Design Approach for Trans – Sahara Gas Pipeline Transmission

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Abstract— Natural gas is poised to occupy a more prominent place in the global energy balance, coupled with other renewable energy sources, natural gas is an energy source of choice for the development of Africa and offers a vehicle for its integration with the world economy. The Economic Sustainability and expected benefits of Nigerian gas reserves are estimated at 5 trillion cubic meters – equal to roughly ten years of consumption of the EU. The modeling approach for long distance natural gas pipeline is optimized and design for the minimum installation cost. In this research paper the minimum cost for the installation of long gas pipeline is obtained by designing the different set of pipeline networks for which the installation costs are estimated and compared with the minimum capital cost. Various available pipeline systems are designed by varying the compressor station spacing for different standard pipe diameters. Accurate installation costs for each obtained pipeline network are estimated by using an advanced cost estimating computer program which uses real and updated data for the estimation. These total costs of the various pipeline networks are compared to obtain a minimum installation cost. The 1000km pipeline is divided into a number of short sections to calculate the pressure drop along the pipeline length. The design pressure is considered according to the 600 flange rating, 1,350 Psig and 90% of the design pressure is envisaged for the maximum operating pressure for the pipeline design.

Keywords— Tran-Sahara, Transmission, Pipeline, Minimum cost.

I. INTRODUCTION

The Trans –Sahara Transmission Gas Pipeline is poised to occupy a more prominent place in the global energy balance, coupled with other renewable energy sources, natural gas is an energy source of choice for the development of Africa and offers a vehicle for its integration with the world economy [6]. The Nigerian Economic Sustainability and the expected benefits of gas reserves are estimated at about 5 trillion cubic meters – equal to roughly ten years of consumption of the EU [6]. In light of growing demand factors from Europe due to the depletion of European gas fields, and the need for an alternative to Russian gas, the demand from Europe is likely to remain high. Hence, the Trans- Sahara Transmission Gas Pipeline (TSGP) would enable these African economies to access a new market for their gas reserves, thereby increasing their incomes [6].

Furthermore, the Nigeria –Algeria pipeline is also called Trans- Sahara Transmission Gas Pipeline that would contribute to eliminating natural gas flaring in Nigeria [6]. The TSGP has the critical advantage of supplying gas to Northern Nigeria, Niger, Southern Algeria, as well as Burkina Faso, and Southern Mali which are currently affected by high energy prices and desertification [6]. The TSGP between Algeria and Nigeria will recover flared gas in Nigeria and bring it through a pipeline toward the European market. This gas is burnt currently in flares and hence, represents a loss of energy equivalent to 220.000 barrels/day for Nigeria and thus has grave consequences on the environment [6].

The TSGP will provide power to the areas, thereby contributing notably to the revival of those areas, where there exists water and fertile land. Also, it will enable several countries in the region to access electricity [6]. The major difficulty in the use of natural gas is transportation and storage because of its low density.

Natural gas which was once an almost embarrassing and unwanted by-product or more correctly a co-production of crude oil production now provides about 1/5th of the entire world primary energy requirement [3]. This remarkable development has taken place in only a few years with increased availability of the gas resources of the country and the construction of long distance, large diameter steel pipeline which has brought these sample supplies of gaseous fuel to domestic, commercial and industrial users many miles away from the field themselves [3].

Considering the high cost of providing the natural gas using pipeline system many researchers have focused their research on minimising the cost of transportation. In this research, a detailed procedure for the optimising long gas pipeline networks is

outlined. The technical choices like the nominal pipeline diameter, pipe wall thickness, valves and fittings, compressors are explained. The cost estimation for the installation costs of the pipeline and the compressor stations are discussed. Based on the cost estimate, an optimum network of the pipeline was selected.

II. DESIGN AND OPTIMISATION OF LONG GAS PIPELINES

Due to the cost of pipeline, compressor stations and its maintenance, different approaches have been adopted by various scholars. This includes Rank Optimisation, Differential Equations, and Optimisation by Linear Programming, Dynamic Optimisation and Mathematical Analysis [17].

