Widths of Channel Routing in VLSI Design

Zhixiang Yin[1], Jianzhong Cui[2], Xianya Geng[3]

[1,2,3]Department of Mathematics and Physics, Anhui University of Science and Technology, P.R.C

[2]Department of Computer Science, HuaiNan Union University,P.R.C

Abstract—*In VLSI design, one of the most important detailed routings is the channel routing. Given a channel with length n in 2-layer Manhattan model, Szeszler proved that the width (number of tracks required for routing) of the channel is at most* $\frac{7}{4}n$*, and this upper bound can be achieved by a linear time algorithm. In this note, we improve the upper bound* $\frac{7}{4}n$ *to* $\frac{3}{2}n$*, which also can be achieved by a linear time algorithm.*

***Keywords*—Channel routing, Graph Theory, VLSI design.**

I. INTRODUCTION

In VLSI design, One of the most important detailed routings is the channel routing [2,5,6]. A channel is defined by a rectangular grid of size $(w+2)\times n$, insisting of horizontal tracks (numbered from 0 to w+1) and vertical columns (numbered from 1 to n), where w is the width and n is the length of the channel. The horizontal tracks numbered 0 and w + 1 are called the top and bottom of the channel, respectively. Points on the top or bottom are called terminals. A net is a set of terminals. The channel routing problem (CRP) is to interconnect all the terminals in the same nets, using minimum possible routing area, that is, minimizing the width w with the length n fixed. If all the terminals of each net are situated on one side, top or bottom, of the channel, we speak of single row routing problem.

An instance of the CRP is a set $\mathrm{N}=\{N_1,\dots,N_t\}$ of pair wise disjoint nets, each containing at least two terminals. The instance is called bipartite if each net N_1 contains exactly two terminals, one on the top and the other on the bottom of the channel. The instance is dense if each terminal belongs to some net. A net is trivial if it consists of two terminals which are situated in the same column of the channel.

II. MAIN RESULTS

A solution of a channel routing problem is said to belong to the Manhattan model if consecutive layers contain wire segments of different directions only. The following results were obtained by Szeszler [4], which completely characterizes solvable CRP instances in 2-layer Manhattan model and gives upper bounds on the widths, in terms of the lengths, of the channels.

Theorem 2.1 (Szeszler [4]) A channel routing problem is not solvable in 2-layer Manhattan model (with a arbitrary width) if and only if it is bipartite, dense and has at least one non-trivial net. Moreover, if an instance is solvable, then it can be solved with width at most $\frac{3}{2}$n in the bipartite, and $\frac{7}{4}$n in the general case, where n is the length of the channel.

From the above theorem, It seems that the general case requires more routing area than the bipartite case. Surprisingly, we find that this is not necessary to be true. In the next theorem, we show that the bound $\frac{7}{4}$n in the above theorem can be improved to $\frac{3}{2}$n, that is, the bound in the general case is the same as that in the bipartite case.

Theorem 2.2 For a solvable channel routing problem with length n (not necessary to be bipartite) in 2-layer Manhattan model, there is a solution with width at most $\frac{3}{2}$n.

Proof. As in [4], we first consider the nets on only one side, top or bottom, of the channel, which is a single row routing problem. Consider the horizontal constraint graph H with vertex set

V (H) ={ fall the nets from one side containing at least two terminals}, and an interval is associated with each net, stretching from its leftmost terminal to its rightmost terminal, there is an edge joining two vertices (nets) in H if and only if the corresponding intervals intersect. By Gallai Theorem [1], the number of tracks in an optimal solution of the single row routing problem is equal to the clique number of its horizontal constraint graph, which can be done in linear time since it is an interval graph. Let S_t and S_b be the number of tracks in an optimal solution for the single row routing problem on the top and bottom of the channel, respectively. Denote by A and B the sets of nets on the top and bottom of the channel, respectively. Since there is a clique of size S_t (S_b) in the horizontal constraint graph for the single row routing problem on the top (bottom) of the channel, and each net in the clique contains at least two terminals (two columns of the channel), we have that

$$|A| \le S_t + (n - 2S_t) = n - S_t$$
$$|B| \le S_b + (n - 2S_b) = n - S_b$$

Without loss of generality, we may assume that $S_t \ge S_b$. Let D be the set of nests which contain at least one terminal from each side of the top and bottom. We construct a bipartite routing problem based on D as follows: if a net N in D contains more than one terminal from the top, we arbitrarily choose one and only one terminal as the terminal of the net; similarly, choose one and only one terminal if it contains more than one terminal from the bottom. In this way, we obtain a bipartite routing problem with D' as its nets, where

$$|D'| = |D| \le \min\{|A|, |B|\} \le \min\{n - S_t, n - S_b\} = n - S_t \quad (2.1)$$

