Cascaded Model-Free Fuzzy Control: an Application to the Coupled Tanks System

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***Abstract*— *This report proposes a design methodology for cascaded model-free fuzzy control systems. The ordinary Mamdani approach is modified in order to use expert knowledge for variable set-point control without any need of the system model. The methodology is successfully tested in a sub-actuated, naturally delayed setup, known as the coupled tanks system, where the water level is maintained at different set points, both in simulation and in real time.*

Keywords— Cascade Systems, Coupled Tanks, Fuzzy Control, Variable Set Point Control.

I. INTRODUCTION

From a control point of view, cascade systems are challenging, since (a) they are sub-actuated, (b) they naturally incorporate delays in the control action, and (c) they usually pursue several set points. Since the common approach within the control community is to consider the mathematical model of the system, obtained from first principles, in order to design an appropriate control law which stabilizes around the origin, very simple systems such as the coupled tanks may be hard to control due to the changing reference level. The usual solution is to adjust the controller, most commonly a PID, to different set points, as has been done with aircraft altitude control [1]. This, of course, requires a proper tuning which is indeed more complex for cascade systems.

Within model-based methodologies, cascade systems have been widely studied through input-output stability methods [2], input-to-state approaches [3, 4], gain scheduling [5], passivity stabilization [6, 7], sliding mode control [8, 9], backstepping[10], and synchronization [11]. Model-free methodologies, relying on expert knowledge and artificial intelligence, have also proposed a variety of solutions to the problem of analyzing and controlling cascade systems: via nonlinear scheduling [12], fuzzy exact representations [13], fuzzy sliding control [14], genetic algorithms [15], and evolutionary laws [16].

This work proposes a design methodology based on model-free fuzzy control, i.e., a Mamdani representation of expert knowledge about the plant for control purposes. In contrast with the ordinary approach which mixes all the controlled variables in a single fuzzy system to calculate the appropriate control law, it is shown that a cascade system is better (and simpler) dealt with if the fuzzy controller "decouples" the cascaded variables as to allow them to vary according to the desired set points.

The following sections are organized as follows: Section II presents the required preliminaries on model-free fuzzy control and cascade systems this work is concerned about; Section III introduces the novel methodology and explains the heuristic as well as numerical advantages of the proposal; Section IV introduces a typical cascade system: the coupled tanks, which is employed in Section V to illustrate how the proposed technique can be successfully implemented for level control of coupled tanks, both in simulation as well as in real-time, with clear advantages over the PID-based controller of the provider and the traditional fuzzy control; Section VI closes the paper by providing some conclusions.

II. PRELIMINARIES

Typically, expert knowledge for fuzzy control is collected in a series of r **IF-THEN** rules of the form.

$$\text{IF } x_1 \text{ IS } A_1^l \text{ AND} \cdots \text{AND } x_n \text{ IS } A_n^l \text{ THEN } u \text{ IS } B^l, \tag{1}$$

with $A_1^l, A_2^l, \ldots, A_n^l$, B^l, $l = 1, 2, \ldots, r$,representing possibly redundant fuzzy sets, each of them characterized by a membership function (MF) $\mu_{A_i^l}(x_i)$ or $\mu_{B^l}(u)$, holding the property $0 \le \mu_{A_i^l}(x_i) \le 1$, $0 \le \mu_{B^l}(u) \le 1$ [17].

This linguistic structure is then translated into a specific calculation of the controller output $u(x)$ which is the control input of the system; should the latter be multi-input, a series of blockswith the form(1) should be proposed. Calculating $u(x)$ from a rule base (1) requires choosing a lot of parameters whose effect cannot be easily cast: number of fuzzy sets, shape, height, and support of each MF, fuzzifier, defuzzifier, fuzzy inference engine, choice of t-norm, s-norm, and fuzzy implication, etc. [18]. Ordinarily, due to computational simplicity, MFs are chosen as triangular or trapezoidal, regularly distributed on the space of interest (i.e., the values of the states and control input the system is supposed to work at) while theMamdani fuzzy inference engine is fed with a singleton fuzzifier and its output post-processed via a center averagedefuzzifier. Mathematically, this is equivalent to the following expression:

$$u(x)=\frac{\sum_{l=1}^{M}\bar{y}^{l}\Pi_{i=1}^{n}\mu_{A_i^l}(x_i)}{\sum_{l=1}^{M}\Pi_{i=1}^{n}\mu_{A_i^l}(x_i)} \tag{2}$$

where $\bar{y}^l$ are the centers (along the values of u) of the fuzzy sets whose membership functions are given by $\mu_{B^l}(u)$.

