Controller Design for Nonlinear Networked Control Systems with Random Data Packet Dropout

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Abstract— *This paper investigates the state feedback control problem of nonlinear networked control systems in presence of data packet dropout. The random data packet dropout is modeled as a Bernoulli distributed sequence with a known conditional probability distribution. The closed-loop system model is obtained through the state augmentation method. The sufficient conditions for the existence of a state feedback controller are derived by means of Lyapunov stability theory such that the closed-loop control system is exponentially stable in the mean-square sense. The maximum bound of the nonlinear item of the control system is also obtained. Then a linear matrix inequality (LMI) based approach for designing such a controller which can be solved by Matlab LMI tool box is presented. Finally, a numerical simulation example is used to demonstrate the effectiveness of the proposed method.*

Keywords—Data Packet Dropout, Exponentially Mean-square Stable, Linear Matrix Inequality, Nonlinear.

I. INTRODUCTION

In networked control system (NCS), control loops are closed by real-time networks. Such kind of control systems bring a lot of advantages, such as reduced system wiring, simple system maintenance and low cost. In the field of industrial control, remote medical treatment, military and other fields, NCS has been widely used [1-2].However, the application of communication networks in feedback control loops makes the NCS analysis and synthesis complex because data packets through networks suffer not only transmission delays but also data packet drop out [3-4].Many scholars have investigated the effect of data packet dropout on the performance of the control system.

In recent years, there are many results for the controller design of linear network control systems respect to the data packet dropout. For the NCS with random data packet drop out, literature [5-6] modeled NCS as a kind of Markov jump system, and the mode dependent controller is designed. In literature [7], if the data packet drop out accrues, the input of the actuator was considered to be zero, and the optimal estimation of NCS was studied. For a class of singular controlled plant, literature [8] modeled closed-loop system as a kind of asynchronous dynamical system, and the H_∞ control problem was investigated. However, there is not much work reported about nonlinear NCS. The state feedback control problem for nonlinear NCS is still very limited.

In this paper, the state feedback control problem for a kind of nonlinear NCS is investigated. Assuming that the data packet dropout obeys Bernoulli distribution, the closed-loop system is modeled as a class of control system which contains stochastic variable. The sufficient condition and design method of the state feedback controller for a class of nonlinear NCS with random data packet drop out are given. Simulation results show that the method proposed in this paper is effective.

II. PROBLEM FORMULATION

A nonlinear controlled plant is given by:

$$x(k+1) = Ax(k) + Bu(k) + f(k, x(k)) \tag{1}$$

Where $x(k) \in R^n$ is system state vector and $u(k) \in R^m$ is the system control input. A and B are all real constant matrices. $f(k, x(k))$ is nonlinear disturbance, satisfying the following Lipschitz condition:

$$f^T(k, x(k)) f(k, x(k)) \le \delta^2 x^T(k) H^T H x(k) \tag{2}$$

Where His real constant matrix, and$\delta \ge 0$is the bound of the nonlinear disturbance.

We consider the circumstance that data packet drop out occurs between sensor and controller. A stochastic variable is defined as follows to describe the process of data packet drop out :

$$\alpha(k)=\begin{cases}0 & \text{data packet dropout occurs}\\ 1 & \text{otherwise}\end{cases}.$$

The mathematical expectation and variance of $\alpha(k)$ are as follows:

$$\text{Prob}\{\alpha(k)=1\}=E\{\alpha(k)\}:=a\ ,\ \text{Prob}\{\alpha(k)=0\}=1-E\{\alpha(k)\}:=1-a\ ,\ \text{Var}\{\alpha(k)\}=E\{(\alpha(k)-a)^2\}=a(1-a):=b^2\ .$$

