Multiuser Resource Allocation Algorithms for Downlink OFDMA-based MIMO Network

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Abstract**—***The problem of simultaneous multiuser resource (subcarriers-and-bits) allocation algorithm in OFDMA-based multiple input-multiple output (MIMO) system has recently attracted significant interest. In this paper, we employ adaptive modulation technique and advanced use of multiple antennas at both the transmitter and receiver to develop four resource allocation schemes. The first scheme assigns subcarrier to the user with best channel gain and employs spatial multiplexing (SM) on the MIMO system to further enhance the throughput. The space-division multiple-access (SDMA) scheme assigns single subcarrier simultaneously to the terminals with pairwise "nearly orthogonal" spatial signatures. In the third scheme, we propose to design the transmit beamformers based on the zero-forcing (ZF) criterion such that the multiuser interference (MUI) is completely removed. Specifically, we propose a low-complexity iterated terminals-selection algorithm in conjunction with the ZF criterion such that the selected ZF (SZF) scheme can be exploited to achieve throughput multiplication. Alternatively, we propose a least-squares (LS) based multiuser resource allocation algorithm to cope with the over-determined system such that all users are allowed to share single subcarrier.*

Keywords—SDMA, OFDMA, MIMO, Selected Zero-Forcing (SZF), Least-Squares (LS).

I. INTRODUCTION

In 4G and future cellular networks, mushrooming users need to share the spectrum to achieve high-rate multimedia communication while ensure the fulfillment of quality-of-service (QoS) requirements. In the downlink (DL) of 4G LTE, orthogonal frequency division multiple access (OFDMA) technique is employed [1]. The advantages of OFDMA includes robust multipath suppression, ability to combat intersymbol interference (ISI), relative low complexity, and flexibility in accommodating many users with widely varying data rates [2]. The rationale for OFDMA is dynamically allocating subcarriers to the user with best channel state. However, the resource allocation algorithms are not specified in the LTE standard, and recently, quite a few scheduling and resource allocation algorithms have been proposed for OFDMA cellular system [3-7]. Resource allocation is essentially a constrained-optimization problem [Chap. 18 of [8]] that either maximizes the overall data rate or minimizes the total transmit power subject to specific constraints, e.g., the users' QoS requirements. The optimum resource allocation algorithm is itself a NP-complete problem whose solution can only be found with exhaustive search [7], which is infeasible for practical situation. In the work of [1], the sum rate of all users is maximized with the constraint of total transmit power and minimum data rate for each user. While the work of [3] aims to allocate the subcarriers and power such that the minimum user's data rate is maximized. Alternatively, proportional fairness scheduling [4] is designed to take advantage of multiuser diversity, while maintaining comparable long-term throughput for all users. In the work of [5], a Lagrangian-based algorithm is proposed to attain a dramatic gain in power efficiency. However, the load of computation is too high to put it into practical use.It is well-known that one of the largest advantages of LTE over incumbent standard is more advanced use of multiple antennas. And the massive multiple input-multiple output (MIMO) technique is apt to be included in future 5G network [9-12]. The key benefits of wireless communication provided by antenna array include:

1) **Spatial diversity:** It can effectively combat fading.
2) **Interference suppression:** The interference can be removed from the desired user as long as the array size exceeds the number of interferers.
3) **Spatial multiplexing:** It allows different signal sources to be sent simultaneously in the same bandwidth.

All the above techniques can effectively increase the capacity or throughput of a wireless communication system. The main focus of this paper is to jointly exploit the three benefits of multi-antenna technology to develop a simple yet efficient resource allocation algorithm in the downlink of an OFDMA-based cellular system.

This paper considers a multiuser MIMO system, in which each cell is consisted of a L-antenna base station (BTS) and K remote terminals (RT) each equipped with Q antennas. Downlink OFDMA system with the assumption that each RT estimates and feedback the instantaneous channel state information (CSI) to the BTS. We apply adaptive modulation according to the CSI to meet the minimum QoS requirement, in which the symbol error rate (SER) is the performance metric. We develop four resource allocation schemes, which allocate subcarriers and bits to each user via different antennas. Specifically, in order to maximize system throughput as well as spectral efficiency, we employ various array processing techniques. The algorithms proposed in this paper attempt to maximize system throughput subject to the constraints of total transmit power and minimum data rate for each user.

The first scheme jointly exploits spatial diversity and spatial multiplexing (SM) technique, in which the BTS designates subcarrier to the user with the highest channel gain between BTS and a specific RT. Since multiple substreams can be sent through the MIMO system, the data rate is expected to be increased. The SM scheme is inherently a single-user algorithm in which each subcarrier can only be assigned to single user at specific time. While in scheme 2, we assign subcarrier to the users with "nearly orthogonal" spatial signatures. We refer it as the space-division multiple-access (SDMA) scheme. Though throughput of the SDMA scheme is expected to be increased, nevertheless, an obvious flaw of this scheme is that spatial channels are rarely orthogonal in practice. In the third scheme, we propose to design prefilters at the BTS that meets the zero-forcing (ZF) criterion to remove the multiuser interference (MUI). Since MUI is removed and each subcarrier can be assigned simultaneously to all the user terminals, therefore throughput multiplication can be achieved. However, in the case of, complete MUI suppression is not possible; we propose two alternative multi-user resource allocation algorithms. The first algorithm selects out of spatial signatures that contribute to the system the maximum throughput. We refer it as the selected ZF (SZF) scheme. Moreover, we propose a low-complexity iterated algorithm to implement the SZF scheme. The second algorithm to overcome is based on the least-squares (LS) criterion. We refer it as the selected LS scheme. The LS scheme is feasible and efficient since each subcarrier can be shared simultaneously to all spatial signatures. Nevertheless, residual MUI is inevitable in the LS scheme.

The rest of this paper is organized as follows. In Section 2, we describe the system model and formulate the problem. Section 3 describes and analyzes the proposed multiuser subcarriers-and-bits allocation schemes. Section 4 focuses on the issue of multiuser resource allocation when the BTS array size is less than the sum of RTs' array size. In Section 5, we demonstrate the system performance and discuss the numerical results. Concluding remarks are finally made in section 6.

Notation: We use upper and lower case boldface letters to denote matrices and vectors, respectively. $[\ ]^T, [\ ]^H$ stand for matrix or vector transpose and complex transpose, respectively. We will use $E\{\ \}$ for expectation (ensemble average), $\|\ \|$ for vector norm, and $\equiv$ for "is defined as". $\|\mathbf{A}\|_F$ denotes the Frobenius norm of matrix **A**. $\mathbf{I}_K$ denotes an identity matrix with size *K*. $\mathbf{e}_k$ denotes the *k*th column vector of an identity matrix. A complex normal variable with mean μ variance σ^2 reads as $\sim N(\mu, \sigma^2)$. $\delta(\cdot)$ is the dirac delta function. $\bar{x}$ denotes the complex conjugate of *x*. $\lfloor x \rfloor$ denotes the floor function, *i.e.*, the maximum integer value that is equal to or less than *x*. $[x]_- = \min\{1, x\}$. $\binom{n}{k}$ stands for the combination of *k* out of *n*. $tr(\mathbf{A})$ denotes the trace (sum of the diagonal elements) of square matrix **A**.

II. SYSTEM MODEL AND PROBLEM FORMULATION

We consider an OFDMA-based cellular network, where in a specific cell, the BTS is equipped with *L* antennas and there are *K* RTs each with *Q* antennas. For each subcarrier, flat-fading channel model is assumed. Since all RTs share the same bandwidth, we attempt to develop an efficient resource allocation algorithm to allocate *N* subcarriers to the *K* users subject to pre-determined constraints.

