Strong chromatic index of graphs: a short survey

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Abstract— *A strong edge coloring of a graph G is an edge coloring such that every two adjacent edges or two edges adjacent to a same edge receive two distinct colors; in other words, every path of length three has three distinct colors in G. The strong chromatic index of G, denoted by $\chi'_S(G)$, is the smallest integer k such that G admits a strong edge coloring with k colors. This survey is an brief introduction to some good results regarding the strong chromatic index of planar graphs, bipartite graphs and so on.*

Keywords— strong edge coloring; strong chromatic index.

I. INTRODUCTION

Strong edge colorability was introduced by Fouquet and Jolivet [21, 22] and used to solve the frequency assignment problem in some radio networks. A strong k *-edge-coloring* of a graph G is a mapping from $E(G)$ to $\{1,2,\cdots,k\}$ such that every two adjacent edges or two edges adjacent to a same edge receive two distinct colors. In other words, the graph induced by each color class is an induced matching. This can also be seen as a vertex 2-distance coloring of the line graph of G. The *strong chromatic index* of G, denoted by $\chi'_S(G)$, is the smallest integer k such that G admits a strong k-*edge-coloring*.

There is a classical conjecture with respect to $\chi'_S(G)$, which is posed by Erdős and Nešetřil:

Conjecture 1.1 (Erdős [18], Erdős [19], Faudree et al. [20]) *If G is a graph with maximum degree Δ then*

$$\chi'_s(G)\le\begin{cases}\frac{5}{4}\Delta^2, & if\quad \Delta\quad is\quad even,\\ \frac{1}{4}\left(5\Delta^2-2\Delta+1\right), & if\quad \Delta\quad is\quad odd.\end{cases}$$

The best known upper bound was given by Molloy and Reed in 1997:

Theorem 1.1 (Molloy and Reed [33]) *For Δ large enough, every graph with maximum degree Δ has* $\chi'_s(G)\le 1.998\Delta^2$.

In 2015, Bruhn and Joos [10] give a stronger bound for the strong chromatic index, for Δ large enough, every graph with maximum degree Δ has $\chi'_s(G)\le 1.93\Delta^2$. For small maximum degrees, $\Delta=3,4$ were studied:

Theorem 1.2 (Andersen [2] Horák et al. [27]) *Every graph with maximum degree $\Delta\le 3$ admits a strong 10-edge-coloring.*

Theorem 1.3 (Cranston [13]) *Every graph with maximum degree $\Delta\le 4$ admits a strong 22-edge-coloring.*

According to conjecture 1.1, the conjectured bound is 20. For graphs with $\Delta\ge 4$, there is a new conclusion [40].

An upper bound for the strong chromatic index of subcubic graphs in terms of the maximum average degree $mad(G)=\max\{\frac{2|E(H)|}{|V(H)|}, H\subseteq G\}$, was given in [26]. More details will be given in the next theorems.

Theorem 1.4 (Wang and Zhao [40]) *If* G *is a graph with girth at least* 2Δ *and* $mad(G) < 2 + \frac{1}{3\Delta - 2}$, *where* $\Delta \geq 4$, *then* $\chi'_S(G) \leq 2\Delta - 1$.

In 1990, Faudree et al. [20] proposed various conjectures on subcubic graphs (graphs with maximum degree at most three).

Conjecture 1.2 *Let* G *be a subcubic graph.*

1. *The strong chromatic index is at most 10.*
2. *If* G *is bipartite, then* $\chi'_S(G) \leq 9$.
3. *If* G *is planar, then* $\chi'_S(G) \leq 9$.
4. *If* G *is bipartite and the degree sum of every edge is at most 5, then* $\chi'_S(G) \leq 6$.
5. *If* G *is bipartite with girth at least 6, then* $\chi'_S(G) \leq 7$.
6. *If* G *is bipartite and its girth is large, then* $\chi'_S(G) \leq 5$.

Then in 2015, P. DeOrsey et al. improve a more general bound of $\chi'_S(G)$ in [15]. P. DeOrsey et al. give some definitions, S_3 be a triangle with pendant edges at each vertex, S_4 be a 4-cycle with pendant edges at two adjacent vertices, and for $k \geq 5$, let S_k be a k-cycle with pendant edges at each vertex.