In this research, the pipeline was designed for two different pressures and the variables considered are the pipeline diameter, pipe thickness, pipeline length, turbines and compressors were adapted according to the American Petroleum Institute Standards [17]. The Actualized Transportation Cost Index(ATCI) was used in analysing the pressures and the optimal diameter which represents the ratio between the major actualized costs and the transportation cost. The costs that are approximately same for all the system like engineering costs are not considered in the economic calculations. Therefore the ATCI does not represent the exact transportation cost but only a convenient index for the economic comparison of technical feasible system [17], considered in the objective function.

This is optimised against the variables like number of compressor stations, pipe segment length, pipe diameter, inlet pressure and outlet pressure for a constant flow rate. They also considered a scenario that the compressor capital cost is a linear function of horsepower with an unchanging capital expenditure for zero power. The pipeline configuration was chosen as the pipeline segment is represented by an arc and the compressor station by a node, the pipelines system is taken horizontally. To solve this problem, a non-linear programming algorithm combined with a branch and bound algorithm is used.



FIGURE 1: THE TRANS –SAHARA GAS PIPELINE TRANSMISSION ROUTE

2.1 LP Optimisation

In this method, a generic formula is given for optimising the gas pipeline network regarding a linear programming. In the cost optimisation of the network, it is considered that the revenues are as negative values which are made by selling the gas at hubs. Cost function and set of constraints formed by different variable are combined to form a linear programming. The variables have to be linear for linear programming. To achieve the objective that is low cost some significant decisions are considered and they are capacity contracts to engage and the amount of gas sent from one place to another in each hour[16].

2.2 Rank Optimisation

Rank optimisation is a mathematical approach for capital minimisation cost, proposed by. An objective function for the main line system of high pressure gas network (10 MPa pressure) has been optimised, and the objective function is

$$\min F = \sum_{i=1}^{n} f_1(D_i, \delta_i, L_i) + \sum_{j=1}^{n} f_2(N_j, \varepsilon_j)$$
(1)

This objective function is also stated as the life cycle cost (total investment cost for pipeline system and its maintenance) of pipeline and compressor. This objective function is the sum of pipeline and compressor investment which comprises both continuous variables (pressure, temperature and length) and discrete variables (pipe diameters, pipe thickness and a number of compressor stations). This method is also called mixed discrete optimisation method.

The rank-optimization method considers different range of variables which are reduced gradually depending on the model characteristics. For example, first the Reynolds number is calculated for the different series of input pipe diameters and thicknesses. Diameters are eliminated from series when their Reynolds number cannot satisfy the flow state.

This is repeated for each diameter in the series to achieve a new and reduced series of pipe diameters. Similarly other parameters are assessed to get the optimal results. For the optimisation of the high pressure pipeline objective function, a program was compiled. Different cases are considered and compiled for the maximum total cost, and the results.

2.3 The Pipeline System Assumptions

The gas transportation system consists of a long pipeline with several compressor stations. It goes through very remote areas and rough terrains from Nigeria to Algeria. The pipeline configuration involved intricate designs, to simplify the design process the assumptions below were made.

- 1. Gas flow rate is constant along the pipeline.
- 2. The supply gas is nearly pure methane.
- 3. Uniform pipe diameter throughout the transportation line.
- 4. To avoid differences in pressure, the compressor altitude is assumed to be constant.

2.4 Technical Considerations

The technical choices for the design of the pipeline and compressors of the pipeline network are explained.

2.4.1 Pipeline Nominal Diameter

A range of standard API 5L (American Institute of Petroleum) specification pipeline diameters from 48 inches to 60 inches were considered. These standard diameter pipes are commercially available and commonly used for the transportation of the fluids.

2.4.2 Pipeline Material

Type of steel adopted for the whole pipeline is X60 under an API-5L system, pipes with longitudinal or spiral factory welds are utilized in the Russian section of the pipeline.

2.4.3 Pipe Wall Thickness

Rather than the commercial values of the pipe thickness under API 5L systems the calculated values are considered. The minimum pipe thickness is calculated by

$$t_m = \frac{Pd_o}{2SFET} + t_c$$

Where, $t_m =$ pipe wall thickness, mm

P = Design pressure, kPa

E = Longitudinal joint factor

d_o= Outside diameter, mm

F = Construction type design factor

- S = Minimum yield strength, kPa
- T = Temperature derating (Temperature < 120 °C, T =1.0)
- t_c= Corrosion allowance, m

Irrespective of operating pressure, the pipe wall thickness should not exceed 1.25 inches (32 mm) as per the X60 grade steel material, otherwise stress relieving each weld will be required.