Let $\left\{h_{k,l,q}^n\right\}_{\substack{l=1,\ldots,L\\q=1,\ldots,Q\\k=1,\ldots,K\\n=1,\ldots,N}}$ be the instantaneous magnitude of the channel gain between the *k*th RT's *q*th antenna and the *l*th antenna of BTS on the *n*th subcarrier, which includesthe effect of path loss, shadowing and fading. For each subcarrier *n*, it can also be regarded as multiple MIMO system with channel matrix $\left\{\mathbf{H}_k^n\right\}_{\substack{k=1,\ldots,K\\n=1,\ldots,N}}$, given by

$$\mathbf{H}_k^n \equiv \begin{bmatrix} h_{k,1,1}^n & h_{k,1,2}^n & \cdots & h_{k,1,Q}^n \\ h_{k,2,1}^n & h_{k,2,2}^n & \cdots & h_{k,2,Q}^n \\ \vdots & \vdots & \ddots & \vdots \\ h_{k,L,1}^n & h_{k,L,2}^n & \cdots & h_{k,L,Q}^n \end{bmatrix} = \begin{bmatrix} \mathbf{h}_{k,1}^n & \mathbf{h}_{k,2}^n & \cdots & \mathbf{h}_{k,Q}^n \end{bmatrix} \quad (1)$$

where $\left\{\mathbf{h}_{k,q}^n\right\}_{q=1,\ldots,Q}$ denotes the L-by-1 channel vector seen by the kth user's qth antenna.

$$\mathbf{h}_{k,q}^n \equiv \begin{bmatrix} h_{k,1,q}^n & h_{k,2,q}^n & \cdots & h_{k,L,q}^n \end{bmatrix}^T \quad (2)$$

Let us consider here a Time-Division Duplexing (TDD) scheme, in which reciprocity between uplink and downlink channels can be assumed. Consequently, according to the method proposed in [12], the BTS can estimate the CSI from the uplink pilot signals transmitted by each user. A schematic illustration of the system under consideration is depicted in Fig. 1. For simplicity, we invoke the following assumptions throughout this paper:

1) The BTS has perfect knowledge of the CSI, which are periodically reported by each user.
2) The channel matrices, $\left\{\mathbf{H}_k^n\right\}_{\substack{k=1,\ldots,K \\ n=1,\ldots,N}}$, stay constant during the resource allocation process.

The problem addressed in this paper is to exploit the available CSI to perform real-time subcarriers and bits allocation. The design goal of the resource allocation algorithm is to maximize the overall throughput (bits per unit time) subject to the following constraints:

1) The QoS for each RT should meet system requirement. In this paper, QoS is measured by SER.
2) The energy consumption at the BTS should be upper-bounded by E_{total}.
3) The data rate allocated for kth user should meet a minimum data rate requirement equal to R_k bits per OFDM symbol.

Please note that the resource allocation algorithms described in this paper is flexible with every attempt to maximize the throughput, thereby, the subcarriers allocated for each user are distributed rather than bunched.

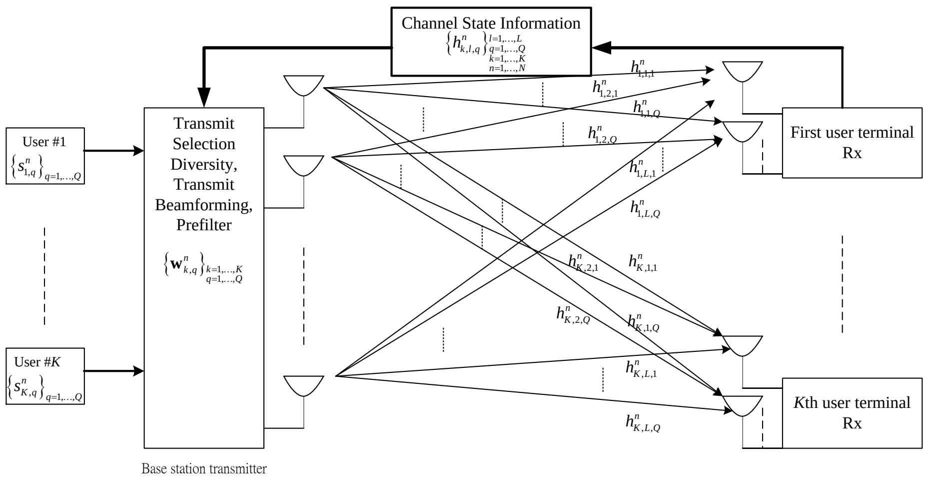

FIG. 1: BLOCK DIAGRAM OF THE MULTIUSER MIMO SYSTEM

III. JOINT SUBCARRIERS AND BITS ALLOCATION ALGORITHMS

In order to achieve the challenging spectral efficiency and user throughput targets, adaptive modulation and coding are also used and included in the specifications of LTE [1]. The rationale of adaptive modulation is to transmit as high a data rate as possible as long as the channel is good. On the other hand, data is transmitted at a lower rate when and where the channel is in poor condition. In this paper, we employ *M-ary* PSK (MPSK) for QoS measurement, though extension to other modulation techniques is without conceptual difficulty. The SER for MPSK modulation is upper bounded by [13]

$$P_{e,MPSK} < 2Q\left(\sqrt{\frac{2E}{N_0}}\sin\frac{\pi}{M}\right) \quad (3)$$

Where E denotes the received symbol energy. N_0 is the one-sided power spectral density of AWGN. The tail function is defined as $Q(x) \equiv \frac{1}{\sqrt{2\pi}} \int_x^{\infty} \exp\left(-\frac{t^2}{2}\right) dt$. Obviously, SER increases with respect to the number of bits per symbol, $m = \log_2 M$. To maximize the throughput while maintaining the QoS requirement, it is usually required that the SER must be lower than a pre-determined threshold, $P_{e,MPSK} \leq \lambda$. Using (3) and after some manipulations, we have

$$M \leq \frac{\pi}{\sin^{-1}\left(\left[\sqrt{\frac{N_0}{2E}} Q^{-1}\left(\frac{\lambda}{2}\right)\right]_{-}\right)} \tag{4}$$

Therefore, the allocated bits per OFDM symbol is upper bounded by

$$m = \left\lfloor \log_2 \left(\frac{\pi}{\sin^{-1}\left(\left[\sqrt{\frac{N_0}{2E}} Q^{-1}\left(\frac{\lambda}{2}\right)\right]_{-}\right)} \right) \right\rfloor \tag{5}$$

The algorithm for achieving optimum solution requires exhaustive search. The number of iterations needed in single antenna scenario (M=Q=1) is about $O\left(K^N\right)$ [7], which is infeasible even for moderate N and K. Therefore in this paper, we attempt to develop suboptimum solutions.

3.1 Single User Spatial Multiplexing (SM) Scheme

In the single user spatial multiplexing (SM) scheme, the BTS transmits multiple substreams simultaneously to a specific RT using the same subcarrier. For a specific n, the BTS first chooses among $\{\mathbf{H}_k^n\}_{k=1,\ldots,K}$ with the largest Frobenius norm

$$k^* = \arg \max_{k\in\{1,\ldots,K\}} \left\|\mathbf{H}_k^n\right\|_F \tag{6}$$

where $\left\|\mathbf{H}_k^n\right\|_F = \sqrt{\sum_{q=1}^{Q}\sum_{l=1}^{L}\left|h_{k,l,q}^n\right|^2}$. Let ρ_k^n be the subcarrier allocation index, that is, $\rho_k^n = 1$ when the nth subcarrier is assigned to the kth RT, $\rho_k^n = 0$, otherwise. That is

$$\text{If } \rho_k^n = 1 \Rightarrow \rho_{k'}^n = 0 \text{ for all } k' \neq k \tag{7}$$