While certain applications benefit from the fuzzy control structure (2), which mixes in a single control law all the states under consideration (see, for instance, the twin rotor MIMO system in [19]), some others may be hard to control without a proper decoupling (for example, the underactuated PENDUBOT system [20]): the latter is the case even when slow dynamics such as those in the coupled tanks system are involved.

Usually, decoupling is based on the fact that, disregarding certain nonlinear terms, the system dynamics can be split into several parts which are normally easier to tackle than the whole coupled nonlinear plant [21]. This sort of approach, though very popular, might still be not enough to deal with cascade systems.

2.1 Problem statement

Provide a model-free fuzzy control methodology for cascade systems, based on an adaptation of the decoupling technique (separate fuzzy control structures) while taking into account the cascade properties of the system.

III. CASCADE FUZZY CONTROL

The sort of cascade system under consideration is that of a series of n systems, where the output of the i -th system is the control input of the $(i+1)$-th block (see Figure 1). Clearly, the traditional fuzzy control approach described above fails to correctly perform under such a situation due to the following reasons:

1) A single control signal $u(x)$ as in (2) is going to be determined for the whole system, which means that the way an output of a block acts as the control input of the next one is ignored.
2) Most of the cascade control systems require variable set points, which obliges the structure in (2) to depend on the error signal $x_r - x(t)$, where x_r is a given set point. Since the output of the i -th block is the control input of the next one, this means that a variable set point should be arranged for the $(i+1)$-th block too: this contrasts with the usual fixed fuzzy structure in (2) where the fuzzy sets are predefined in shape and location in a compact set of the state space under consideration.
3) The naturally induced time delay of the cascade system will be thoroughly ignored if (2)is employed.
4) If a set point is fixed for a given state x_i within a certain block, it might constrain the set points of the following blocks, a condition that will be dismissed if a *vector* set point x_r is used for the whole state vector, as traditional fuzzy control demands.
5) The equilibria in cascade systems is usually different from the origin and may require a feed forward control action [22].

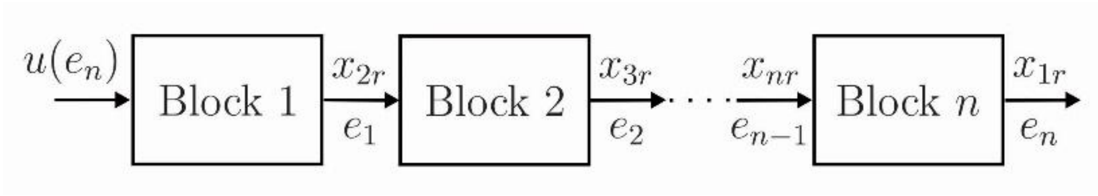

FIGURE 1. CASCADE SYSTEM

Thus, taking into account the aforementioned drawbacks of the ordinary fuzzy control methodology, our proposal is as follows:

1) Define the error we are interested at as the final block in the cascade system, where x_{rn} is the desired set point:

$$e_n(t) = x_{rn} - x_n(t). \tag{3}$$

2) Define fuzzy sets for $e_n(t)$ based on the allowed ranges of the state $x_n(t)$ and the expert knowledge about the plant.

3) Define fuzzy sets for the control input u at the beginning of the block series and proceed to construct a fuzzy controller of the form (2) which depends exclusively on $e_n(t)$, i.e, $u(x) \equiv u(x_n)$.

4) Define the error of the i-th block as $e_i(t) = x_{ir} - x_i(t)$, where x_{1r} is defined according to the $(i-1)$-th block, for $i = 1, \ldots, \text{n}-1$, except for the first block where x_{ir} is generated by the operations performed at the n-th block.

5) Define fuzzy sets for $e_i(t)$ based on the allowed ranges of the state $x_i(t)$, $i = 1, 2, \ldots, n-1$; they will help defining the reference of the next block.