2.4.4 Valves and Flanges

Standard ANSI (American National Standard Institute) class rating valves and flanges are considered. The isolation valves are used in places like either side of the river, railway and major road crossings, the also serve the purpose of maintenance of the pipeline network. This isolation valves can operate against full operating pressure. The selection of valves depends on the location and the fluid flow. The standard flanges used, and that can withstand more than the maximum operating pressure for safety. However, these flange joints can be the source of the leak in the pipeline.

2.4.5 Pipe Roughness

Pipe roughness for the X60 grade steel is taken as 0.0018 inches. The relative roughness is calculated by

$$Relative roughness = \varepsilon/D \tag{3}$$

Where, ε = pipe roughness, mm

D = pipe inside diameter, mm.

2.4.6 Design Pressure

The design pressure is the maximum operating pressure the pipeline willbe subjected. Design pressure is usually, 110% of the maximum operating pressure. Most of the pipelines are designed considering 600 lb flange rating or 900 lb flange rating, the same flange rating i.e. 1,350 psig is found [15].

2.4.7 Compressibility Factor

The compressibility factor, Z, is a dimensions number that represents a gas's deviation from ideal gas behaviour. It is estimated from other physical properties. For the conditions in the design, the estimated compressibility factor value is 0.96 [15].



(2)

2.4.8 Pressure Drop

The pressure drop is calculated over short sections along the pipeline length because the density of the gas is not constant. The gas flow in the pipeline is considered as turbulent flow. For a turbulent flow pressure drop along the pipe is calculated by this equation [15].

$$P_{2} = \sqrt{P_{1}^{2} - \frac{3086Q^{2}\gamma LTZ}{\left[\frac{T_{s}}{P_{s}}E4\log_{10}\left(\frac{3.7d}{\varepsilon}\right)\right]^{2}d^{5}}}$$
(4)

Where, Ts = Standard temperature (288.9 °K) Ps = Standard pressure (101.56 kPa)

E = Pipeline efficiency, use 1.0	d = Pipe internal diameter, mm
e = Pipe roughness, mm	P1 = Inlet pressure, kPa
P2 = Outlet pressure, kPa	Z = Compressibility factor
L = Pipeline length, m	T = Average flowing temperature, °K

2.4.9 Gas compressor power

The gas compressor power is calculated in four stages the mass flow rate the pressure head developed by the compressor, the compression power the total power including the losses.

2.4.10 Mass Flow rate

The flow rate of the gas is normally specified in meter cube per hour, but mass flow rate is specified in kilograms per hour. And the mass flow rate is calculated by [15].

$$w = QM/2.842R \tag{5}$$

Where, w = Mass flow, kg/hr

Q = Volume flow, m3/hr

M = Molecular weight

R = Gas constant, 8.314

2.5 Pressure Head

Pressure head is the energy required to compress the given amount of gas to the desired outlet pressure. The pressure head is calculated by:

$$H_{IS} = \frac{ZRT}{M(k-1)/k} \left[\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} - 1 \right]$$

Where, H_{IS} = Head, N.m/kg,	Z = Compressibility
R = Gas constant, 8.314	T = Inlet temperature, °C
P1 = Inlet pressure, kPa (abs)	P2 = Outlet pressure, kPa (abs)
M = Molecular weight	k = Isentropic exponent, Cp/Cv.

2.5.1 Gas Power

The gas power is the actual compressor power required to compress the gas to the required output pressure. It is calculated by:

(6)

$$G_P = \frac{w H_{IS}}{\eta_{IS} 3,600} \tag{7}$$

Where, $G_p = \text{Gas power, kW}$ w = Mass flow, kg/hr

 H_{IS} = Head, N.m/Kg h η_{IS} = Isentropic efficiency

2.5.2 Total Power

The total power is the gas power and including other power losses due to some mechanical reasons like seals and bearings. It is also called as break power.

The mechanical losses are estimated by:

$$Losses = 0.75 * G_P^{0.4}$$
(8)

And the brake power is calculated by

 $B_P = G_P + mechanical \ losses$

2.5.3 Discharge Temperature

This is the outlet temperature of the discharged gas from the compressor. The gas temperature rises due to the work done on the gas and. The discharge temperature should not exceed 180 $^{\circ}$ C [15].

