

# Numerical Calculation and Analysis of the Normal Relationship of the Dual-Frequency Altimeter Backscatter Coefficient

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**Abstract**—To improve effectively sea surface wind speed retrieval accuracy when it is raining, it is very interesting to find out a more accurate normal relationship between Ku band backscatter coefficient  $\sigma_{ku}^0$  and C band backscatter coefficient  $\sigma_c^0$  for a dual frequency altimeter (13.58GHz/5.25GHz). In this paper an electromagnetic scattering model with a rough surface profiles is established. In terms of the established rough sea surface model  $\sigma_{ku}^0$  and  $\sigma_c^0$  are calculated by electromagnetic numerical methods of method of moment (MOM) and Kirchoff approximation (KA) when the rms of the rough surface is various. The simulation results are in a good agreement by these two different methods: MOM and KA. Those calculated results show that the relationship between  $\sigma_{ku}^0$  and  $\sigma_c^0$  obtained by calculating electromagnetic scattering from Gaussian random rough sea surface profiles exists positive differences from the normal relationships while those obtained by calculating electromagnetic scattering from Exponential rough sea surface profiles exist negative differences from those. It is obvious that the calculated relationship can be in a good agreement with an empirical normal relationship by adjusting compositions of those ocean spectrums. It is a good way to find out the normal relationship between  $\sigma_{ku}^0$  and  $\sigma_c^0$  by theoretical calculation and analysis, and also is an effective way to study the ocean spectrum and sea states.

**Keywords**— Dual frequency altimeter, Normal relationship, Backscatter coefficient, MOM, KA.

## I. INTRODUCTION

The Topex/Poseidon satellite, which was developed by National Aeronautics and Space Administration (NASA) and the French Space Agency (CNES) was launched on August 10,1992, carrying the first dual frequency altimeter (13.58GHz/5.25GHz). It was because of the presence of the dual-frequency altimeter, removing rainfall influence on wind speed inversion had a substantial progress. It was known that the prime effect of rainfall was the attenuation of the return pulse and there was an order of magnitude larger at Ku band than at C-band, in addition to attenuation, rain can also change the shape of the return pulse and hence the measurement of the significant wave height (swh) and sea surface height (ssh) can also be affected. In terms of Topex/Poseidon measurements, a statistical relationship (or normal relationship) between  $\sigma_{ku}^0$  and  $\sigma_c^0$  was presented in [1]. With the normal relationship the rainfall influence on  $\sigma_{ku}^0$  has to be eliminated in order to improve wind speed retrieval. After that many normal relationships [2-4] were put forward to be used to define a new rain flag to detect all rain events, as well as to improve wind speed retrieval. Those presented normal relationships are all empirical and statistical in terms of measurements provided from a certain altimeter. For  $\sigma_{ku}^0$  and  $\sigma_c^0$  there is a correlation on the physical mechanism, but now it is difficult to theoretically reduce the formula, And thus many empirical relationships were usually proposed and adopted.

The main motivations for this study are firstly, to better understand the normal relationship between  $\sigma_{ku}^0$  and  $\sigma_c^0$  when rain is not present, and secondly, to show that the differential effect of oceanic spectrums on Ku and C band microwave signals can be used to define a better and possibly operational normal relationship. A further goal is to investigate the possibility of using dual-frequency altimeter data to improve sea surface wind speed retrieval [5,6].

## II. ELECTROMAGNETIC MODEL AND NUMERICAL CALCULATION

The electromagnetic model is established by Monte Carlo simulation. Numerical calculations are carried out by MOM and KA.

**2.1 Electromagnetic scattering model**

**2.1.1 Ocean spectrum**

It is more convenient to supposing that the ocean spectrum case is still a Gaussian process with the ocean spectral density. For 1-D spectrum, the Gaussian spectrum is

$$W(k) = \frac{h^2 l}{2\sqrt{\pi}} \exp\left(-\frac{k^2 l^2}{4}\right) \tag{1}$$

If the ocean spectrum case is an exponential process, it may be given as

$$W(k) = \frac{h^2 l}{\pi(1+k^2 l^2)} \tag{2}$$

where  $k$  is space wave number,  $h$  and  $l$  are RMS height and correlation length. Other spectrums are not studied in this paper.