Clearly, the steps above "propagate" the error at the final block to the first one and from there to the others as to define the set points and fuzzy sets accordingly. Formally, this decouples the system dynamics in blocks while preserving its connectivity through the next set point value.

IV. THE COUPLED TANKS SYSTEM

The coupled tanks system model is given by [22]:

$$\begin{aligned} \dot{x}_1(t) &= \frac{K_p u(t)}{A_{t1}} - \frac{A_{o1}\sqrt{2gx_1(t)}}{A_{t1}}, \\ \dot{x}_2(t) &= \frac{A_{o1}\sqrt{2gx_1(t)}}{A_{t2}} - \frac{A_{o2}\sqrt{2gx_2(t)}}{A_{t2}}, \end{aligned} \tag{4}$$

where x_1 is the water level of the first tank, x_2 is the water level of the second tank, $u(t)$ is the control input corresponding to the pump voltage, K_p is the gain coupling the pump voltage with the volumetric inflow rate to tank 1, A_{t1} is the cross area of tank 1, A_{o1} is the cross-section area of tank 1 outlet, A_{t2} is the cross area of tank 2, and A_{o2} is the cross-section area of tank 2 outlet, and $g = 9.81\, m/s^2$ corresponds to gravity. A schematic of the plant setup is shown in Figure 2.

Our objective is to find a fuzzy controller that performs variable set point control through the model-free fuzzy methodology proposed above; then, compare it with two other methodologies: (a) a traditional fuzzy controller implemented with fixed fuzzy sets via the Fuzzy Toolbox MATLAB [23], and (b) a PID controller proposed in [22].

The fuzzy controller is based on the system errors, though –as explained in the previous section– the first one is not a difference between the desired level and the actual level measure. Indeed, we have

$$e_1 = x_{1r} - x_1,\; e_2 = x_{2r} - x_2,$$

with x_{2r} as the desired level of the second tank and $x_{1r} = L_r$, where L_r comes from the second tank as explained later.

The corresponding ranges for the first and the second errors are chosen as

$$e_1 \in [-1,1],\ e_2 \in [-1,1],$$

whereas for the pump voltage and the reference generated by the second controller the following ranges are specified:

$$V_p \in [0,22],\ L_r \in [0,22].$$

Identical three fuzzy sets are defined for both errors; they have the following membership functions and linguistic meanings (seeFigure 3):

$$\text{Negative: } \mu_n(e_i) = \begin{cases} 1, & e_i < -1 \\ -e_i, & -1 \le e_i \le 0, \\ 0, & e_i > 0 \end{cases} \quad \text{Zero: } \mu_c(e_i) = \begin{cases} 0, & |e_i| > 1 \\ e_i + 1, & -1 \le e_i \le 0, \\ -e_i + 1, & 0 < e_i \le 1 \end{cases} \quad \text{Positive: } \mu_p(e_i) = \begin{cases} 0, & e_i < 0 \\ e_i, & 0 \le e_i \le 1. \\ 1, & e_i > 1 \end{cases}$$

According to the proposed methodology, the membership functions of the pump voltage are variable: they depend on the desired set point α which is given by [22]:

$$\alpha = \frac{A_{o1}\sqrt{2gL_r}}{K_p}, \tag{5}$$

Thus, the corresponding variable-shape membership functions of the fuzzy sets associated with the pump voltage are shown in Figure 4. Note that the main values of the negative and center fuzzy sets are variable.

As for the second controller, a variable reference λ is used (see Figure 5); it comes simply from the first reference:

$$\lambda = x_{1r} \tag{6}$$

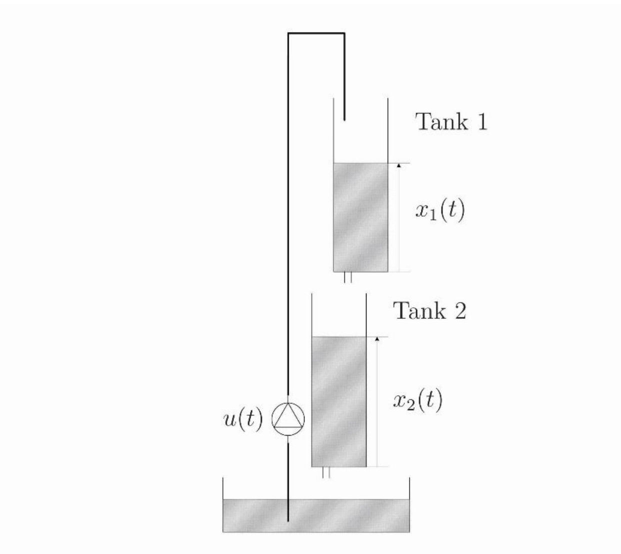

FIGURE 2. THE COUPLED TANKS SYSTEM

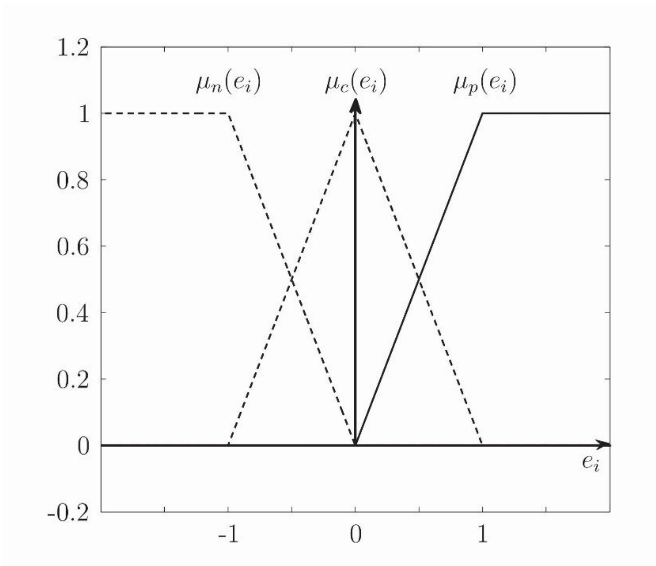

FIGURE 3. MEMBERSHIP FUNCTIONS FOR e_1 AND e_2

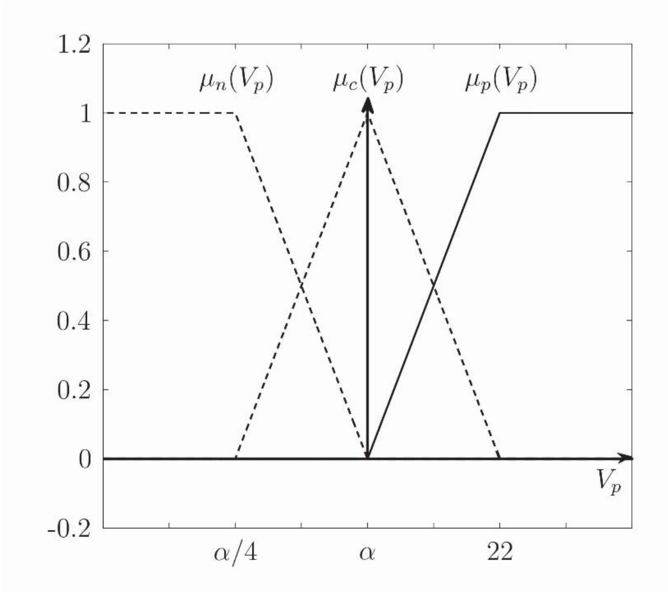

FIGURE 4. MEMBERSHIP FUNCTIONS OF THE PUMP VOLTAGE

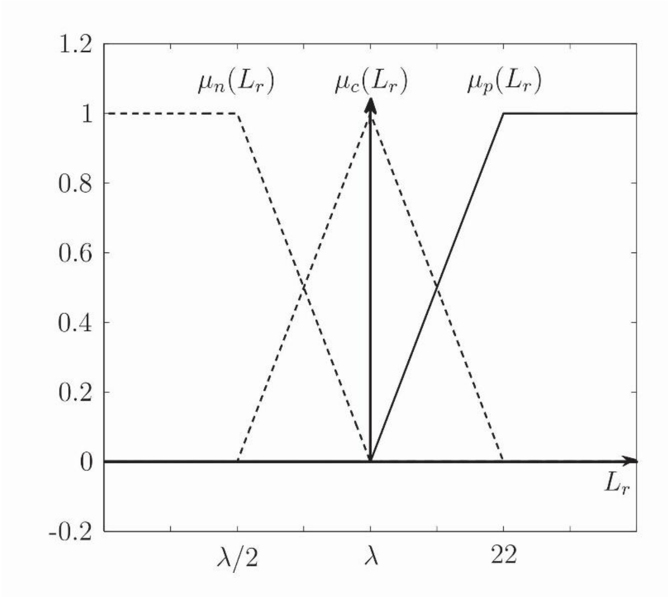

FIGURE 5. MEMBERSHIP FUNCTIONS OF THE GENERATED REFERENCE L_r

In contrast with ordinary fuzzy methodologies which use a single fuzzy rule base which is fed with n-uples of the states in order to determine the control input $u(x)$, our proposal splits the fuzzy controllers in n fuzzy rule bases. Since $n=2$ in the coupled tanks system, we have a fuzzy rule base for e_1 and a second one for e_2: they are given in Table 1 and Table 2, respectively, where the antecedents are fuzzy sets e_n (negative), e_c (around zero), and e_p (positive). The rule consequents for e_1 are L_a (high level in tank 1), L (around the reference L_r), and L_b (low level in tank 1 around half of the reference L_r). The rule consequents for e_2 are V_a corresponding to high values of the pump voltage, V about the average value, and V_b about a fourth of the average value V. No time derivatives (level rate of change) are taken into account due to the lack of proper sensors: results show this decision does not affect the control effectiveness.

TABLE 1
FUZZY RULE BASE FOR e_1

IF $V_p = e_2$ IS	THEN V_1 IS
e_n	V_a
e_c	V
e_p	V_b

TABLE 2
FUZZY RULE BASE FOR e_2

IF $V_p = e_2$ IS	THEN V_1 IS
e_n	V_a
e_c	V
e_p	V_b

V. SIMULATION AND REAL-TIME RESULTS

As stated before, one of the approaches employed to contrast our proposal is that of traditional fuzzy control based on fixed membership functions, more specifically implemented via MATLAB Tool box fuzzy [23]. As expected, such a rigid structure does not allow variable set point references to be properly followed as shown for simulation in Figure 6, where a steady state error of approximately 0.13 cm appears independently of the reference $x_{2r} = x_{ref}$. Due to this unacceptable steady-state error, the fuzzy controller thus designed was not implemented in real time.