The pipeline is divided into many sections of different lengths so that some of the different pipeline networks are obtained.

For each pipeline network pressure drop is calculated for those different short sections because the gas density varies along the pipe line length.

The inlet pressure of the compressor is calculated for each pipeline network.

After obtaining the inlet pressure for each compressor, the pressure head is calculated for each pipeline network.

By using the compressor head and other parameters the gas compression power is calculated.

The total mechanical losses due to the seals and bearings of the compressor are calculated.

Finally, the discharge temperature of the gas is calculated. The given procedure is repeated for the different pipeline diameters. After repeating the procedure a number of different pipeline networks are obtained. For each network, the pipeline installation costs and the compressor installation costs are estimated.

III. OPTIMISING THE PIPELINE SYSTEM

By following the technical design procedure, several pipeline systems are obtained for different standard diameters. For each pipeline network the pipeline installation cost and the compressor station installation cost are determined. The total investment cost for the each network was calculated. Then the costs of all the networks are compared to find the minimum total cost. For a fixed diameter the network with the minimum installation cost is the best available pipeline network.

3.1 Basic Data for Pipeline Design

In the pipeline design, the parameters involved are length, material, diameter, thickness, internal diameter and the inside roughness of the pipe.

The total pipeline length considered is 3000 km. While design calculations are done per every1000 km. The 1000km length is divided into number of short sections to calculate the pressure drop along the pipeline length. So the flow rate considered for the system is 2250 MM SCFD. The design pressure is considered according to the 600 flange rating i.e. 1,350 Psig. 90% of the design pressure is considered for the maximum operating pressure for the pipeline design[15].

For the pipeline diameters Standard API 5L (American Institute of Petroleum) specification pipeline diameters from 48 inches to 60 inches(1219mm,1524 mm)was considered for the pipeline networks design.

Pipe material used for the pipeline is X60 grade steel material under API-5L system specification, pipes with longitudinal or spiral factory welds are adopted in the construction of the pipeline.

The pipe roughness is the height of projections in the pipeline, and for the material steel, the roughness is 0.0018 inch, (0.04572mm).

The internal diameter and the wall thickness are calculated for individual diameters. In the design of the compressor, the parameters involved are molecular weight of the gas, specific gravity of the gas, compressibility, operating temperature, discharge temperature, isentropic efficiency, specific heat ratio, isentropic exponent, mass flow rate, gas constant, volume flow rate and inlet pressure of the compressor. In this design, of the pipeline network the discharge pressure is fixed and it is 1215 psig[15].

The gas transported is taken as pure methane so the molecular weight of the gas is 16.402. The Compressibility of the methane gas is estimated, (see figure 2 for the compressibility factor graph) and it is 0.96. Specific gravity of methane gas is calculated from its molecular weight and it is 0.566. The specific heat ratio of the methane gas is $C_p/C_v = 1.31$. The average fluid temperature is taken as (288°K). The isentropic efficiency is of the compressor is 78%. The isentropic exponent is calculated by using specific heat ratio of the methane gas.

IV. COST ESTIMATION CALCULATION

Costs for the pipeline installation and compressor installation are estimated by using an onshore version of IHS CERA cost estimating package QUE\$TORTM 9.9, with the cost data for West Siberia. The cost of 1000 km long pipelines with standard diameters between 48 and 60 inches diameter (1219 mm, 1524 mm) were modelled with the fixed inlet pressure of 1215 psia.

The cost of the compressors stations was estimated for inlet pressures from 500 psia to 1150 psia and a fixed discharge pressure of 1230 psia. The gas flow rate was set at 2250 MM SCFD. Due to the large gas flows, 5 running units were required with a sixth as a spare unit.

4.1 Pipeline cost estimation

Several factors were taken into consideration in estimating the cost of pipeline installation; some of these costs include material cost, construction costs and some other costs.