In the SM scheme, the fact that each RT has Q receive antennas can be exploited in the spatial domain by transmitting up to Q independent symbol streams simultaneously for each user. In what follows, the outputs of the antenna array at the BTS can be described as

$$\mathbf{x}_k^n = \sum_{q=1}^{Q} \mathbf{w}_{k,q}^n s_{k,q}^n \tag{8}$$

where $\{s_{k,q}^n\}_{q=1,\ldots,Q}$ is the symbol to be detected by the kth user's qth receive antenna. We let the transmitted symbol energy be normalized to be 1, *i.e.*, $E\left\{\left|s_{k,q}^n\right|^2\right\} = 1$, throughout the paper. $\{\mathbf{w}_{k,q}^n\}_{q=1,\ldots,Q}$ is the L-by-1 transmit weight vector designated for $\{s_{k,q}^n\}_{q=1,\ldots,Q}$, respectively. The received Q-by-1 vector signal at the kth RT yields

$$\mathbf{y}_k^n = \mathbf{H}_k^{nT}\mathbf{x}_k^n + \mathbf{n}_k^n \tag{9}$$

where $\mathbf{n}_k^n$ is the zero-mean Gaussian noise vector with covariance matrix $\frac{N_0}{2}\mathbf{I}_Q$. For each subcarrier, we aim at transmitting Q symbols simultaneously to the kth user. The design goal of $\{\mathbf{w}_{k,q}^n\}_{q=1,\ldots,Q}$ is to avoid interference to other symbols. We assume L>Q, which is usually the case, and employ singular-value-decomposition (SVD) [chap. 7 of [8]] to decompose $\mathbf{H}_k^{nT}$ as

$$\mathbf{H}_k^{nT} = \mathbf{U}_k^n \mathbf{\Sigma}_k^n \mathbf{V}_k^{nH} \tag{10}$$

where $\mathbf{U}_k^n \in \mathbb{C}^{Q\times Q}, \mathbf{V}_k^n \in \mathbb{C}^{L\times L}$ are unitary matrices, and $\mathbf{\Sigma}_k^n \in \mathbb{C}^{Q\times L}$ is a zero matrix except for the square roots of Qnonzero eigenvalues of the matrix $\mathbf{H}_k^{nH}\mathbf{H}_k^n$ on thediagonals. We denote each diagonal element by $\left\{\sqrt{\sigma_{k,q}^n}\right\}_{q=1,\dots,Q}$. Let $\mathbf{v}_{k,q}^n$ be the qth column vectors of $\mathbf{V}_k^n$, then bythe SVD theorem, the first Q columns of $\mathbf{V}_k^n$ are orthonormal basis of the column space of $\mathbf{H}_k^n$.

$$\text{span}\left\{\mathbf{v}_{k,1}^n, \cdots, \mathbf{v}_{k,Q}^n\right\} = \text{CSP}\left\{\mathbf{H}_k^n\right\} \tag{11}$$

Therefore, we choose the weightvectors $\left\{\mathbf{w}_{k,q}^n\right\}_{q=1,\dots,Q}$ as

$$\mathbf{w}_{k,q}^n = \eta_{k,q}^n \mathbf{v}_{k,q}^n; q = 1, \dots, Q \tag{12}$$

where $\eta_{k,q}^n$ denotes the energy normalization factor in order to meet the transmit energy constraint. In what follows, the received signal vector at the kth RT can be obtained as

$$\mathbf{H}_k^{nT}\mathbf{x}_k^n = \left(\mathbf{U}_k^n \mathbf{\Sigma}_k^n \mathbf{V}_k^{nH}\right)\mathbf{x}_k^n = \mathbf{U}_k^n \begin{bmatrix} \eta_{k,1}^n \sqrt{\sigma_{k,1}^n} s_{k,1}^n \\ \eta_{k,2}^n \sqrt{\sigma_{k,2}^n} s_{k,2}^n \\ \vdots \\ \eta_{k,Q}^n \sqrt{\sigma_{k,Q}^n} s_{k,Q}^n \end{bmatrix} \tag{13}$$

The optimum receiver is composed of a bank of matched-filters (correlators) with column vectors of $\mathbf{U}_k^n$ as template signals

$$\begin{aligned} \mathbf{U}_k^{nH}\mathbf{y}_k^n &= \mathbf{U}_k^{nH}\left(\mathbf{H}_k^{nT}\mathbf{x}_k^n + \mathbf{n}_k^n\right) \\ &= \mathbf{U}_k^{nH}\mathbf{U}_k^n \begin{bmatrix} \eta_{k,1}^n \sqrt{\sigma_{k,1}^n} s_{k,1}^n \\ \eta_{k,2}^n \sqrt{\sigma_{k,2}^n} s_{k,2}^n \\ \vdots \\ \eta_{k,Q}^n \sqrt{\sigma_{k,Q}^n} s_{k,Q}^n \end{bmatrix} + \mathbf{U}_k^{nH}\mathbf{n}_k^n \\ &= \begin{bmatrix} \eta_{k,1}^n \sqrt{\sigma_{k,1}^n} s_{k,1}^n \\ \eta_{k,2}^n \sqrt{\sigma_{k,2}^n} s_{k,2}^n \\ \vdots \\ \eta_{k,Q}^n \sqrt{\sigma_{k,Q}^n} s_{k,Q}^n \end{bmatrix} + \tilde{\mathbf{n}}_k^n \end{aligned} \tag{14}$$

It is easy to show that $\tilde{\mathbf{n}}_k^n \equiv \mathbf{U}_k^{nH}\mathbf{n}_k^n$ is still white with the same covariance matrix. It is evident from (14) that the Q symbols, $\left\{s_{k,q}^n\right\}_{q=1,\dots,Q}$, can be simultaneously detected. The received symbol energy for the kth user's qth receiver at the nth subcarrier can be calculated by

$$E_{k,q}^n = E\left\{\left|\eta_{k,q}^n \sqrt{\sigma_{k,q}^n} s_{k,q}^n\right|^2\right\} = \sigma_{k,q}^n {\eta_{k,q}^n}^2 \tag{15}$$

Substituting (15) into (5), the bits that can be sent to the kth user's qth antenna can be obtained as

$$m_{k,q}^n = \left\lfloor \log_2\left(\frac{\pi}{\sin^{-1}\left(\left[\frac{1}{\eta_{k,q}^n\sqrt{\sigma_{k,q}^n}}\sqrt{\frac{N_0}{2}}Q^{-1}\left(\frac{\lambda}{2}\right)\right]_-\right)}\right)\right\rfloor \tag{16}$$

Based on (12), the energy consumption can be calculated as

$$E_{SM} = \sum_{k=1}^{K}\sum_{n=1}^{N}\sum_{q=1}^{Q}\rho_k^n E\left\{\left|s_{k,q}^n\right|^2\right\}\left\|\mathbf{w}_{k,q}^n\right\|^2 = \sum_{k=1}^{K}\sum_{n=1}^{N}\sum_{q=1}^{Q}\rho_k^n \eta_{k,q}^{n\ 2} \le E_{total} \tag{17}$$

To determine $\left\{\eta_{k,q}^n\right\}$ under the constraint $E_{SM} \le E_{total}$, we may choose $\left\{\eta_{k,q}^n\right\}$ as

$$\eta_{k,q}^n = \sqrt{\frac{E_{total}}{N}} \tag{18}$$

Substituting (18) into (16), the energy constraint and channel condition of the SM scheme are related by

$$m_{k,q}^n = \left\lfloor \log_2\left(\frac{\pi}{\sin^{-1}\left(\left[\frac{1}{\sqrt{\sigma_{k,q}^n}}\sqrt{\frac{NN_0}{2E_{total}}}Q^{-1}\left(\frac{\lambda}{2}\right)\right]_{-}\right)}\right)\right\rfloor \tag{19}$$

There then, the throughput of the kth user can be calculated by

$$T_k = \sum_{n=1}^{N}\sum_{q=1}^{Q}\rho_k^n m_{k,q}^n; \quad k = 1,\dots,K \tag{20}$$

And the overall throughput is $T = \sum_{k=1}^{K} T_k$.