**2.1.2 Random Rough Surface**

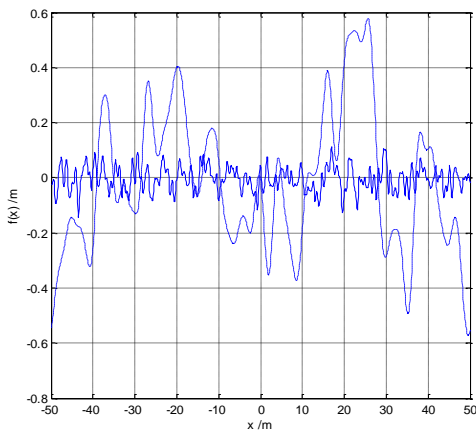
We generate Gaussian or exponential rough surface with Monte Carlo simulation and Fourier Transform. For 1-D case, we have

$$f(x) = \frac{1}{L} \sum_{i=-N/2+1}^{N/2} F(k_i) \exp(jk_i x) \tag{3}$$

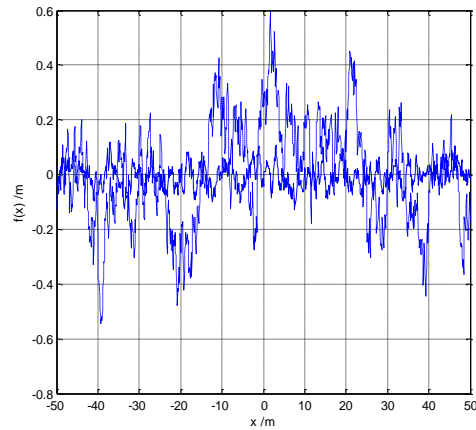
where

$$F(k_i) = \frac{2\pi}{\sqrt{2\Delta k}} \sqrt{W(k_i)} \begin{cases} N(0,1) + jN(0,1), & i \neq 0, N/2 \\ N(0,1), & i = 0, N/2 \end{cases} \tag{4}$$

Some parameters were defined in [7]. We can see that the exponential spectrum surface of fig.2.2 is more than spiky than the Gaussian surface of fig.2.1.



**FIG.2.1 1D GAUSSIAN ROUGH SURFACE PROFILE --  $h=0.221m, l=2.21m$ ; —  $h=0.0442m, l=4.42m$**



**FIG.2.2 1D EXPONENTIAL ROUGH SURFACE PROFILE --  $h=0.221m, l=2.21m$ ; —  $h=0.0442m, l=4.42m$**

**2.2 MOM**

Consider an incident wave  $\psi_{in}(\vec{r})$  impinging upon a rough sea surface (fig.2.1 or fig.2.2) with height profile  $z=f(x)$ . The wave function  $\psi(\vec{r})$  can be given as

$$\psi(\vec{r}) = \psi_{in}(\vec{r}) + \psi_s(\vec{r}) \tag{5}$$

Where  $\psi_s(\vec{r})$  is the scattered wave and position vector  $\vec{r} = x\hat{x} + z\hat{z}$ . In terms of wave equation and Green's theorem, we can obtain

$$\begin{aligned} & \psi_{in}(\vec{r}') + \int_s ds \hat{n} \cdot [\psi(\vec{r}) \nabla g(\vec{r}, \vec{r}') - g(\vec{r}, \vec{r}') \nabla \psi(\vec{r})] \\ & = \begin{cases} \psi(\vec{r}') & \vec{r}' \in V_0 \\ 0 & \vec{r}' \in V_1 \end{cases} \end{aligned} \tag{6}$$

Note that in equation (6),  $\bar{r}$  is on the sea surface while  $\bar{r}'$  is in region  $V_0$  above the sea surface or in region  $V_1$  under the sea surface. Applying electromagnetic field boundary conditions, we can obtain

$$\sum_{n=1}^N A_{mn} U_n + \sum_{n=1}^N B_{mn} \psi_n = b_m \tag{7}$$

$$\sum_{n=1}^N A_{mn}^{(1)} \rho U_n + \sum_{n=1}^N B_{mn}^{(1)} \psi_n = 0 \tag{8}$$

Some quantities in equation (7) and equation (8) can be defined in[7].

### 2.3 KA

From equation (1) and the second Green's theorem, the scattered field  $\psi_s(\bar{r})$  can be given by[8]

$$\begin{aligned} \psi_s(\bar{r}) &= \psi(\bar{r}) - \psi_{in}(\bar{r}) \\ &= \int_c \left[ \psi(\bar{r}') \frac{\partial g(\bar{r}, \bar{r}')}{\partial \hat{n}'} - g(\bar{r}, \bar{r}') \frac{\partial \psi(\bar{r}')}{\partial \hat{n}'} \right] dl' \end{aligned} \tag{9}$$

Where

$$\psi(\bar{r}') = \psi_{in}(\bar{r}') [1 + R(\bar{r}')] \tag{10}$$

$$\frac{\partial \psi(\bar{r}')}{\partial \hat{n}'} = \frac{\partial \psi_{in}(\bar{r}')}{\partial \hat{n}'} [1 - R(\bar{r}')] \tag{11}$$

position vector  $\bar{r}'$  is on the rough sea surface and  $R(\bar{r}')$  is Fresnel reflection coefficient other parameters are defined in[8-9].