When data packet dropout occurs, the actuator continues to use the previous cycle control signal, so NCS with data packet dropout can be described as:

$$x(k+1)=Ax(k)+\alpha(k)Bu(k)+(1-\alpha(k))Bu(k-1)+\ f(k,x(k)) \tag{3}$$

Assume the state feedback control law is:

$$u(k)=Kx(k) \tag{4}$$

Substituting (4) into (3), we have the following closed-loop system:

$$x(k+1)=(A+\alpha(k)BK)x(k)+(1-\alpha(k))BKx(k-1)+\ f(k,x(k)) \tag{5}$$

At sampling time k , if we augment the state-variable as $\xi(k)^T=\begin{bmatrix}x(k)^T & x(k-1)^T\end{bmatrix}^T$,the closed-loop system (5) can be written as:

$$\xi(k+1)=\mathrm{A}(k)\xi(k)+EF(k,\xi(k)) \tag{6}$$

Where $\mathrm{A}(k)=\begin{bmatrix}A+\alpha(k)B & (1-\alpha(k))B\\ I & 0\end{bmatrix}$, $E=\begin{bmatrix}I\\0\end{bmatrix}$, $F(k,\xi(k))=f(k,G\xi(k))$, $G=\begin{bmatrix}I & 0\end{bmatrix}$.

Before proceeding, we need the following definition.

Definition 1 The closed-loop NCS (6) is said to be exponentially mean-square stable if there exist constants $\mu>0$ and $0<\rho<1$ such that $\varepsilon\{\|\xi_k\|^2\}<\mu\rho^k\varepsilon\{\|\xi_k\|^2\}$ holds for all.

Lemma 1[9] Let T_1, T_2 be symmetric matrices. The conditions on $x^TT_1x<0$ such that $x^TT_2x<0$ hold if there exist a scalar $\varepsilon>0$ such that $T_1-\varepsilon T_2<0$.

III. CONTROLLER DESIGN

Theorem 1 Take as given communication channel parameters $\text{Prob}\{\alpha(k)=0\}=1-a$,The closed-loop NCS(6) is exponentially mean-square stable if there exist positive definite matrices P,Q and nonnegative real scalar $\varepsilon>0$ such that the following matrix inequality holds:

$$\begin{bmatrix}\Xi_{11}+\varepsilon\delta^2H^TH & * & *\\ \Xi_{21} & \Xi_{22} & *\\ \Xi_{31} & \Xi_{32} & \Xi_{33}-\varepsilon I\end{bmatrix}<0 \tag{7}$$

where

$\Xi_{11}=(BK)^TP(A+aBK)+b^2(BK)^TPBK+Q-P$, $\Xi_{21}=(1-a)(BK)^TP(A+aBK)-b^2(BK)^TPBK$, $\Xi_{31}=IP(A+aBK)$,

$\Xi_{22}=(1-a)^2(BK)^TPBK+b^2(BK)^TPBK-Q$, $\Xi_{32}=(1-a)IPBK$, $\Xi_{32}=P$.

Proof:

Define a Lyapunov function

$$V(k)=\xi(k)^T S\xi(k)=\xi(k)^T\begin{bmatrix}P & 0\\ 0 & Q\end{bmatrix}\xi(k)^T =$$

$$x(k)^T Px(k)+x(k-1)^T Qx(k-1)$$

Where P and Q are positive definite matrices, we have:

$$E\{V_{k+1}\}-E\{V_k\}$$
$$=E\{[(A+\alpha(k)BK)x(k)+(1-\alpha(k))BKx(k-1)+f(k,x(k)]^T P$$
$$[(A+\alpha(k)BK)x(k)+(1-\alpha(k))BKx(k-1)+f(k,x(k)]+$$
$$x(k)^T Qx(k)-x(k)^T Px(k)-x(k-1)^T Qx(k-1)$$
$$=E\{[(A+\alpha BK)x(k)+(\alpha(k)-\alpha)BKx(k)-(\alpha(k)-\alpha)BKx(k-1)+(1-\alpha)BKx(k-1)+f(k,x(k)]^T P$$
$$[(A+\alpha BK)x(k)+(\alpha(k)-\alpha)BKx(k)-(\alpha(k)-\alpha)BKx(k-1)+(1-\alpha)BKx(k-1)+f(k,x(k)]$$
$$x(k)^T Qx(k)-x(k)^T Px(k)-x(k-1)^T Qx(k-1)$$

$$=\phi(k)^T\begin{bmatrix}\Xi_{11} & * & *\\ \Xi_{21} & \Xi_{22} & *\\ \Xi_{31} & \Xi_{32} & \Xi_{33}\end{bmatrix}\phi(k) \tag{8}$$

Where $\phi(k)^T=\begin{bmatrix}x(k)^T & x(k-1)^T & f(k,x(k))^T\end{bmatrix}^T$.

