF for determining the path with zero slack. TABLE 2 shows the calculation of ES, LS, EF, and LF. The results show that the critical path is

$$A - B - C - E - F - J - L - N$$

as these activities have a zero slack. This path has a completion time of 44 days, which is the shortest completion time in the whole project.

TABLE 2
ACTIVITIES CODE WITH THEIR ES, EF, LS, LF AND SLACK

Activity Code	Earliest Start (ES)	Earliest Finish (EF)	Latest Start (LS)	Latest Finish (LF)	Slack (LS-ES)	Critical
A	0	2	0	2	0	Yes
B	2	6	2	6	0	Yes
C	6	16	6	16	0	Yes
D	16	22	20	26	4	No
E	16	20	16	20	0	Yes
F	20	25	20	25	0	Yes
G	22	29	26	33	4	No
H	29	38	33	42	4	No
I	16	23	18	25	2	No
J	25	33	25	33	0	Yes
K	33	37	34	38	1	No
L	33	38	33	38	0	Yes
M	38	40	42	44	4	No
N	38	44	38	44	0	Yes

PERT could be used to find the probability of a project which can be accomplished within 44 days. Mean and variance for every activity that are computed by using (4) is shown in TABLE 3.

TABLE 3
ACTIVITY CODE AND THE CALCULATION OF MEAN AND VARIANCE

Activity	Optimistic Estimate (*a*)	Most Likely Estimate (*m*)	Pessimistic Estimate (*b*)	Critical	Mean	Variance
A	1	2	3	Yes	2	1/9
B	2	3.5	8	Yes	4	1
C	6	9	18	Yes	10	4
D	4	5.5	10	No	6	1
E	1	4.5	5	Yes	4	4/9
F	4	4	10	Yes	5	1
G	5	6.5	11	No	7	1
H	5	8	17	No	9	4
I	3	7.5	9	No	7	1
J	3	9	9	Yes	8	1
K	4	4	4	No	4	0
L	1	5.5	7	Yes	5	1
M	1	2	3	No	2	1/9
N	5	5.5	9	Yes	6	4/9

The mean is calculated by adding up all the mean values of the critical activities, as given below:

$$\mu = 2 + 4 + 10 + 4 + 5 + 8 + 5 + 6 = 44.$$

The variance is calculated by adding up all the variance values of the critical activities as follows:

$$\sigma^2 = \frac{1}{9} + 1 + 4 + \frac{4}{9} + 1 + 1 + 1 + \frac{4}{9} = 9.0.$$

By substituting the mean and the variance that are calculated from (4) into (5) to calculate the z-value, the probability that the project will be done in 44 days is 0.91.

Besides finding the probability of a given duration of the project, the corresponding duration of the project can be known by giving the probability of the time estimated. For this purpose, five values of probability and the duration are randomly selected as shown in TABLE 4. Their respective duration and probability are calculated to show the best minimum completion time for the project, where the results with some given probabilities can be obtained by calculating the duration and vice versa.

TABLE 4
CALCULATION RESULTS OF GIVEN PROBABILITY AND DURATION

Given Probability	Duration Calculated
0.90	50
0.50	40
0.88	50
0.75	50
0.60	40

(a) Duration Calculated

Given Duration	Probability Calculated
40	0.84
42	0.50
44	0.91
47	0.98
48	0.25

(b) Probability Calculated

As a result, the shortest completion time, which close with mean and high probability (higher than 90%), is selected. That is, the shortest completion time is 44 days with 91% completion.

V. CONCLUDING REMARKS

In this paper, the probabilistic completion time of a project scheduling was discussed. The minimum possible completion time for the project mentioned is 44 days with the probability of 0.91. Uncertainty in the project can be minimized by applying CPM and PERT. The project manager will be able to cope with these methods as they know the use of these two approaches. CPM is more suitable for construction process as the project has fairly accurate in the time estimation. However, if it is dealing with large and high capital construction project, PERT would be a better choice. For the future usage, it is more challenging if it is including the cost in the project and taking the real data from the real-world implementing projects. This suggestion would be more practical if the knowledge can be applied in more challenging environment and increasing the value of the study.

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