Let D_1 denote the set of nets in D which contain exactly one terminal on the top and let D_2 the set of nets in D which contain at least two terminals on the top. Set $|D_1| = d_1$ and $|D_2| = d_2$. Clearly $d_1 + d_2 = |D| \le n - S_t$. As seen before, each net in the clique of the horizontal constraint graph for the single row routing problem contains at least two terminals, we see that

$$d_1 \le n - 2S_t \quad (2.2)$$

Suppose that $D' = \{N_1, N_2, \ldots, N_t\}$, where $d = |D'|$. Consider the vertical constraint graph F, which is a directed graph based on D', with vertex set $V(F) = D'$ and there is a directed edge $N_i = N_j$, from N_i to N_j, if and only if there is a column in which the terminal on the top belongs to N_i and the terminal on the bottom belongs to N_j. Since each N_i contains only one terminal from top or bottom, each vertex of F has both in-degree and out-degree at most one. Hence, each nonempty component of F is either a directed path or a directed cycle. As shown in [4], if the nonempty component is a directed path of t vertices (nets), then we only need t tracks for these t nets.

Suppose that a nonempty component of F is a directed cycle $C = (N_{i_1} N_{i_2} \ldots N_{i_t} N_{i_1})$. If C does not contain any net from D_2, then as shown in [4], we need $t+1$ tracks for these t nets. Otherwise, without loss of generality, let $N_{i_t} \in D_2$. So N_{i_t} contains at least two terminals a_i and a_j on the top of the channel. We may assume that a_i is the terminal used in D' and a_j is another terminal, where a_i and a_j are in the columns i and j, respectively. Let $b_i(b_j)$ be the terminal on the bottom in the column $i(j)$. So b_i is a terminal of N_{i_1}. Now, instead of using a_i as a terminal of N_{i_t} in D', we use a_j as a terminal of N_{i_t} and denote this modified D' by D''.

If b_j is not a terminal of any net in D'', then C becomes a directed path in D'', and thus we only need t tracks for these t nets. If b_j is a terminal of some net N_k in D'', since each vertex of V has both in-degree and out-degree at most one, we have that $k \ne \{i_1, i_2, \ldots, i_t\}$, and N_k is the initial vertex of a directed path P of another component of

F. Suppose that P contains p nets. Then, C and P, together with the directed edge from N_{i_t} to N_k, form a directed path in D'', and so we only need $t+p$ tracks for these $t+p$ nets on the path.

From the solutions above, for a component C with t vertices (nets), where $t \geq 2$, if C does not contain any net from D_2, then we need $t+1$ tracks for the t net of C; Otherwise, we need t tracks (here we have taken into account of the S_t tracks used in the optimal solution for the single row routing problem on the top of the channel). That is to say, using this solution at most d_1 nets belong to directed cycles and at least d_2 nets belong to directed paths. So by Theorem 2.1, routing this new bipartite routing problem need at most $d_2 + \frac{3}{2}d_1$ tracks. It follows that the total number of tracks required is at most

$$w \leq d_2 + \frac{3}{2}d_1 + S_t + S_b = d + \frac{1}{2}d_1 + S_t + S_b$$

Using (2.1) and (2.2), we have that

$$\begin{aligned} w &\leq n - S_t + \frac{1}{2}(n - 2S_t) + S_t + S_b \\ &= \frac{3}{2}n - S_t + S_b \leq \frac{3}{2}n \end{aligned}$$

This completes the proof of Theorem 2.2.

Based on the arguments in the proof of Theorem 2.2, it is easy to construct a linear time algorithm to give a solution for a channel routing problem, using at most $\frac{3}{2}n$ tracks, where n is the length of the channel.

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