The other trajectory shown in Figure 6 is the one generated by a PID controller, which comes along with the coupled tanks system by Quanser [22]: it has obviously been tuned as to reach the variable set point references in minimum time while keeping an overshoot below the 15 per cent.

We now turn our attention to the cascade model-free fuzzy control we are proposing. Its variable membership shapes as shown in Figure 4 and Figure 5, allow variable set point tracking as shown in the simulation results of Figure 7. No further adaptation is required as these fuzzy sets (and consequently, their corresponding controllers) are automatically adjusted. For the sake of comparison, the simulation corresponding to the PID controller of the Quanser provider is also included: our approach clearly achieves the same tracking quality as the PID without the overshooting of the latter.

The real time implementation of the proposed controller produces the results shown in Figure 8: though small, a steady-state error is still observed, an error that is not present in simulation (included in the same figure for comparison). The real time shortcoming is due to the time delay characteristics of the coupled tanks system. This time delay can be noticed in the control action plotted in Figure 9: recall that the water pumped up has to reach the first tank as its outflow has to control the second tank level, which is the one we are interested at.

Time delays can be formally analyzed via Lyapunov functionals [24]. Nevertheless, in the spirit of the proposed approach, the steady-state error can be driven to zero by simply using the same variable membership functions in wider ranges: when $[-3,3]$ is used instead of $[-1,1]$, we obtain the tracking in Figure 10: it obviously has improved over the first approach.