Theorem 1.5 (P. DeOrsey et al. [15]) *Let* G *be a subcubic graph.*

1. *If* G *does not contain* S_3, S_4 *or* S_7, *and* $mad(G) < 2 + \frac{1}{7}$, *then* $\chi'_S(G) \leq 5$
2. *If* G *is planar and has girth at least 30, then* $\chi'_S(G) \leq 5$.

When $\Delta(G) = 4$, the conclusion of Theorem 1.4 may be improved.

Theorem 1.6 (P. DeOrsey et al. [15]) *Let* G *be a subcubic graph.*

1. *If* G *has girth at least 7 and* $mad(G) < 2 + \frac{2}{13}$, *then* $\chi'_S(G) \leq 7$.
2. *If* G *is planar and has girth at least 28, then* $\chi'_S(G) \leq 7$.

Regarding subcubic graphs with restriction on girth, there is a conclusion.

Theorem 1.7 (Wang and Zhao [40]) *If* G *is a subcubic graph with girth at least 9 and* $mad(G) < \frac{48}{23}$, *then* $\chi'_S(G) \leq 5$.

Theorem 1.8 (Hocquard and Valicov [26]) *Let* G *be a subcubic graph (a graph with Δ≤3).*

1. *If* $mad(G) < \frac{15}{7}$, *then* $\chi'_S(G) \leq 6$.
2. *If* $mad(G) < \frac{27}{11}$, *then* $\chi'_S(G) \leq 7$.
3. *If* $mad(G) < \frac{13}{5}$, *then* $\chi'_S(G) \leq 8$.
4. *If* $mad(G) < \frac{36}{13}$, *then* $\chi'_S(G) \leq 9$.

Then in 2013, Hocquard et al. gave a stronger result of Theorem 1.8.

Theorem 1.9 (Hocquard et al. [25]) *Let* G *be a subcubic graph,* *1. If* $mad(G) < \frac{7}{3}$ *, then* $\chi'_S(G) \le 6$.

2. If $mad(G) < \frac{5}{2}$ *, then* $\chi'_S(G) \le 7$.

3. If $mad(G) < \frac{8}{3}$ *, then* $\chi'_S(G) \le 8$.

4. If $mad(G) < \frac{20}{7}$ *, then* $\chi'_S(G) \le 9$.

II. MAIN RESULTS WITH RESPECT TO PLANAR GRAPHS

In 1977, K. Appel and W. Haken proved every planar map is four colorable. In 1990, Faudree et al. gave an another conclusion for planar graphs.

Theorem 2.1 (Faudree et al. [20]**)** *If* G *is a planar graph, then* $\chi'_S(G) \le 4\Delta + 4$ *. Moreover for every integer* ≥ 2 *there exists a planar graph* G *with* $\Delta(G) = \Delta$ *and* $\chi'_S(G) = 4\Delta - 4$.

The above conclusion is on general planar graphs, and regarding cubic planar graphs, there is another conjecture mentioned in [20, 29].

Conjecture 2.1 (Faudree et al. [20] Jensen et al. [29]) If G is subcubic planar graph, then $\chi'_S(G) \le 9$.

Respecting planar subcubic graphs with restriction of girth, Hocquard and Valicov gave some good results in 2011.

Theorem 2.2 (Hocquard and Valicov [26]) Let G be a planar subcubic graph with girth g ,

1. If $g \ge 30$, then $\chi'_S(G) \le 6$.

2. If $g \ge 11$, then $\chi'_S(G) \le 7$.

3. If $g \ge 9$, then $\chi'_S(G) \le 8$.

4. If $g \ge 8$, then $\chi'_S(G) \le 9$.

Then in 2015, Philip DeOrsey et al. [15] make some improvements based on the above theorem, if G is a subcubic planar graph and $girth(G) \ge 30$, then $\chi'_S(G) \le 5$, in which the upper bound of $\chi'_S(G)$ is reduced from 6 to 5.

The strong chromatic index was also studied for graphs with large girth.

Theorem 2.3 (Hudák et al. [28]) Let G be a planar graph with girth at least 6 and maximum degree $\Delta \ge 4$. Then, $\chi'_S(G) \le 3\Delta + 5$.