From (2), we can get:

$$\phi(k)^T\begin{bmatrix}-\delta^2 H^T H & * & *\\ 0 & 0 & *\\ 0 & 0 & I\end{bmatrix}\phi(k)\le 0 \tag{9}$$

According to lemma 1, if there exists a scalar $\varepsilon>0$ such that

$$\begin{bmatrix}\Xi_{11}+\varepsilon\delta^2 H^T H & * & *\\ \Xi_{21} & \Xi_{22} & *\\ \Xi_{31} & \Xi_{32} & \Xi_{33}-\varepsilon I\end{bmatrix}<0 \tag{10}$$

then the following inequality subject to (9) holds.

$$\begin{bmatrix}\Xi_{11} & * & *\\ \Xi_{21} & \Xi_{22} & *\\ \Xi_{31} & \Xi_{32} & \Xi_{33}\end{bmatrix}<0$$

Since $S>0$,it is clear that

$\lambda_{min}(s)\varepsilon\{\|\eta_k\|^2\}\le\varepsilon\{\eta_k^T S\eta_k\}\le\lambda_{max}(s)\varepsilon\{\|\eta_k\|^2\}$. Because $\varepsilon\{\xi_{k+1}^T S\xi_{k+1}\}-\varepsilon\{\xi_k^T S\xi_k\}<0$, there exists $0<\gamma<\lambda_{\max}(S)$ such that $\varepsilon\{\xi_{k+1}^T S\xi_{k+1}\}\le-\gamma\{\|\xi_k\|^2\}+\varepsilon\{\xi_k^T S\xi_k\}\le\left(1-\frac{\gamma}{\lambda_{max}(S)}\right)\varepsilon\{\xi_k^T S\xi_k\}$.Consequently, we can get

$$\lambda_{\min}(s)\varepsilon\{\|\eta_k\|^2\}\le\varepsilon\{\eta_k^T S\eta_k\}\le(1-\frac{\gamma}{\lambda_{\max}(S)})\varepsilon\{\xi_k^T S\xi_k\}\le$$

$$(1-\frac{\gamma}{\lambda_{\max}(S)})^k \varepsilon\{\xi_0^T S\xi_0\} \le \lambda_{\max}(S)(1-\frac{\gamma}{\lambda_{\max}(S)})^k \varepsilon\{\| \xi_0 \|^2\}$$

That is $\varepsilon\{\| \xi_{\text{k}} \|^2\} < \mu\rho^k \varepsilon\{\| \xi_k \|^2\}$,

where $\mu = \frac{\lambda_{\max}(S)}{\lambda_{\min}(S)}$, $\rho = 1-\frac{\gamma}{\lambda_{\max}(S)}$.

Theorem 2 Take as given communication channel parameters $\text{Prob}\{\alpha(k)=0\}=1-a$, the systems (6) is exponentially mean square stable if there exist positive definite matrices M>0,N>0, matrix Y and a scalar $\varphi > 0$ satisfying the following LMI

$$\begin{bmatrix} \Sigma & * \\ \Pi & \Lambda \end{bmatrix} < 0 \tag{11}$$

where

$\Sigma = diag(N-M,-N,-I)$, $\Lambda = diag(-M,-M,-\varphi I)$, $\Pi = \begin{bmatrix} AM & (1-a)BY & I \\ bBY & -bBY & 0 \\ HM & 0 & 0 \end{bmatrix}$ and the controller is given by $u(k) = YM^{-1}x(k)$.