The joint subcarriers and bits allocation for the SM scheme is equivalent to finding $\left\{\rho_k^n\right\}_{\substack{k=1,\dots,K\\ n=1,\dots,N}}$ to satisfy the following constrained-optimization problem

$$\arg\max_{\rho_k^n} \sum_{k=1}^{K}\sum_{n=1}^{N}\sum_{q=1}^{Q}\rho_k^n m_{k,q}^n$$

Subject to

1) $T_k = \sum_{n=1}^{N}\sum_{q=1}^{Q}\rho_k^n m_{k,q}^n \ge R_k \quad ;\forall k = 1,\dots,K$

2) $\sum_{k=1}^{K}\sum_{n=1}^{N}\rho_k^n = N$

3) if $\rho_k^n = 1 \Rightarrow \rho_{k'}^n = 0, \quad \forall k' \ne k$

We implement the SM scheme as follows:

Step 1: Starting from the first subcarrier, the BTS assigns the subcarrier to the user with the highest Frobenius norm of channel matrix, which corresponds to the maximum bits to be transmitted, among all users in that subchannel. Let W be the user set, $W = \{1, 2, \dots, K\}$, we have

$$k^* = \arg\max_{k \in W}\left\|\mathbf{H}_k^n\right\|_F$$

Since each subcarrier can only be assigned to single user, hence, as soon as subcarrier n has been assigned to user k^*, we set

$$\rho_{k^*}^n = 1, \rho_k^n = 0, \text{ for all } k \ne k^*$$

Step 2: Perform SVD on $\mathbf{H}_{k^*}^{n\ T}$ to obtain $\left\{\mathbf{v}_{k^*,q}^n\right\}_{q=1,\dots,Q}$

Step 3: Calculate $\left\{m_{k^*,q}^n\right\}_{q=1,\dots,Q}$ using (19), and then calculate $T_{k^*} = \sum_{i=1}^{n}\sum_{q=1}^{Q}\rho_{k^*,q}^i m_{k^*,q}^i$

Step 4: Check whether user k^* meets the minimum bit rate constraint. If $T_{k^*} < R_{k^*}$, which means more subchannels should be assigned to user k^*, go back to step 1 and check for the next subcarrier. On the other hand, if $T_{k^*} \geq R_{k^*}$, we temporarily exclude user k^* and go back to step 1 to assign the subchannels to those users that have not attained the minimum bit rate constraint.

Step 5: As long as all the users meet the minimum bit rate constraint, remove all the exclusions, redo step 1 until all the subcarriers have been allocated.

3.2 SDMA Scheme

Different from the SM scheme, a subcarrier can be assigned simultaneously to multiple users in the SDMA scheme. In the proposed SDMA scheme, we systematically assigns "most orthogonal" user terminals to the same subchannel. It is plausible to measure the orthogonality of two spatial signatures at subcarrier n by the crosscorrelation magnitude, $\left|\mathbf{h}_{k,q}^{n\ H}\mathbf{h}_{k',q'}^{n}\right| \quad ;\forall(k,q)\neq(k',q')$.

Let W^n be the chosen set of spatial signatures for the nth subcarrier that are pairwise approximately orthogonal, which means

$$\left|\mathbf{h}_{k,q}^{n\ H}\mathbf{h}_{k',q'}^{n}\right| \approx 0 \quad ;\forall(k,q),(k',q')\in W^n \tag{21}$$

Since the users in W^n are allowed to transmit simultaneously on subcarrier n without causing interference to each other at the receiving end, the subcarrier allocation index for the nth subcarrier becomes

$$\rho_{k,q}^{n} = \begin{cases} 1;(k,q)\in W^n \\ 0;\text{otherwise} \end{cases} \tag{22}$$

Therefore, the transmitted signal at the BTS on subcarrier n can be described as

$$\mathbf{x}^n = \sum_{(k,q)\in W^n} \mathbf{w}_{k,q}^{n} s_{k,q}^{n} \tag{23}$$

Since the channel vectors are pairwise orthogonal, the weight vector for transmit beamforming can be designed as the complex conjugate of the spatial signature vector

$$\mathbf{w}_{k,q}^{n} = \eta_{k,q}^{n}\bar{\mathbf{h}}_{k,q}^{n};(k,q)\in W^n \tag{24}$$

For a specific $(k,q)\in W^n$, the received signal at the kth user's qth antenna can be obtained as

$$\begin{aligned} y_{k,q}^{n} &= \mathbf{h}_{k,q}^{n\ T}\left(\sum_{(k',q')\in W_n} \mathbf{w}_{k',q'}^{n} s_{k',q'}^{n}\right) + n_{k,q}^{n} \\ &= \eta_{k,q}^{n} s_{k,q}^{n}\left\|\mathbf{h}_{k,q}^{n}\right\|^2 + \mathbf{h}_{k,q}^{n\ T}\left(\sum_{(k',q')\neq(k,q)} \eta_{k',q'}^{n} s_{k',q'}^{n}\bar{\mathbf{h}}_{k',q'}^{n}\right) + n_{k,q}^{n} \\ &\approx \eta_{k,q}^{n} s_{k,q}^{n}\left\|\mathbf{h}_{k,q}^{n}\right\|^2 + n_{k,q}^{n} \end{aligned} \tag{25}$$

Please note that in writing (25), we have neglected the residual MUI, based on the fact of (21). Since several users can simultaneously share single subchannel, the overall throughput is expected to be increased compared with the SM scheme.

The energy consumption of the SDMA scheme can be calculated as

$$\begin{aligned} E_{SDMA} &= \sum_{n=1}^{N}\sum_{(k,q)\in W^n} \rho_{k,q}^{n} E\left\{\left|s_{k,q}^{n}\right|^2\right\}\left\|\mathbf{w}_{k,q}^{n}\right\|^2 \\ &= \sum_{n=1}^{N}\sum_{(k,q)\in W^n} \rho_{k,q}^{n}\eta_{k,q}^{n\ 2}\left\|\mathbf{h}_{k,q}^{n}\right\|^2 \leq E_{total} \end{aligned} \tag{26}$$

Let $N' = \sum_{n=1}^{N}\sum_{(k,q)\in W^n} \rho_{k,q}^{n}$, be the sum of the number of elements of the subset $\{W^n\}_{n=1,\ldots,N}$. We may determine $\{\eta_{k,q}^{n}\}$ by

$$\eta_{k,q}^{n} = \frac{1}{\left\| \mathbf{h}_{k,q}^{n} \right\|} \sqrt{\frac{E_{total}}{N'}} \tag{27}$$

Substituting (27) into (5), we obtain the number of bits that can be sent to the *k*th RT's *q*th antenna at *n*th subcarrier provided that $(k,q) \in W^n$.

$$m_{k,q}^{n} = \left\lfloor \log_2 \left(\frac{\pi}{\sin^{-1}\left(\left[\frac{1}{\left\| \mathbf{h}_{k,q}^{n} \right\|} \sqrt{\frac{N' N_0}{2E_{total}}} Q^{-1}\left(\frac{\lambda}{2} \right) \right]_{-} \right)} \right) \right\rfloor \tag{28}$$

Henceforth, the throughput of each user yields

$$T_k = \sum_{n=1}^{N} \sum_{(k,q)\in W^n} \rho_{k,q}^{n} m_{k,q}^{n}; \quad k = 1,\ldots,K \tag{29}$$

The algorithm of the proposed SDMA scheme is proceeded in the following:

Step 1: ***Grouping***: Starting from the first subcarrier, the BTS first separates and groups all the spatial signatures $\left\{ \mathbf{h}_{k,q}^{n} \right\}_{q=1,\ldots,Q}$ into a set of quasi-orthogonal subsets based on a predetermined criterion,

$$\left| \mathbf{h}_{k,q}^{n\;H} \mathbf{h}_{k',q'}^{n} \right| \le \alpha \quad ; \forall (k,q) \ne (k',q') \tag{30}$$

where α is a small positive value. We denote the quasi-orthogonal subsets for *n*th subcarrier as $\left\{ W_i^n \right\}_{i=1,\ldots}$.