Very recently, Bensmail et al. gave a better bound of Theorem 2.3 in [4] , if G is a planar graph with with girth $g \ge 6$ then $\chi'_S(G) \le 3\Delta + 1$, which made some improvements for the upper bound of $\chi'_S(G)$.

Then Bensmail et al. gave us some conclusions, considering the upper bound of $\chi'_S(G)$ in some general conditions.

Theorem 2.4 (Bensmail et al. [4]). Let G be a planar graph with maximum degree Δ and girth g . If G satisfies one of the following conditions below, then $\chi'_S(G) \le 4\Delta$

- $\Delta \ge 7$,

- $\Delta \geq 5, \; and \;\; g \geq 4,$
- $g \geq 5.$

If G is planar with large girth and large maximum degree Δ, then we have $\chi_S'(G) \leq 3\Delta$. In 2013, Hocquard et al. [25] gave an conclusion for planar graphs, if G is a planar graph with $\Delta \leq 3$ containing neither induced 4-cycles, nor induced 5-cycles, then $\chi_S'(G) \leq 9$.

Theorem 2.5 (Hudák et al. [28]). Let G be a subcubic planar graph with girth at least 6, then $\chi_S'(G) \leq 9$.

If the girth is large enough, then the upper bound can be strengthened to $2\Delta - 1$. Borodin and Ivanova [8] has proved: every planar graph with maximum degree Δ is strong $(2\Delta - 1)$-colorable if its girth is at least $40\left\lfloor \frac{\Delta}{2} \right\rfloor + 1$.

Theorem 2.6 (Chang et al. [11]). Let F_Δ be the family of planar graphs with maximum degree at most Δ. Every graph in F_Δ with girth at least $10\Delta + 46$ admits a strong $(2\Delta - 1)$ -edge-coloring when $\Delta \geq 4$.

Guo et al. [24] improve the girth to $10\Delta + 46$. Very recently, Wang and Zhao make some improvements based on this conclusion in [40], reducing the girth to $10\Delta - 4$, and every graph in F_Δ still admits a strong $(2\Delta - 1)$-edge-coloring.

III. STRONG CHROMATIC INDEX OF HALIN GRAPHS

Definition 3.1 *Let T be a tree without vertices of degree two. Consider a plane embedding of T, and connect the leaves of T by a cycle that crosses no edges of T. A graph that is constructed in this way is called a Halin graph.*

Suppose G is a Halin graph of order $2h+2$ with a caterpillar T as its characteristic tree, $h \geq 1$. We name the vertices along the spine P_h by $1, 2, \cdots, h$. The vertices adjacent with 1 are named by 0 and $1'$. The vertices adjacent with h are named by $h+1$ and h'. Other leaf adjacent with i is named by i', $2 \leq i \leq h-1$. Note that $0, 1', \cdots, h', h+1$ are vertices lying on the adjoint cycle C_{h+2}. We shall use this vertex labeling through this paper. Let G_h be the set of all cubic Halin graphs whose characteristic trees are caterpillars of order $2h+2$.

In 2006, Shiu et al. proved the following theorem, and came up with the conjecture 3.1.

Theorem 3.1 (Shiu et al. [36]) *For $h \geq 4$ and $G \in G_h$, we have $6 \leq \chi_s'(G) \leq 8$.*

There is a general bounds for the strong chromatic index of cubic Halin graphs. Shiu et al. in [36] mentioned that any cubic Halin graphs G contains at least two triangles. It is easy to see that $G \in G_h$ for some $h \geq 2$ if and only if G contains only two triangles. In 2006, Shiu et al. [36] proved that if G is a cubic Halin graph, then $6 \leq \chi_s'(G) \leq 9$ and the bounds are sharp.

A complete cubic Halin graph is a cubic Halin graph whose characteristic tree is a complete cubic tree, in which all leaves are at the same distance from the root vertex. Next, we introduce some conclusion respecting the strong chromatic index of the complete cubic Halin graph. A cubic tree is a tree in which all interior vertices are of degree 3. For $n \geq 0$, a complete cubic tree T_n is a cubic tree of height $n+1$ with a root vertex v_0 such that all its leaves are at the same distance $n+1$ from v_0. The level of a vertex is defined to be the distance from the root vertex to that vertex. For any edge $e = uv$ of T_n, assume v is a child of u. The level of e is defined to be the level of v. Therefore, T_n has $n+1$ levels.