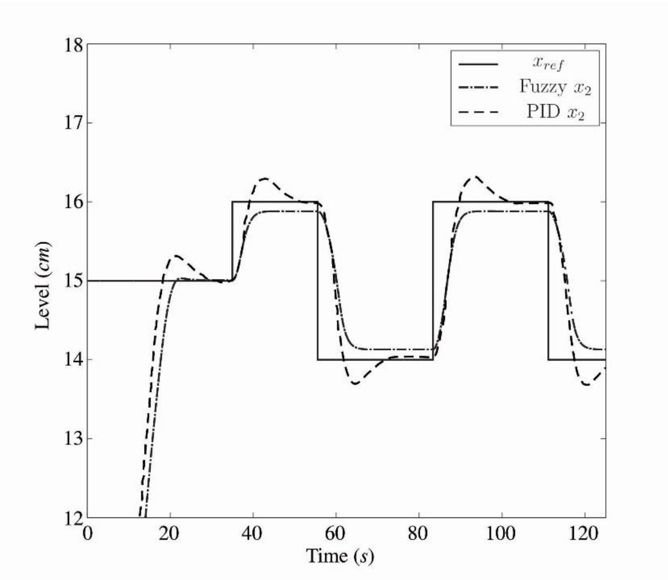

FIGURE 6. SIMULATION RESULTS: "TOOLBOXFUZZY" VS PID

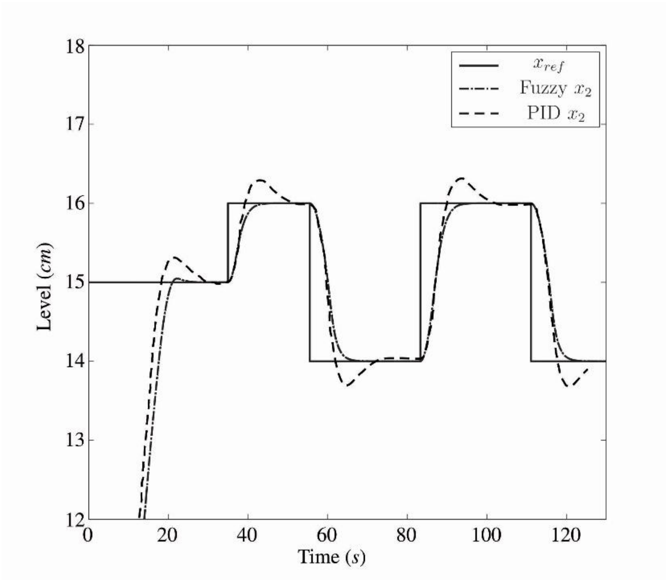

FIGURE 7. SIMULATION RESULTS: CASCADED FUZZY CONTROLLER VS PID

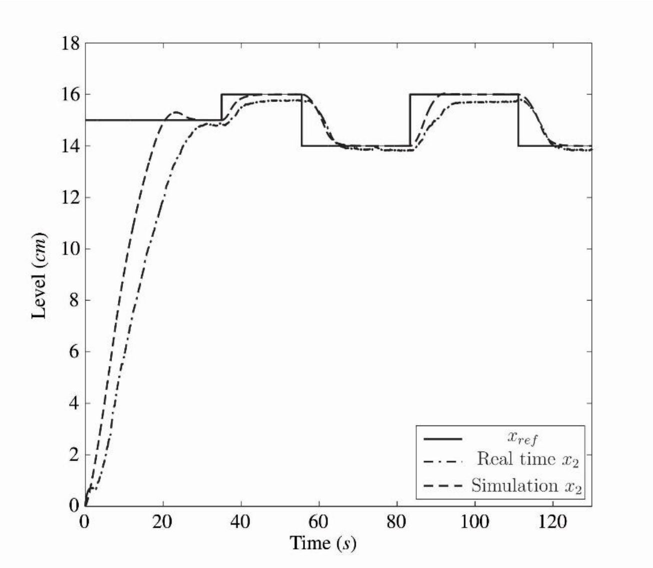

FIGURE 8. REAL TIME RESULTS: FIRST ATTEMPT

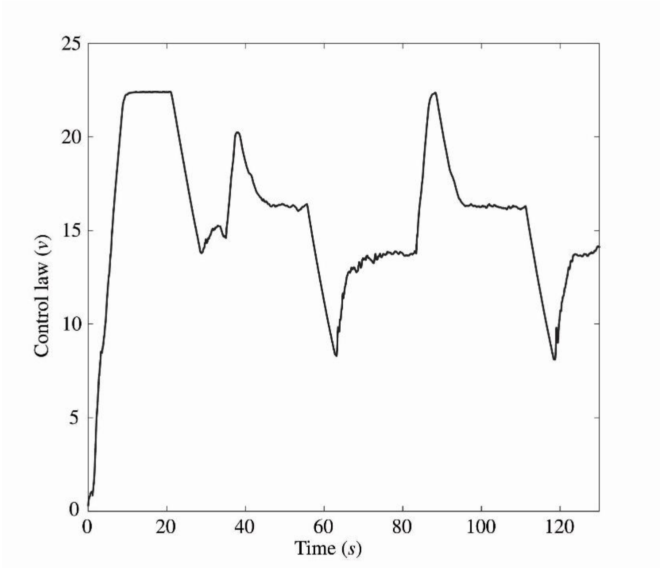

FIGURE 9. CONTROL SIGNAL FOR THE FIRST CASE

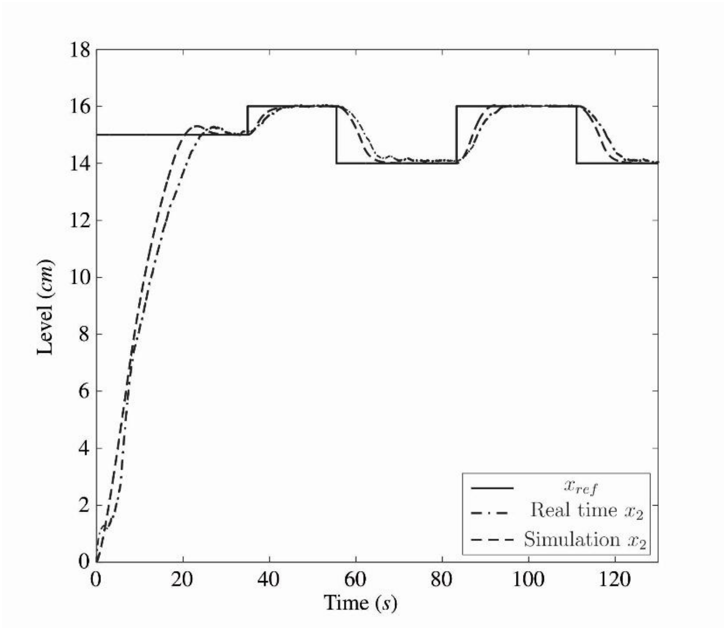

FIGURE 10. REAL TIME RESULTS: SECOND ATTEMPT

VI. CONCLUSION

A novel cascaded model-free fuzzy control has been presented: it allows dealing with cascaded systems by introducing adaptable shape membership functions in order to deal with variable set point goals. By subsequently transmitting the reference from one block to next one, the cascaded fuzzy methodology is able to pick the right shape and "center" for the fuzzy sets as to track the desired set point at the last block. The methodology has been successfully implemented in the coupled tanks system, where it has performed better than the provider PID control and the traditional fuzzy control structure. Time delays have also been appropriately dealt with by simply expanding the error intervals.

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REFERENCES

[1] P. Korba, "A Gain-Scheduling Approach to Model-Based Fuzzy Control," Ph.D. dissertation, University of Duisburg, Duisburg, Germany, 2000.

[2] H. Khalil, Nonlinear Systems. New Jersey, USA: Pearson Education, 2000.

[3] H. Ito, "A Lyapunov Approach to Cascade Interconnection of Integral Input-to-State Stable Systems," IEEE Transactions on Automatic Control, vol. 55, no. 3, pp. 702–708, 2010.

[4] W. Lin and Q. Gong, "A Remark on Partial-State Feedback Stabilization of Cascade Systems Using Small Gain Theorem," IEEE Transactions on Automatic Control, vol. 48, no. 3, pp. 497–499, 2003.

[5] W. Lin and R. Pongvuthithum, "Global Stabilization of Cascade Systems by C◦ Partial-State Feedback," IEEE Transactions on Automatic Control, vol. 47, no. 8, pp. 1356–1362, 2002.

[6] R. Ortega, J. Loría, P. Nicklasson, and H. Sira-Ramirez, Passivity-based Control of Euler-Lagrange Systems: Mechanical, Electrical and Electromechanical Applications. London: Springer, 1998.