Proof:

(7) Is equivalent to

$$\begin{bmatrix} \tilde{\Xi}_{11} + \delta^2 H^T H & * & * \\ \tilde{\Xi}_{21} & \tilde{\Xi}_{22} & * \\ \tilde{\Xi}_{31} & \tilde{\Xi}_{32} & \tilde{\Xi}_{33} - I \end{bmatrix} < 0 \tag{12}$$

Where

$\tilde{\Xi}_{11} = (BK)^T \tilde{P}(A+aBK) + b^2 (BK)^T \tilde{P}BK + \tilde{Q} - \tilde{P}$, $\tilde{\Xi}_{21} = (1-a)(BK)^T \tilde{P}(A+aBK) - b^2 (BK)^T \tilde{P}BK$,

$\tilde{\Xi}_{31} = I\tilde{P}(A+aBK)$, $\tilde{\Xi}_{22} = (1-a)^2 (BK)^T \tilde{P}BK + b^2 (BK)^T \tilde{P}BK - \tilde{Q}$, $\tilde{\Xi}_{32} = (1-a)I\tilde{P}BK$, $\tilde{\Xi}_{32} = \tilde{P}$,

$\tilde{P} = \frac{P}{\varepsilon}$, $\tilde{Q} = \frac{Q}{\varepsilon}$,

And (12) can be written as

$$\begin{bmatrix} b^2 (BK)^T \tilde{P}BK + \tilde{Q} - \tilde{P} + \delta^2 H^T H & * & * \\ -b^2 (BK)^T \tilde{P}BK & b^2 (BK)^T \tilde{P}BK - \tilde{Q} & * \\ IP(A+aBK) & 0 & -I \end{bmatrix} + \begin{bmatrix} (A+aBK)^T \\ (1-a)(BK)^T \\ I \end{bmatrix} \tilde{P} \begin{bmatrix} A+aBK & (1-a)(BK) & I \end{bmatrix} < 0 .$$

By Schur complement, one can obtain

$$\begin{bmatrix} b^2 (BK)^T \tilde{P}BK + \tilde{Q} - \tilde{P} + \delta^2 H^T H & * & * & * \\ -b^2 (BK)^T \tilde{P}BK & b^2 (BK)^T \tilde{P}BK - \tilde{Q} & * & * \\ IP(A+aBK) & 0 & -I & * \\ A+aBK & (1-a)BK & I & -\tilde{P}^{-1} \end{bmatrix} < 0$$

Similarly, using Schur again we can get

$$\begin{bmatrix} \Delta_{11} & * \\ \Delta_{21} & \Delta_{22} \end{bmatrix} < 0 \tag{13}$$

Where

$$\Delta_{11} = diag(\tilde{Q} - \tilde{P}, -\tilde{Q}, -I),\ \Delta_{22} = diag(-\tilde{P}, -\tilde{P}, -\varphi I),\ \Delta_{21} = \begin{bmatrix} A & * & * \\ bBK & -bBK & * \\ H & 0 & 0 \end{bmatrix}.$$

Performing a congruence transformation to (13) by $diag\{\tilde{P}^{-1}, \tilde{P}^{-1}, I, I, I, I, I\}$, and let $M = \tilde{P}^{-1}$, $N = MQM$, $Y = KM$

$\delta^{-2} = \varphi$, (11) can be obtained, which complete the proof.

Corollary 1 If the following optimization problem has solution

$$\min_{M>0, N>0, \varphi>0, Y} \varphi \quad s.t. (11)$$

and the solution is φ_{min} ,the maximum bound of nonlinear disturbance in (2)is $\delta_{max} = \sqrt{\varphi_{min}}^{-1}$.

Remark 1 the above convex optimization with LMI constraints can be solved by Matlab LMI toolbox objective function minimization problem solver mincx.

IV. SIMULATION EXAMPLE

Consider the following nonlinear controlled plant

$$x(k+1) = \begin{bmatrix} 1 & -2 \\ 0.3 & 0.6 \end{bmatrix} x(k) + \begin{bmatrix} -1 \\ 0.2 \end{bmatrix} u(k) + \begin{bmatrix} 0.1x_1(k)\sin x_1(k) \\ 0.2x_2(k)\cos x_2(k) \end{bmatrix}.$$

Assume that the initial state of the system is $x(0) = [3 \quad -1]^T$ and the Probability of the data packet dropout is $\Pr ob\{\alpha(k) = 0\} = 0.2$ and the matrix H is a unit matrix. According to Theorem 2, and using Matlab LMI tool, we can get

$Y = [0.4610 \quad -0.2155]$, $M = \begin{bmatrix} 40.0981 & 29.5003 \\ 29.5003 & 63.9610 \end{bmatrix}$. Hence, the controller gain matrix is

$K = YM^{-1} = [0.0212 \quad -0.0131]$.