A complete cubic Halin graph H_n is a cubic Halin graph whose characteristic tree is T_n. Clearly, $H_0 \cong K_4$. Also when $n \geq 1$, H_n is not a necklace, since H_n is a C_4-free graph (a C_4-free is a graph that does not contain a 4-cycle). In [37] Shiu and Tam gave the results for complete cubic Halin graph, $\chi'_s(H_1) = 7$, $\chi'_s(H_n) = 6$ for $n = 0$ or $n \geq 2$.

For $h \geq 1$, a cubic Halin graph N_{e_h}, called a necklace. Its characteristic tree T_h consists of the path $v_0, v_1, \cdots, v_{h+1}$, and leaves $v'_1, v'_2, \cdots, v'_h$ such that the unique neighbor of v'_i in T_h is v_i for $1 \leq i \leq h$ and vertices $v_0, v'_1, \cdots, v'_h, v_{h+1}$ are in order to form the adjoint cycle C_{h+2}. Following is a conjecture based on the above conclusions.

Conjecture 3.1 (Shiu et al. [37]) *If G is a cubic Halin graph that is different from any necklace, then $\chi'_s(G) \leq 7$.* Lih et al. gave the theorem respecting cubic Halin graph:

Theorem 3.2 (Lih et al. [31]) *If a cubic Halin graph $G = T \bigcup C$ is different from N_{e_2} and N_{e_4}, then $\chi'_s(G) \leq 7$.*

Theorem 3.3 (Lai et al. [30]) *If a Halin graph $G = T \bigcup C$ is different from N_{e_2} and any wheel W_n, $n \not\equiv 0 \pmod 4$, then $\chi'_s(G) \leq \chi'_s(T) + 3$.*

In fact, Theorem 7.1 gives a strong edge-coloring of necklace. Shiu et al. provided another conclusion for necklace and determine the strong chromatic index of it.

Theorem 3.4 (Shiu et al. [36]) *Suppose $h \geq 1$,*

$$\chi'_s\left(N_{e_h}\right) \leq \begin{cases} 6, & if \quad h \quad is \quad odd, \\ 7, & if \quad h \geq 6 \quad and \quad is \quad even, \\ 8, & if \quad h = 4, \\ 9, & if \quad h = 2. \end{cases}$$

IV. STRONG CHROMATIC INDEX OF DEGENERATE GRAPHS

Definition 4.1 *If every subgraph H of G has a vertex v, such that the degree of v is at most k, then G is a k-degenerate graph.*

With respect to $\chi'_S(G)$ of k-degenerate graph, Dębski et al. [14] gave an upper bound $(4k-1)\Delta(G) - k(2k+1)$ in 2013. And then, Yu [42] obtained an improved upper bound $(4k-2)\Delta(G) - 2k^2 + k + 1$. In 2014, Wang give a better result for the upper bound, which is the best result now.

Theorem 4.1 (Wang [39]) *If G is a k-degenerate graph with maximum degree Δ, and $k \leq \Delta$, then $\chi'_S(G) \leq (4k-2)\Delta(G) - 2k^2 + 1$.*

Conjecture 4.1 (Chang and Narayanan [12]) *There exists an absolute constant c such that for any k-degenerate graphs G with maximum degree Δ, $\chi'_S(G) \leq ck^2\Delta$. Furthermore, the k^2 may be replaced by k.*

Wang also investigates a class of graphs whose all 3^+-vertices induce a forest, and gave the theorem as follows.

Theorem 4.2 (Wang [39]) *If G is a graph such that all its 3^+-vertices induce a forest, then $\chi'_S(G) \leq 4\Delta(G) - 3$.*

A graph is 2-degenerate if every subgraph has minimum degree at most two. Outplanar graphs, non-regular subcubic graphs, and planar graphs with girth at least six are all 2-degenerate graphs. Chang and Narayanan (2013, [12]) proved that a 2-degenerate graph with maximum degree Δ has strong chromatic index at most $10\Delta(G) - 10$. They actually proved a stronger statement.

Theorem 4.3 (Chang and Narayanan [12]) *Let* G *be a 2-degenerate graph with maximum degree* Δ, *let* $B=\{1,2,\cdots,5\Delta-5\}$ *and* $B'=\{1',2',\cdots,(5\Delta-5)'\}$. *Then* G *has a strong edge coloring with the colors from* $B\bigcup B'$ *such that*

(a) Every pendant edge (if any) is colored with a color in B.