Step 2: ***Selecting***: Denoting the sum norm of each vector in W_i^n as $\left\| W_i^n \right\|$, the BTS assigns the *n*th subcarrier to all the users in W_i^n, which yields the highest sum norm

$$i^* = \arg\max_i \left\| W_i^n \right\| \tag{31}$$

And we immediately set the subcarrier allocation index for the *n*th subcarrier as

$$\rho_{k,q}^{n} = \begin{cases} 1; (k,q) \in W_{i^*}^{n} \\ 0; \text{otherwise} \end{cases}$$

Step 3: Step 1 and 2 are proceeded until all the subcarriers have been allocated, *i.e.*, $\left\{ W_{i^*}^{n} \right\}_{n=1,\ldots,N}$, $\left\{ \rho_{k,q}^{n} \right\}$ are created and $N' = \sum_{n=1}^{N} \sum_{(k,q)\in W_{i^*}^{n}} \rho_{k,q}^{n}$.

Step 4: Calculate $m_{k,q}^{n}$ using (28), and then calculate the throughput of each user by (29).

Step 5: ***Reallocating***: Starting from the first user, check whether the minimum bit rate constraint is attained. If $T_k \ge R_k$, check for the next user, otherwise, increase the threshold value from α to $\alpha + \Delta$, where Δ is a small positive value, redo step 2~5 until $T_k \ge R_k; k = 1,\ldots,K$.

If all users' bit rates exceed the minimum bit rate constraint, then the subcarrier and bits allocation process has been completed. The overall throughput can be obtained as

$$T = \sum_{k=1}^{K} T_k = \sum_{k=1}^{K} \sum_{n=1}^{N} \sum_{(k,q)\in W_{i^*}^{n}} \rho_{k,q}^{n} m_{k,q}^{n} \tag{32}$$

3.3 Zero-Forcing (ZF) Scheme

The zero-forcing (ZF) scheme is an extension of the SM scheme, in which each subcarrier may be shared by all the RTs. To attain this goal, the weight vector of ZF scheme is selected based on the zero-forcing criterion such that each user's transmission does not interfere with other users' data. Alternatively, a user's own data symbols are spatially multiplexed by the Q receiving antennas. To mitigate MUI, the weight vector should meet the following criterion

$$\mathbf{h}_{k',q'}^{n\ T}\mathbf{w}_{k,q}^{n} = \eta_{k,q}^{n}\delta(k-k',q-q') = \begin{cases}\eta_{k,q}^{n}; k=k', q=q' \\ 0; \text{otherwise}\end{cases} \tag{33}$$

Upon defining the L-by-KQ matrix, $\mathbf{H}^n \equiv \begin{bmatrix}\mathbf{H}_1^n & \mathbf{H}_2^n & \cdots & \mathbf{H}_K^n\end{bmatrix}$, (33) can be rewritten as a compact form

$$\mathbf{H}^{nT}\mathbf{w}_{k,q}^{n} = \eta_{k,q}^{n}\mathbf{e}_{(k-1)Q+q} \tag{34}$$

where $\mathbf{e}_{(k-1)Q+q}$ denotes the $((k-1)Q+q)$th column vector of $\mathbf{I}_{KQ}$. If the array size of BTS satisfies $L \geq KQ$, which is usually the case in massive MIMO [11,12] system, we have infinitely many solutions since (34) is an underdetermined system. Assuming that $\mathbf{H}^n$ is full column rank, then the minimum-norm solution of (34) can be obtained as

$$\mathbf{w}_{k,q}^{n} = \eta_{k,q}^{n}\mathbf{H}^{n}\left[\mathbf{H}^{nT}\mathbf{H}^{n}\right]^{-1}\mathbf{e}_{(k-1)Q+q} \tag{35}$$

The received signal at the kth RT's qth receiver yields

$$y_{k,q}^{n} = \mathbf{h}_{k,q}^{n\ T}\left(\sum_{l=1}^{K}\sum_{r=1}^{Q}s_{l,r}^{n}\mathbf{w}_{l,r}^{n}\right) + n_{k,q}^{n} = \eta_{k,q}^{n}s_{k,q}^{n} + n_{k,q}^{n} \tag{36}$$

As revealed by (36), though multiple users are simultaneously transmitted using the same subcarrier, only the desired signal retains. Consequently, the MUI is completely removed by transmit beamforming and each subcarrier can be assigned simultaneously to all the user terminals.

Since for arbitrary n, all user terminals are allowed to share a subchannel at the same time, hence, the subcarrier allocation index for the ZF scheme is

$$\rho_{k,q}^{n} = 1; \forall k = 1,\ldots,K, n = 1,\ldots,N, q = 1,\ldots,Q \tag{37}$$

The energy consumption of the ZF scheme can be calculated from (37) and (35) as

$$\begin{aligned} E_{ZF} &= \sum_{k=1}^{K}\sum_{n=1}^{N}\sum_{q=1}^{Q}\rho_{k,q}^{n}\left\|\mathbf{w}_{k,q}^{n}\right\|^{2} \\ &= \sum_{k=1}^{K}\sum_{n=1}^{N}\sum_{q=1}^{Q}\eta_{k,q}^{n\ 2}\left\|\mathbf{H}^{n}\left[\mathbf{H}^{nT}\mathbf{H}^{n}\right]^{-1}\mathbf{e}_{(k-1)Q+q}\right\|^{2} \\ &= \sum_{k=1}^{K}\sum_{n=1}^{N}\sum_{q=1}^{Q}\eta_{k,q}^{n\ 2}\left[\mathbf{H}^{nT}\mathbf{H}^{n}\right]^{-1}\left((k-1)Q+q,(k-1)Q+q\right) \end{aligned} \tag{38}$$

Since E_{ZF} should be upper-bounded by E_{total}, we may select $\{\eta_{k,q}^{n}\}$ as

$$\eta_{k,q}^{n} = \sqrt{\frac{E_{total}}{KNQ\left[\mathbf{H}^{nT}\mathbf{H}^{n}\right]^{-1}\left((k-1)Q+q,(k-1)Q+q\right)}} \tag{39}$$

Substituting the symbol energy $E_{k,q}^{n} = E\left\{\left|\eta_{k,q}^{n}s_{k,q}^{n}\right|^{2}\right\} = \eta_{k,q}^{n\ 2}$ and (39) into (5), we obtain the allowable bits that can be sent to user k's qth antenna at subcarrier n as

$$m_{k,q}^{n} = \left\lfloor \log_2\left(\frac{\pi}{\sin^{-1}\left(\left[\sqrt{\frac{KNQN_0\left[\mathbf{H}^{nT}\mathbf{H}^{n}\right]^{-1}\left((k-1)Q+q,(k-1)Q+q\right)}{2E_{total}}}Q^{-1}\left(\frac{\lambda}{2}\right)\right]_{-}\right)}\right)\right\rfloor \tag{40}$$

Exploiting (40), the overall throughput of the ZF scheme can be calculated by

$$T_{ZF} = \sum_{k=1}^{K} T_k = \sum_{k=1}^{K}\sum_{n=1}^{N}\sum_{q=1}^{Q} m_{k,q}^{n} \tag{41}$$

However, in heavy-loaded or bursty traffic system, the assumption that $L \geq KQ$ no longer satisfies. Complete MUI suppression is not possible, we propose two algorithms to cope with this problem in the preceding section.

IV. MULTIUSER RESOURCE ALLOCATION IN THE CASE OF $L<KQ$

4.1 Selected ZF (SZF) Scheme

Consider the case $L < KQ$, it is not possible to use ZF scheme since (34) becomes an overdetermined system and the matrix $\mathbf{H}^{nT}\mathbf{H}^{n}$ in (35) is now singular. To make the ZF scheme feasible, one needs to select L out of the KQ column vectors of $\mathbf{H}^{n}$. We refer it as the selected ZF (SZF) scheme. For each subcarrier, the optimum SZF algorithm first requires to generate $\binom{KQ}{L}$ square matrices. Each possible matrix is substituted into (40) and (41) to evaluate the corresponding throughput, and the best (largest) one is chosen subject to the minimum rate constraint. However, since the optimum solution can only be found with exhaustive search, it is computationally prohibitive for most practical values of L and (or) KQ.