### III. SIMULATION RESULTS

In terms of equation (7), equation (8) and equation (9) we can calculate scattering from the rough sea surface with Gaussian or exponential distribution. In order to avoid artificial reflection from the two endpoints of the rough surface, we taper the incident wave so that the incident wave decays to zero in a Gaussian manner for large  $x$ .

In fig.3.1, the results by mom and KA are in a good agreement and valid. The rough surface is Gaussian. The scattering from the rough surface with Gaussian distribution is more than the normal relationship does, while the scattering from the rough surface with exponential distribution is less than the normal relationship, as shown in fig.3.2. Note that the scattering from the rough surface with Gaussian and exponential distribution is among some normal relationships. It is obvious that a normal relationship only stands for a practical sea state with one or several ocean spectrums. From fig.3.1 and fig.3.2 it is obvious that the normal relationship between  $\sigma_{ku}^0$  and  $\sigma_c^0$  is relative to several ocean spectrums.

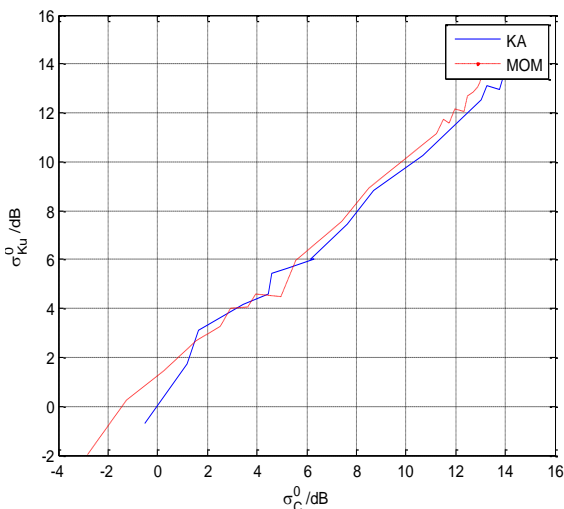


FIG.3.1 RELATIONSHIP BETWEEN  $\sigma_{ku}^0$  and  $\sigma_c^0$

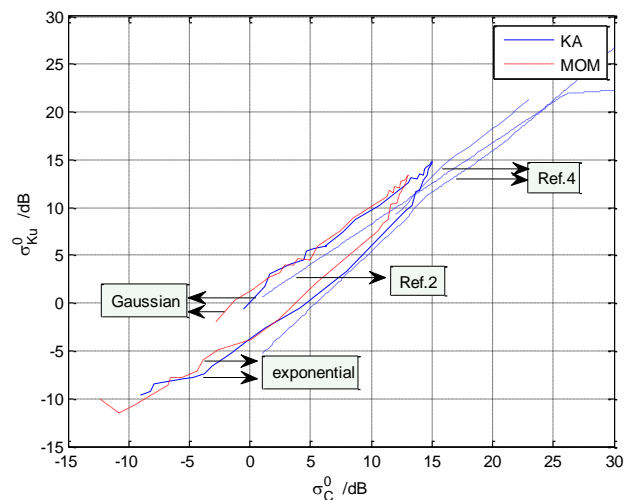


FIG.3.2 THE CALCULATED RELATIONSHIP AND SEVERAL NORMAL RELATIONSHIPS

All in all, for different oceans and sea states there perhaps exists different normal relationships with different ocean spectrums.

#### IV. CONCLUSION

Applying MOM and KA, we finish simulating scattering from Gaussian random rough sea surface profiles and exponential random rough sea surface profiles with various rms height  $h$ , respectively. The relationship between  $\sigma_{ku}^0$  and  $\sigma_c^0$  is presented and can be in a good agreement with the normal relationships by adjusting or simulating ocean spectrum compositions. The relationship between  $\sigma_{ku}^0$  and  $\sigma_c^0$  from Gaussian and exponential distribution is among those normal relationships. Our simulating is a good way to study the normal relationships between  $\sigma_{ku}^0$  and  $\sigma_c^0$ .

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