(b) If a pendant edge is colored with a color $c\in B$, *then no edge within distance 1 to the edge is colored with* c'.

Luo and Yu [32] improved the upper bound to $8\Delta-4$, and they gave a stronger statementas follows.

Theorem 4.4 (Luo and Yu [32]) *Let* G *be a 2-degenerate graph with maximum degree* Δ, *let* $B=\{1,2,\cdots,4\Delta-2\}$ *and* $B'=\{1',2',\cdots,(4\Delta-2)'\}$. *Then* G *has a strong edge coloring with the colors from* $B\bigcup B'$ *such that*

(a) Every pendant edge (if any) is colored with a color in B.

(b) If a pendant edge is colored with a color $c\in B$, *then no edge within distance 1 to the edge is colored with* c'.

(c) No pair of colors $\{c,c'\}$ *appears at the same vertex.*

Then in 2014, Wang gave a result which is stronger than the above results. The result may be raised from $8\Delta-4$ to $6\Delta-7$.

V. STRONG CHROMATIC INDEX OF CHORDLESS GRAPHS

Definition 5.1 *A graph is said to be chordless if there is no cycle in the graph that has a chord. Let* G *be any graph such that all its cycle lengths are multiples of some fixed integer* $t\geq 3$. *Then it can be easily seen that the graph* G *has to be chordless.*

In 1990, Faudree et al. [20] considered a particular subclass of chordless graphs, namely the class of graphs in which all the cycle lengths are multiples of four, and asked whether the strong chromatic index of these graphs can be bounded by a linear function of the maximum degree. While Basavaraju and Francis give a better bound of $\chi'_S(G)$, and the result is as follows.

Theorem 5.1 (Basavaraju et al. [3]) *If* G *is a chordless graph with maximum degree* Δ, *then* $\chi'_S(G)\leq 3\Delta$.

In [12], Chang et al. also studied the strong chromatic index of a chordless graph.

Theorem 5.2 (Chang et al. [12]) *If* G *is a chordless graph with maximum degree* Δ, *then* $\chi'_S(G)\leq 8\Delta-8$.

The proof is very similar to the one for 2-degenerate graphs, they also proved if chordless G has maximum degree Δ, then $\chi'_S(G)\leq 6\Delta-2$. While, Dębski et al. gave a better result for strong chromatic index of chordless graphs in [14], every chordless graph G of maximum degree Δ satisfies $\chi'_S(G)\leq 4\Delta-3$.

VI. STRONG CHROMATIC INDEX OF BIPARTITE GRAPHS

In this section, we focus on strong edge-coloring of *bipartite graphs*, which are graphs whose vertex set admits a bipartition into two independent sets. In 1990, Faudree, Gyárfás, Schelp and Tuza presented a conjecture:

Conjecture 6.1 (Faudree et al. [20]) *For every bipartite graph* G, *we have* $\chi'_S(G)\leq\Delta^2$.

The $\chi'_S(G)$ of above conjecture is bounded by Δ^2, in which Δ is the maximum degree of bipartite graph G. In 1993, Brualdi and Quinn Massey proposed a conjecture , which made a progress of Conjecture 6.1.

Conjecture 6.2 (Brualdi and Quinn Massey [9]) *For every bipartite graph* G *with bipartition* A *and* B, *we have* $\chi'_S(G)\leq\Delta(A)\Delta(B)$.

We can know that if G is a bipartite cubic graph without cycle of length 4, then $\chi'_S(G) \le 7$. Compared with Conjecture 6.1, the upper bound of chromatic index decrease from 9 to 7. If G is a bipartite cubic graph with sufficiently large girth, then $\chi'_S(G) \le 5$, that is to say, compared with Conjecture 6.1, the upper bound of chromatic index decrease from 9 to 5.

In the spirit of Conjecture 6.2, we define a (d_A, d_B)-bipartite graph to be a bipartite graph with parts A and B, such that $\Delta(A) \le d_A$ and $\Delta(B) \le d_B$. Conjecture 6.2 is still widely open, it is first known to hold whenever G is subcubic bipartite $[38]$: for every $(3,3)$-bipartite graph G, we have $\chi'_S(G) \le 9$. In 1993, Brualdi et al. give a stronger bound of Conjecture 6.2 on the condition of without cycle of length 4, and the content of this conclusion is as follows.