The objective of the proposed SZF algorithm is to allocate each subcarrier to L spatial signatures selected from $\mathbf{H}^{n}$ such that the throughput is maximized with reduced complexity. Such is accomplished as follows:

Step 1: Initialization

For each subcarrier (start from the first subcarrier, n=1), the BTS first assigns the subcarrier to the vector channel with the highest gain, which corresponds to the maximum bits to be transmitted, among all users and all antennas in that subchannel. Let W be the joint user antenna index set, $W = \{(k,q)\}_{\substack{k=1,2,\ldots,K \\ q=1,2,\ldots,Q}}$, we have

$$\left(k^*, q^*\right) = \arg\max_{(k,q)\in W} \left\|\mathbf{h}_{k,q}^{n}\right\| \tag{42}$$

Step 2: Set i=1,

$$W^{(1)} = W \setminus \left(k^*, q^*\right)$$

Set $\mathbf{h}^{(1)} = \mathbf{h}_{k^*,q^*}^{n}$ and construct $\mathbf{H}^{(1)} = \left[\mathbf{h}^{(1)}\right]$

Step 3: While $i < L$, set $i=i+1$

The BTS chooses each additional spatial signature based on the criterion that the throughput is maximized after the new spatial signature is added:

$$\left(k^*, q^*\right) = \arg\max_{(k,q)\in W^{(i)}} T\left(\left[\mathbf{H}^{(i)} \quad \mathbf{h}_{k,q}^{n}\right]\right) \tag{43}$$

where the objective function is as defined in (40), (41). As revealed by (43), we use total throughput for the considered subcarrier as the design metric, hence, the spatial signature which leads to maximum throughput is added. It is evident that maximizing $T\left(\left[\mathbf{H}^{(i)} \quad \mathbf{h}_{k,q}^{n}\right]\right)$ is equivalent to minimizing the trace (sum of the diagonal elements) of $\left(\left[\mathbf{H}^{(i)} \quad \mathbf{h}_{k,q}^{n}\right]^T \left[\mathbf{H}^{(i)} \quad \mathbf{h}_{k,q}^{n}\right]\right)^{-1}$. In what follows, we may use the following criterion instead of (43)

$$\left(k^*, q^*\right) = \arg\min_{(k,q)\in W^{(i)}} tr\left\{\left(\left[\mathbf{H}^{(i)} \quad \mathbf{h}_{k,q}^{n}\right]^T \left[\mathbf{H}^{(i)} \quad \mathbf{h}_{k,q}^{n}\right]\right)^{-1}\right\} \tag{44}$$

After the new spatial signature is assigned to the considered subcarrier, the BTS updates the record as

$$W^{(i+1)} = W^{(i)} \setminus \left(k^*, q^*\right)$$

Set $\mathbf{h}^{(i+1)} = \mathbf{h}^{n}_{k^*,q^*}$ and construct $\mathbf{H}^{(i+1)} = \left[\mathbf{H}^{(i)} \quad \mathbf{h}^{(i+1)}\right]$

Step 4: Calculation

Step 2 is repeated until the *L*-by-*L* matrix, $\mathbf{H}^{(L)}$, is constructed, the BTS records the indices of the column (channel) vectors, $W^n \subset W$. Therefore, the bits that can be sent by the *l*th channel can then be calculated as

$$m^n_{k,q} = \left\lfloor \log_2\left(\frac{\pi}{\sin^{-1}\left(\left[\sqrt{\frac{NLN_0\left[\mathbf{H}^{(L)T}\mathbf{H}^{(L)}\right]^{-1}(l,l)}{2E_{total}}}Q^{-1}\left(\frac{\lambda}{2}\right)\right]_{-}\right)} \right) \right\rfloor \tag{45}$$

Henceforth, the throughput of each user yields

$$T^n_k = \sum_{i=1}^{n} \sum_{(k,q)\in W^n} m^i_{k,q}; \qquad k = 1,\ldots,K \tag{46}$$

Step 5: Based on the results obtained from (46), the BTS checks whether user k^* meets the minimum bit rate constraint. If $T^n_{k^*} < R_{k^*}$, which means more subchannels should be assigned to user k^*, go back to step 1 and check for the next subcarrier. On the other hand, if $T_{k^*} \geq R_{k^*}$, we temporarily exclude user k^*

$$W = W \setminus \left\{\left(k^*, q\right)\right\}_{q=1,\ldots,Q}$$

Set*n*=*n*+1, go back to step 1 to assign the subchannels to those users that have not attained the minimum bit rate constraint.

Step 6: As long as all the users meet the minimum bit rate constraint, remove all the exclusions, redo step 1 until all the subcarriers have been allocated.

Iterative method of evaluating (44)

In view of the SZF algorithm, a lot of matrix inverse are required to implement (44). The complexity hinders the applicability of this scheme. We aim at developing an iterated method to reduce the load of computations. To begin with, we define the (*i*+1)-by-(*i*+1) matrix, $\mathbf{A}^{(i+1)}$, as

$$\mathbf{A}^{(i+1)} \equiv \left[\mathbf{H}^{(i)} \quad \mathbf{h}^{(i+1)}\right]^T \left[\mathbf{H}^{(i)} \quad \mathbf{h}^{(i+1)}\right] = \begin{bmatrix} \mathbf{H}^{(i)T}\mathbf{H}^{(i)} & \mathbf{H}^{(i)T}\mathbf{h}^{(i+1)} \\ \mathbf{h}^{(i+1)T}\mathbf{H}^{(i)} & \mathbf{h}^{(i+1)T}\mathbf{h}^{(i+1)} \end{bmatrix} \tag{47}$$

Upon defining $\mathbf{a}^{(i+1)} = \mathbf{H}^{(i)T}\mathbf{h}^{(i+1)}$, (47) can be rewritten into the recursive form

$$\mathbf{A}^{(i+1)} = \begin{bmatrix} \mathbf{A}^{(i)} & \mathbf{a}^{(i+1)} \\ \mathbf{a}^{(i+1)T} & \left\|\mathbf{h}^{(i+1)}\right\|^2 \end{bmatrix} \tag{48}$$

Exploiting the matrix inversion lemma [Chap. 4 of [8]], and the method proposed in [15], we have

$$\left(\mathbf{A}^{(i+1)}\right)^{-1} = \begin{bmatrix} \left(\mathbf{A}^{(i)}\right)^{-1} & \mathbf{0}_i \\ \mathbf{0}_i^T & 0 \end{bmatrix} + \frac{1}{\left(\left\|\mathbf{h}^{(i+1)}\right\|^2 - \mathbf{a}^{(i+1)T}\mathbf{b}^{(i+1)}\right)} \begin{bmatrix} \mathbf{b}^{(i+1)}\mathbf{b}^{(i+1)T} & -\mathbf{b}^{(i+1)} \\ -\mathbf{b}^{(i+1)T} & 1 \end{bmatrix} \tag{49}$$

where $\mathbf{b}^{(i+1)} \equiv \left(\mathbf{A}^{(i)}\right)^{-1}\mathbf{a}^{(i+1)}$, $\mathbf{0}_i$ is a zero vector with size *i*. As we can observe from (49), each time when we attempt to calculate $\left(\mathbf{A}^{(i+1)}\right)^{-1}$ in (44), $\left(\mathbf{A}^{(i)}\right)^{-1}$ is already available. The complexity can be extensively reduced.