By Matlab LMI toolbox objective function minimization problem solver mincx, the maximum bound of nonlinear disturbance in (2) is obtained as $\delta_{\max} = 0.2277$. The process of data packet drop out and the state trajectory of the closed-loop system is shown by the Fig.1 and Fig.2, respectively.

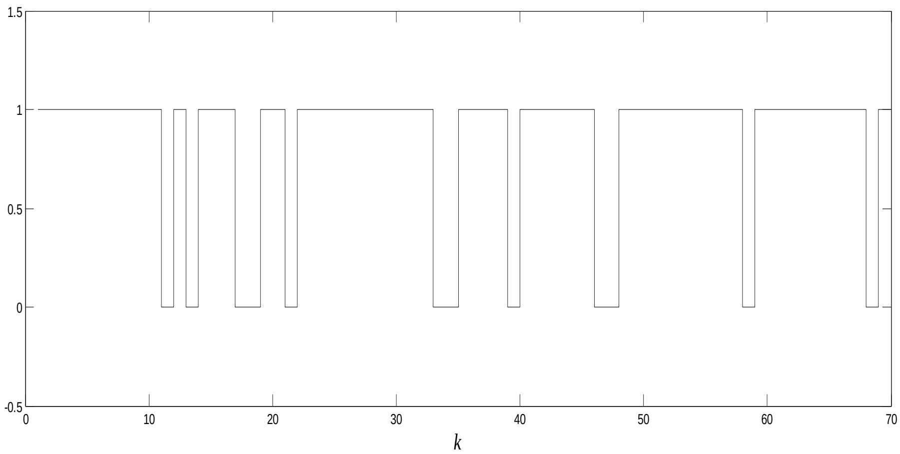

FIG.1 THE RANDOM DATA PACKET DROPOUT DISTRIBUTION

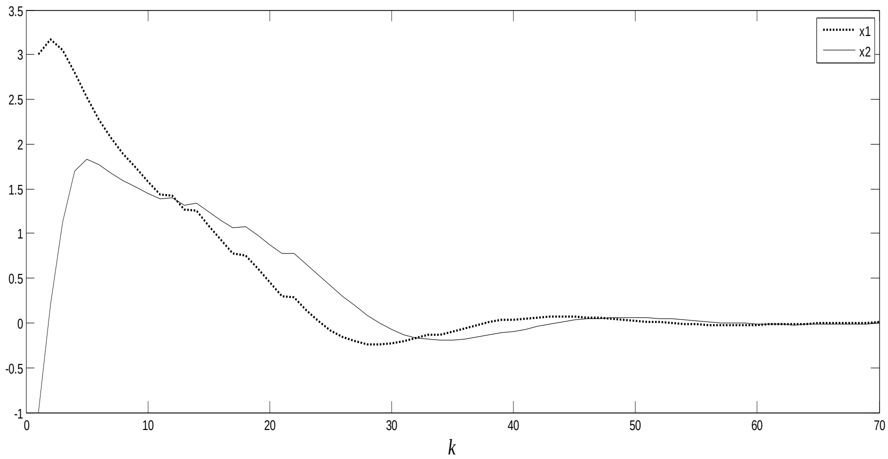

FIG.2 THE STATE TRAJECTORY OF THE CLOSED-LOOP SYSTEM

From the figure above, we can see that the closed-loop system is still stable under the designed controller even if there exists data packet dropout.

V. CONCLUSION

In this paper, the sufficient condition and design method of the state feedback controller for a class of nonlinear NCS with random data packet drop out are given. The controller gain matrix and the maximum bound of the nonlinear item of the control system can be obtained by use of the relevant tool box of Matlab. Simulation results show that the method proposed in this paper is effective.

VI. ACKNOWLEDGEMENTS

This work was supported by the National Natural Science Foundation of China under grant nos. 61174029,61573136,61503136.

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