[7] Z. Chen and J. Huang, "Dissipativity, Stabilization, and Regulation of Cascade-Connected Systems," IEEE Transactions on Automatic Control, vol. 49, no. 5, pp. 635–650, 2004.

[8] M. Wang, J. Zhao, and G. Dimirovski, "Variable Structure Control Method to the Output Tracking Control of Cascade Non-Linear Switched Systems - Brief Paper," IET Control Theory Applications, vol. 3, no. 12, pp. 1634–1640, 2009.

[9] S. F. Yang, "Variable Control Scheme in the Cascade Processes," Expert Systems with Applications, vol. 37, no. 1, pp. 787–798, 2010.

[10] H. Ye,W. Gui, and Z. Jiang, "Backstepping Design for Cascade Systems with Relaxed Assumption on Lyapunov Functions," IET Control Theory Applications, vol. 5, no. 5, pp. 700–712, 2011.

[11] Y. Zhou, Z. Hua, C. Pun, and C. Chen, "Cascade Chaotic System with Applications," IEEE Transactions on Cybernetics, vol. 45, no. 9, pp. 2001–2012, 2015.

[12] J. Catalao, S. Mariano, V. Mendes, and L. Ferreira, "Scheduling of Head-Sensitive Cascaded Hydro Systems: A Nonlinear Approach," IEEE Transactions on Power Systems, vol. 24, no. 1, pp. 337–346, 2009.

[13] Z. Lendek, R. Babuska, and B. De Schutter, "Stability of Cascaded Fuzzy Systems and Observers," IEEE Transactions on Fuzzy Systems, vol. 17, no. 3, pp. 641–653, 2009.

[14] R. Wai, M. Kuo, and J. Lee, "Design of Cascade Adaptive Fuzzy Sliding-Mode Control for Nonlinear Two-Axis Inverted-Pendulum Servomechanism," IEEE Transactions on Fuzzy Systems, vol. 16, no. 5, pp. 1232–1244, 2008.

[15] S. Oh, S. Jung, and W. Pedrycz, "Design of Optimized Fuzzy Cascade Controllers by Means of Hierarchical Fair Competition-Based Genetic Algorithms," Expert Systems with Applications, vol. 36, no. 9, pp. 11 641 – 11 651, 2009.

[16] L. Coelho and C. M.A., "Adaptive Cascade Control of a Hydraulic Actuator with an Adaptive Dead-Zone Compensation and Optimization Based on Evolutionary Algorithms," Expert Systems with Applications, vol. 38, no. 10, pp. 12 262 – 12 269, 2011.

[17] L. Wang, A Course in Fuzzy Systems and Control. Upper Saddle River, NJ, USA: Prentice-Hall, Inc., 1997.

[18] K. M. Passino, S. Yurkovich, and M. Reinfrank, Fuzzy Control. Menlo Park, California: Addison Wesley Longman, Inc., 1998.

[19] T. Gonzalez, P. Rivera, and M. Bernal, "Nonlinear Control for Plants with Partial Information Via Takagi-Sugeno Models: An Application on the Twin Rotor MIMO System," in 2012 9th International Conference on Electrical Engineering, Computing Science and Automatic Control (CCE), 2012, pp. 7–12.

[20] O. Begovich, E. N. Sanchez, and M. Maldonado, "Takagi-Sugeno Fuzzy Scheme for Real-Time Trajectory Tracking of an Underactuated Robot," IEEE Transactions on Control Systems Technology, vol. 10, no. 1, pp. 14–20, 2002.

[21] Twin Rotor MIMO System Advanced Technical Manual, Feedback Instruments LTD., Crowborough, UK, 33-007-4M5, 1997.

[22] J. Apkarian, H. Lacheray, and A. Abdossalami, Coupled Tanks Experiment for MATLAB®/Simulink® Users, Quanser, Markham, Ontario, Canada, L3R 5H6, 2012.

[23] I. MathWorks and W. C. Wang, Fuzzy Logic Toolbox: for Use with MATLAB: User's Guide. Mathworks, Incorporated, 1998.

[24] V. L. Kharitonov and K. Gu, Stability of Time-Delay Systems. New York, USA: Birkhauser, 2003.