4.2 Least-Squares (LS) algorithm

Assuming that $\mathbf{H}^n$ is full (row) rank, the least-squares (LS) solution [8] of an overdetermined system of (34) becomes

$$\mathbf{w}_{k,q}^n = \eta_{k,q}^n \left[\mathbf{H}^n\mathbf{H}^{nT}\right]^{-1}\mathbf{H}^n\mathbf{e}_{(k-1)Q+q} \tag{50}$$

It follows that the energy consumption of the LS scheme can be calculated from (50) as

$$\begin{aligned} E_{LS} &= \sum_{k=1}^{K}\sum_{n=1}^{N}\sum_{q=1}^{Q}\rho_{k,q}^n\left\|\mathbf{w}_{k,q}^n\right\|^2 \\ &= \sum_{k=1}^{K}\sum_{n=1}^{N}\sum_{q=1}^{Q}\eta_{k,q}^{n\ 2}\left\|\left[\mathbf{H}^n\mathbf{H}^{nT}\right]^{-1}\mathbf{H}^n\mathbf{e}_{(k-1)Q+q}\right\|^2 \\ &= \sum_{k=1}^{K}\sum_{n=1}^{N}\sum_{q=1}^{Q}\eta_{k,q}^{n\ 2}\mathbf{M}^n\left((k-1)Q+q,(k-1)Q+q\right) \end{aligned} \tag{51}$$

where $\mathbf{M}^n \equiv \mathbf{H}^{nT}\left[\mathbf{H}^n\mathbf{H}^{nT}\right]^{-2}\mathbf{H}^n$. Since E_{LS} should be upper-bounded by E_{total}, we may select $\left\{\eta_{k,q}^n\right\}$ as

$$\eta_{k,q}^n = \sqrt{\frac{E_{total}}{KNQ\mathbf{M}^n\left((k-1)Q+q,(k-1)Q+q\right)}} \tag{52}$$

The received signal at the kth RT's qth receiver is

$$\begin{aligned} y_{k,q}^n &= \mathbf{h}_{k,q}^{n\ T}\left(\sum_{l=1}^{K}\sum_{r=1}^{Q}s_{l,r}^n\mathbf{w}_{l,r}^n\right)+n_{k,q}^n \\ &= \eta_{k,q}^n s_{k,q}^n + \sum_{l=1}^{K}\sum_{\substack{r=1 \\ (l,r)\neq(k,q)}}^{Q} s_{l,r}^n i_{l,r}^n + n_{k,q}^n \end{aligned} \tag{53}$$

where $i_{l,r}^n \equiv \mathbf{h}_{k,q}^{n\ T}\mathbf{w}_{l,r}^n$ is the residual MUI. The signal-to-interference-plus-noise ratio (SINR) can be obtained from (53) as

$$\gamma_{k,q}^n = \frac{\eta_{k,q}^{n\ 2}}{\sum_{l=1}^{K}\sum_{\substack{r=1 \\ (l,r)\neq(k,q)}}^{Q}\left|i_{l,r}^n\right|^2+\frac{N_0}{2}} = \frac{\dfrac{E_{total}}{KNQ\mathbf{M}^n\left((k-1)Q+q,(k-1)Q+q\right)}}{\sum_{l=1}^{K}\sum_{\substack{r=1 \\ (l,r)\neq(k,q)}}^{Q}\left|i_{l,r}^n\right|^2+\frac{N_0}{2}} \tag{54}$$

The allowable bits that can be sent to user k's qth antenna at subcarrier n can be obtained as

$$m_{k,q}^n = \left\lfloor \log_2\left(\frac{\pi}{\sin^{-1}\left(\left[\frac{1}{\sqrt{\gamma_{k,q}^n}}Q^{-1}\left(\frac{\lambda}{2}\right)\right]_{-}\right)}\right)\right\rfloor \tag{55}$$

Exploiting (55), the overall throughput of the LS scheme can be calculated by

$$T_{LS} = \sum_{k=1}^{K}T_k = \sum_{k=1}^{K}\sum_{n=1}^{N}\sum_{q=1}^{Q}m_{k,q}^n \tag{56}$$

In summary, the main difference between SZF and LS schemes resides in how they deal with MUI. SZF relies on complete MUI suppression, thus a maximum of L spatial signatures are chosen to carry information bits. While in LS scheme, all the KQ spatial signatures convey information bits, nevertheless, the inevitable residual MUI may degrade SER as well as the bits that can be sent through each channel.

V. PERFORMANCE EVALUATION

We set the target QoS (SER upper bound) to be $\lambda = 10^{-4}$ throughout all the simulation examples. For each subcarrier, we generate the channel vector $\mathbf{h}_{k,q}^n = \sqrt{\alpha_{k,q}^n}\mathbf{g}_{k,q}^n$, in which $\mathbf{g}_{k,q}^n$ denotes small-scale fading and $\alpha_{k,q}^n$ denotes large-scale fading coefficient (lognormal distribution and geometric decay). We assume $\mathbf{g}_{k,q}^n \sim \mathbb{C}N\left(\mathbf{0},\mathbf{I}_L\right)$ and are statistically independent across users. The result in each simulation example is obtained from the average of 100 independent trials. Unless otherwise mentioned, we set the total energy consumption constraint at the BTS is set such that the energy to noise ratio for each user is $\frac{E_{total}}{NN_0} = 10dB$. The predetermined threshold, α, and step size, Δ, for the SDMA scheme is set to be 1 and 0.5, respectively. To treat users fairly, we set equal rate constraint on each user, *i.e.*, $R_1 = R_2 = \cdots = R_K = R$ bits per OFDM symbol. A plausible criterion to measure the performance of the subcarriers and bits allocation algorithms is the bandwidth (spectral) efficiency, which is defined as the ratio of the data rate in bits per second to the effectively utilized bandwidth. In this paper, we define the bandwidth efficiency as the overall throughput normalized by the total number of subcarriers

$$c \equiv \frac{T}{N} = \frac{\sum_{k=1}^{K}\sum_{n=1}^{N}\sum_{q=1}^{Q}\rho_{k,q}^{n} m_{k,q}^{n}}{N} \tag{57}$$

In the first simulation example, we aim at comparing the performance of the proposed SZF with the optimum SZF schemes, where the number of subcarriers, the number of RTs and the array size at each RT are *N*=64, *K*=10 and *Q*=2, respectively. Fig. 2 presents the bandwidth efficiency with respect to the number of BTS antennas (varying from *L*=4 to *L*=16), where the SDMA and SM schemes are also provided for comparison. As depicted in Fig. 2, the bandwidth efficiency of the SZF scheme increases in accordance with larger *L*, whereas, the SDMA and SM schemes are insensitive to *L*. It results from that the SM scheme is essentially based on single-user algorithm. Somewhat surprisingly, the SM scheme outperforms SDMA scheme. It may due to the fact that the number of "nearly orthogonal" channel vectors is rare in case that *L* is small. We have also verified from Fig. 2 that the proposed iterated SZF scheme is close to optimum SZF scheme, nevertheless, the complexity of the proposed algorithm is extensively reduced.