Theorem 6.1 (Brualdi et al. [9]) *Let G be a bipartite graph with bipartition X Y and without any cycle of length four. If the maximum degree of a vertex of X is 2 and the maximum degree of a vertex of Y is Δ, then $\chi'_S(G) \le 2\Delta$.*

What we want is that there is no other restrictions about Theorem 6.1, and in 2008, Nakprasit [34] solved the case where one part of the bipartition is of small maximum degree namely at most 2, without the restriction of C_4-free, $\chi'_S(G) \le 2\Delta$ for every $(2,\Delta)$-bipartite graph.

Let F_D denote the graph obtained from a 5-cycle by adding $D-2$ new vertices and joining them to a pair of nonadjacent vertices of the 5-cycle. In 2014 Nakprasit [35] proved that if a loopless muitgraph G has $d(x)+d(y) \le D+2$ with $\min\{d(x), d(y)\} \le 2$ for ang edge xy of G, and G is not F_D, then $\chi'_S(G) \le 2D$.

Theorem 6.2 (Bensmail et al. [5]) *For every $(3,\Delta)$-bipartite graph G, we have $\chi'_S(G) \le 4\Delta$.*

The above theorem is a new result based on Conjecture 6.2, in which the conclusion may not be the best, and in the conjecture the upper bound of $\chi'_S(G)$ is 3Δ for $(3,\Delta)$-bipartite graph, while the above result is 4Δ.

REFERENCES

[1] I. Algor, N. Alon. The star arboricity of graphs. Discrete Mathematics, 75 (1989) 11–22.

[2] L. D. Andersen. The strong chromatic index of a cubic graph is at most 10. Discrete Mathematics, 108 (1992) 231–252.

[3] M. Basavaraju, and M. C. Francis. Strong chromatic index of chordless graphs. Journal of Graph Theory, 80 (2015) 58–68.

[4] J. Bensmail, A. Harutyunyan, H. Hocquard, P. Valicov. Strong edge-colouring of sparse planar graphs. Discrete Applied Mathematics, 179 (2014) 229–234.

[5] J. Bensmail, A. Lagoutte, P. Valicov. Strong edge-coloring of $(3,\Delta)$-bipartite graphs. arxiv:1412.2624v1 (2014+).

[6] M. Bonamy, H. Hocquard, S. Kerdjoudj, and A. Raspaud. Incidence coloring of graphs with high maximum average degree. arXiv:1412.6803v2 (2015+).

[7] M. Bonamy, B. Lévěque and A. Pinlou. 2-Distance coloring of sparse graphs. Journal of Graph Theory, 77 (2014) 190–218.

[8] O. V. Borodin and A. O. Ivanova. Precise upper bound for the strong edge chromatic number of sparse planar graphs. Discussiones Mathematicae Graph Theory, (2013) (4) 759–770.

[9] A. T. Brualdi and J. J. Quinn Massey. Incidence and strong edge colorings of graphs. Discrete Mathematics, 122 (1993) 51–58.

[10] H. Bruhn and F. Joos. A stronger bound for the strong chromatic index. arXiv:1504.02583v1 (2015+).

[11] G. J. Chang, M. Montassier, A. Pêcher and A. Raspaud. Strong chromatic index of planar graphs with large girth. Discussiones Mathematicae Graph Theory, 34 (2014) (4) 723–733.

[12] G. J. Chang and N. Narayanan. Strong chromatic index of 2-degenerate graphs. Journal of Graph Theory, 73 (2013) (2) 119–126.

[13] D. W. Cranston. Strong edge-coloring of graphs with maximum degree 4 using 22 colors. Discrete Mathematics, 306 (2006) (21) 2772–2778.

[14] M. Dębski, J. Grytczuk and M. ŠleszyŃska-Nowak. Strong chromatic index of sparse graphs. Information Processing Letters, 115 (2015) 2 326–330.

[15] P. DeOrsey, J. Diemunsch, M. Ferrara, N. Graber, S. G. Hartke, S. Jahanbekam, B. Lidický, L. Nelsen, D. Stolee and E. Sullivan. On the strong chromatic index of sparse graphs, in preparation.