Diameter Size	48 inch	52 inch	56 inch	60 inch
	1219 mm	1321 mm	1422 mm	1524 mm
Materials	Cost, MM \$USD	Cost, MM \$USD	Cost, MM \$USD	Cost, MM \$USD
Line pipe	515.000	598.200	686.950	827.250
Fittings	41.200	47.856	54.956	66.180
Coating	59.500	67.500	73.000	73.000
Crossings	0.237	0.237	0.237	0.237
Civil Materials	86.500	96.500	96.500	96.500
Freight	63.219	72.926	82.048	95.685
Total Materials	765.656	883.219	993.691	1,158.852

4.2 Material costs

Material costs include line pipe, fittings, coating, crossings, civil materials and freight, The costs are presented in table 4.1.

4.3 Construction Costs

Construction costs include lay pipe, install crossings, testing, the right of way and mob/demob:, project management, insurance and contingency, pipeline installation, and the costs.

V. RESULTS AND DISCUSSIONS

This chapter provides an analysis of the results obtained for the installation cost of the long gas pipeline network. The compressor installation cost was estimated for the inlet pressures from 500 psia to 1150 psia by using the IHS CERA cost estimating package QUE\$TORTM 9.9.



FIGURE V.1 INLET PRESSURE VS COMPRESSOR COST POWER



FIGURE 5.3 COMPRESSOR DISCHARGE TEMPERATURE COST

FIGURE V.4 PIPE DIAMETER VS. INSTALLATION

The minimum pipeline installation cost and compressor station cost of each standard pipe diameter from 48 inch to 60 inches are shown in the table 5.2, and they are plotted on a graph (figure 5.3).

From the plot of pipeline station installation cost in figure 5.3 it is observed that, when the diameter of the pipeline increases, there is a proportional increase in the pipeline installation cost. This increase in cost is mainly due to increase in the material costs of the pipeline construction as shown in the table 4.10.

From the plot of compressor station installation cost in figure 5.3 it can be observed that increase in the pipeline diameter has an inversely proportional effect on the overall compressor station installation cost.

As the diameter of the pipeline increases the pressure drop decreases. In the 48 inch diameter pipeline the pressure drop is high, so more compressor stations and more power is needed to transport the gas to the destination. The combined effect of more stations and more power at each station makes the 48 inch option high capital cost.

The total cost of installing the compressor stations required for each configuration decreases with an increase of pipeline diameter due to the reduction in the number of stations required and a reduction in the duty of each station. However, the pipeline costs increase with increase in diameter.



FIGURE 5.5: TOTAL MINIMUM INSTALLATION COST

The minimum total installation costs of each diameter, showed in table 5.2 were plotted on a graph (figure 5.4). This graph shows that the minimum total installation cost is around 3,660 \$USD at 53.5 inch diameter and the optimum standard diameter is 52 inch (1321 mm) with the minimum cost of 3,668.558 million \$USD for 1000 km of pipeline installation.

The total installation costs for 48 inches (1219 mm) are high because, the pipeline cost is low but the compressor cost due to the large number of high duty stations, resulting in the high total cost.

For the 60 inches (1524 mm) pipe diameter the total cost of the compressor plants is low but the cost of pipeline is very high so the overall cost is high. The 60 inches pipeline (1524 mm) is particularly expensive as there are very few pipe mills that can manufacture this large diameter heavy wall pipe.



FIGURE .V.6 NUMBER OF COMPRESSOR STATIONS VS INSTALLATION COST

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VI. CONCLUSION

In remote areas operating compressor stations can be difficult so they can either be operated unmanned or possibly relocated slightly to place them in a more accessible location. It can be observed that, as the diameter of a pipe increases the number of compressor stations decreases. In the pipeline costs, material costs are proportional to pipeline diameter, while construction and design costs are approximately constant. Therefore minimising the pipeline diameter will reduce the pipeline total installed cost by a significant amount. As the 1524 mm pipe diameter has very small compressor station cost but it has very high pipeline cost. There are very few manufacturers who can supply 1524 mm diameter pipelines and, therefore this size pipeline is disproportionately expensive. This is due to smaller diameter have more pressure drop than larger diameter pipelines, so for the same compression power the gas can be transported for longer distance with larger diameter pipe than smaller diameter pipes. From the figure 5.1, it can be observed that the cost of compressor for the pressure 500 psia and 600 psia is high, so instead of these compressors, the compressor station spacing is altered, and 700 psia pressure compressors can be replaced to reduce the compressor cost

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