Fig. 3 presents the bandwidth efficiency with respect to the number of user terminals (ranging from *K*=3 to *K*=45), where *N*=300, and the array size at the BTS and each RT are *L*=60 and *Q*=3, respectively. As shown in the figure, the bandwidth efficiency is in ascending order from SM scheme to SDMA scheme, then LS scheme, and SZF scheme is the best. Please note that as *K*<20, (*i.e.*, *KQ*<*L*), LS as well as SZF schemes coincide with the regular ZF, which is applied in underdetermined system. We can also observe that the performance of the SZF scheme increases rapidly in accordance with *K* while LS scheme slightly degrades for larger *K*. This arises from the fact that both algorithms allow multiple user terminals to share a subchannel at the same time. Thereby, larger *K* extensively increases the throughput for SZF scheme, whereas the residual MUI resulted from larger *K* degrades the performance of the LS scheme. Fig. 4 compares the bandwidth efficiency with respect to the number of BTS antennas, where *N*=256, *K*=20, and *Q*=5. As revealed by Fig. 4, the SM and SDMA schemes are insensitive to the variation of *L*, nevertheless, performance discrepancy between the multiuser algorithms (SZF and LS) and other schemes is obvious for larger *L*. This may due to the facts that larger *L* corresponds to larger degrees-of-freedom to suppress MUI. Specifically, Fig. 4 reveals that LS outperforms SZF scheme when *L* is small. This is due to the fact that a maximum of *L* spatial signatures are chosen to carry information bits in SZF, whereas LS allows all the *KQ* channels to send information bits. Fig. 5 presents the bandwidth efficiency with respect to $\frac{E_{total}}{NN_0}$ (ranging from 6 to 15 dB), where *N*=256, *K*=20, *Q*=5 and *L*=60, respectively.As revealed by the figure, only SZF scheme has obvious improvement by increasing the dedicated power at each subchannel. The aim of the final simulation example is to evaluate performance improvement invoked by the spatial diversity at the RT. We present the bandwidth efficiency with respect to the number of RT's antennas (ranging from *Q*=1 to *Q*=19) in Fig. 6, where *N*=400, *K*=20, and *L*=200, respectively.As verified by the numerical results, the bandwidth efficiency of the SM and SZF schemes has obvious improvement by inducing larger antenna array at the RT.Please note that as*Q*<10, LS as well as SZF schemes coincide with the regular ZF.The LS scheme, though simple in structure, suffers from MUI for larger Q. Specifically, throughput for the SZF scheme decreases as $Q \approx 10$. This may result from ill-condition for matrix $\mathbf{H}_k^n$ as *KQ* approaches *L*.As a whole, the proposed SZF scheme outperforms the other three schemes to a large extent under different scenarios.

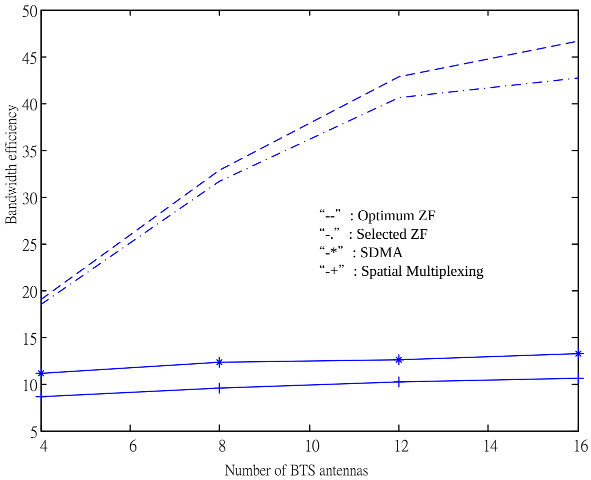

FIG. 2: Bandwidth efficiency with respect to the number of BTS antennas.

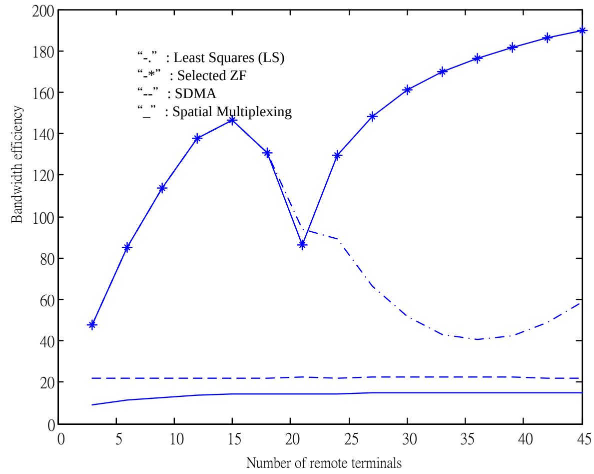

FIG. 3: Bandwidth efficiency with respect to the number of RTs, *N*=300, *L*=60 and *Q*=3.

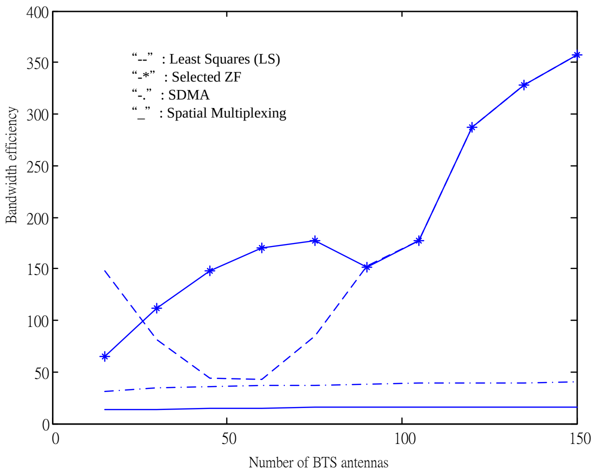

FIG. 4: Bandwidth efficiency with respect to the number of BTS antennas, *N*=256,*K*=20, *Q*=5.

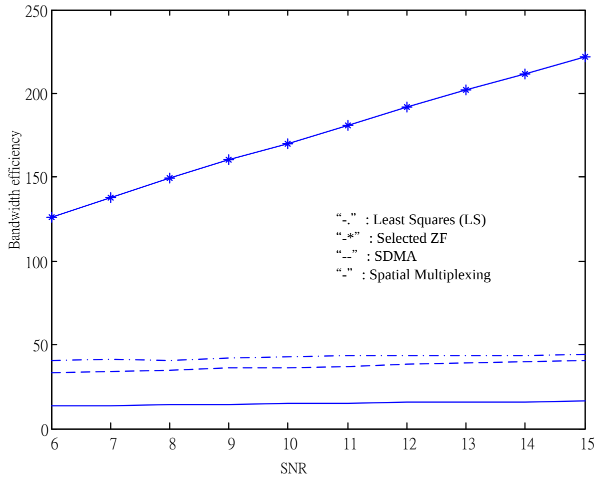

FIG. 5: Bandwidth efficiency with respect to $\frac{E_{total}}{NN_0}$, where *N*=256, *K*=20, *Q*=5 and *L*=60.

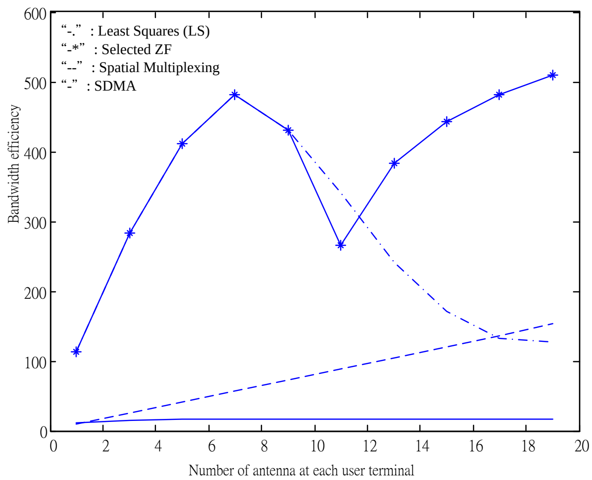

FIG. 6: Bandwidth efficiency with respect to the number of RT's antennas, where *N*=400, *K*=20 and *L*=200.

VI. CONCLUSION

In this paper, we have proposed four resource allocation algorithms in OFDMA based multiuser MIMO system, namely, SM, SDMA, SZF and LS schemes. All the proposed algorithms are channel aware in the sense that they adapt to the channel conditions. Specifically, the SDMA and SZF algorithms judiciously select co-channel users based on their spatial signatures to enable significant improvement in the system throughput. Analytical and simulation results demonstrated that employing the antenna array at the BTS and RT has extensively increased the spectral efficiency. Furthermore, since the proposed algorithms are simple yet reliable, it is plausible to apply it for resource allocation in practical multiuser OFDMA-based LTE system, where multiple antenna transmission and reception are available.

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