[16] M. H. Dolama and E. Sopena. On the maximum average degree and the incidence chromatic number of a graph. Discrete Mathematics and Theoretical Computer Science, 7 (2005) 203–216.

[17] M. H. Dolama, E. Sopena and X. Zhu. Incidence coloring of k-degenerate graphs. Discrete Mathematics, 283 (2004) 121–128.

[18] P. Erdős. Problems and results in combinatorial analysis and graph theory. Discrete Mathematic, 72 (1988) (1-3) 81–92.

[19] P. Erdős and J. Nešetřil. [Problem]. In: G. Halász and V. T. Sós (eds.), Irregularities of Partitions, Springer, Berlin, (1989) 162–163.

[20] R. J. Faudree, R. H. Schelp, A. Gyárfás and Zs. Tuza. The strong chromatic index of graphs Ars Combinatoria, 29 (1990) (B) 205–211.

[21] J. L. Fouquet and J. L. Jolivet. Strong edge-colorings of graphs and applications to multi-kgons. Ars Combinatoria, 16 (1983) (A) 141–150.

[22] J. L. Fouquet and J. L. Jolivet. Strong edge-coloring of cubic planar graphs, in Progress in graph theory (Waterloo, Ont., 1982), Academic Press, Toronto, ON, 1984, pp. 247–264.

[23] G. Yu. Strong edge-colorings for k-degenerate graphs. Graphs and Combinatorics, DOI: 10.1007/s00373–014–1478–1.

[24] Z. Guo, X. Zhao and Y. Yin. Strong edge coloring of planar graphs using $2\Delta-1$ colors, (2015+) submitted.

[25] H. Hocquard, M. Montassier, A. Raspaud and P. Valicov. On strong edge-colouring of subcubic graphs. Discrete Applied Mathematics, 161 (2013) (16-17) 2467–2479.

[26] H. Hocquard, P. Valicov. Strong edge colouring of subcubic graphs. Discrete Applied Mathematics, 159 (2011) (15) 1650–1657.

[27] P. Horák, H. Qing, and W.T. Trotter. Induced matchings in cubic graphs. Journal of Graph Theory, 17 (1993) 151–160.

[28] D. Hudák, B. Lužar, R. Soták and R. Škrekovski. Strong edge coloring of planar graphs. Discrete Mathematics, 324 (2014) (6) 41–49.

[29] T. R. Jensen and B. Toft. Graph coloring problems. John Wiley Sons Inc, 1995.

[30] H. H. Lai, K. W. Lih and P. Y. Tsai. On strong chromatic index of Halin graph. Discrete Mathematics, 312 (2012) 1536–1541.

[31] K. W. Lih and D. D. F. Liu. On the strong chromatic index of cubic Halin graphs. Applied Mathematics Letters, 25 (2012) (5) 898–901.

[32] R. Luo and G. Yu. A note on strong edge-colorings of 2-degenerate graphs, eprint arXiv:1212.6092 (2012).

[33] M. Molloy and B. Reed. A bound on the strong chromatic index of a graph. Journal of Combinatorial Theory, Series B, 69 (1997) (2) 103–109.

[34] K. Nakprasit. A note on the strong chromatic index of bipartite graphs. Discrete Mathematices, 308 (2008) 3726–3728.

[35] K. Nakprasit, K. Nakprasit. The strong chromatic index of graphs and subdivisions. Discrete Mathematices, 317 (2014) 75–78.

[36] W. C. Shiu, P. C. B. Lam and W. K. Tam. On strong chromatic index of Halin graph. Journal of Combinatorial Mathematics and Combinatorial Computing, 57 (2006) 211–222.

[37] W. C. Shiu, and W. K. Tam. The strong chromatic index of complete cubic Halin graphs. Applied Mathematics Letters, 22 (2009) 754–758.

[38] A. Steger and M. L. Yu. On induced machings. Discrete Mathematics, 120 (1993) 291–295.

[39] T. Wang. Strong chromatic index of k-degenerate graphs. Discrete Mathematics, 330 (2014) 17–19.

[40] T. Wang and X. D. Zhao. Odd graph and its application on the strong edge coloring, arXiv: 1412.8358v2.