Adaptive Nonparametric CUSUM Control Chart with Variable Sampling Interval Strategy

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Abstract— *In this paper, we propose a nonparametric CUSUM control chart for detecting a range of shifts in the location parameters based on previous research. This control chart is dynamically adaptive, ranks method-based nonparametric and self-starting; it can monitor various sizes of shifts in difference distributions simultaneously; and it can be used to monitor processes at the start-up stages. This control chart is designed with variable sampling interval technology, which makes it more intelligent and sensitive. Simulation study of reference parameters values and performance comparisons are introduced in detail, so as to conveniently apply this chart to practical production process monitoring. An illustrative chemical example is also present to demonstrate the well implementation of this chart.*

Keywords— Variable sampling interval, Adaptive control chart, CUSUM, Nonparametric scheme.

I. INTRODUCTION

Statistical process control (SPC) is an important quality control tool that uses statistical methods to monitor and control processes. One major concern of SPC is to analyze the stability of a production process, and to give a warning signal when the process has some changes, that is, the process has shifts happened. The control performances of a control chart are generally evaluated with the average run length (ARL), which denotes the average number of observations from the process beginning to giving a warning signal. A control chart with a smaller ARL when the process has a shift is better, under the condition of a same ARL when the process is steady. This technology has currently been adopted in many fields, such as health care, finance, biotechnology and chemical engineering. More introductions about SPC technology and its applications can be found in, for example, Lucas and Saccucci (1990) [1], Thomas and William (1991) [2], Woodall (2000) [3], Montgomery (2009) [4], Zou et al. (2009) [5], Shardt et al. (2012) [6].

CUSUM (Page 1954) [7] is an effective control chart for detecting small and moderate shifts; it is proposed based on the sequential probability ratio test. The conventional CUSUM control chart is generally designed by assuming known the size of shifts. However, to predetermine the exact size of shifts is a difficult thing in most practical process settings. In order to detect both small and large shifts, Lucas (1982) [8] proposed a combined Shewhart-CUSUM chart which is designed by way of adding the Shewhart control limit to a conventional CUSUM control chart. Where after, the emergence of the adaptive control chart resolves this problem significantly. The main design idea of adaptive charts is that the parameters value setting is decided by sample observations, so that the chart can dynamically monitor various sizes of shifts. Sparks (2000) [9] proposed an adaptive CUSUM chart by an one-step-ahead formula to dynamically forecast the parameters values of the CUSUM chart. Shu and Jiang (2006) [10] calculated the chart ARLs of Sparks (2000) by Markov-chain. With that Shu et al. (2008) [11] proposed another adaptive CUSUM chart by improving the dynamically forecasting tool that Sparks (2000) used. Other adaptive charts are discussed by Wu et al. (2009) [12], Li and Wang (2010) [13], Capizzi and Masarotto (2012) [14], among others.

However, these control charts mentioned above often assume that sample observations follow a known parametric distribution, most commonly follow normal distribution. But in most practical processes, the distribution form of sample observations is unknown or not normal. Through the literature we known some nonparametric CUSUM control charts can monitor processes with unknown sample distribution form. For example, Bakir and Reynolds (1979) [15] proposed a nonparametric CUSUM chart based on the Wilcoxon signed-ranks statistic. McDonald (1990) [16] proposed another nonparametric CUSUM chart based on sequential ranks of observations. Qiu and Hawkins (2001) [17] discussed a multivariate nonparametric CUSUM chart based on the sequential ranks method that McDonald used. More recently, based on ranks, Liu et al. (2014) [18] proposed an adaptive nonparametric CUSUM (ANC) chart for detecting variables which shift size and distribution form are both unknown.

Variable sampling interval (VSI) control charts can intelligently change sampling intervals according to sampling results, so as to improve the efficiency of process monitoring. There are also some examples of CUSUM control charts with VSI technology. Luo et al. (2009) [19] proposed an adaptive CUSUM chart with VSI technology. Li and Qiu (2014) [20] take a brief overview about a p-value CUSUM control charts with VSI features. Other charts with VSI features can be found in Reynolds et al. (1988) [21], Prabhu et al. (1993) [22], Zhou et al. (2012) [23], Liu et al. (2015) [24], among others. In general, a chart with VSI feature detects process shifts faster than fixed sampling interval (FSI) charts. However, the VSI technology is rarely used in nonparametric control charts. Control chart with VSI technology is a challenge when the size of shifts and the underlying distribution form are both unknown. To increase the detecting efficiency of shifts from conventional CUSUM control charts, we will discuss an adaptive nonparametric procedure with a VSI technology based on the conventional two-side CUSUM control chart.

The remainder of this article is organized as follows. In Section 2, we provide a brief introduction to some basic methods and the VSI-ANC chart. In Section 3, we discuss some properties of the VSI-ANC chart and give a performance comparison. An example to demonstrate well implementation with a real dataset of a chemical process is presented in Section 4. In the last Section, we provide several concluding remarks.

II. METHODS FOR VSI-ANC CHART

In this section, we provide a brief introduction to some basic methods and the VSI-ANC chart.

2.1 Conventional Two-sided CUSUM Statistic

The CUSUM control chart, which is derived by Page (1954) [7], is based on the sequential probability ratio test, and it is efficient in detecting small and moderate shifts. Suppose that X_t is the observation of the tth sample. Let the the probability density function of X_t be denoted by $f(X_t;\theta)$, where θ denotes the interesting process parameter. To detect a change in θ, let θ_0 and θ_1 represent the in-control(IC) and out-of-control(OC) parameters, respectively. The CUSUM chart is defined as follows:

$$C_t = \max(0, C_{t-1} + Z_t), \tag{1}$$

in which the initial value is $C_0 = \mu$, where μ is the mean value of observation X_t, typically set as 0, and Z_t is the log-likelihood ratio statistic.

$$Z_t = \log \frac{f(X_t;\theta_1)}{f(X_t;\theta_0)}, \tag{2}$$

When the process observation X_t is assumed to be independent identically distributed (i.i.d.) random samples and follows a normal distribution which mean and variance are 0 and σ respectively, the two-side CUSUM statistic is:

$$\begin{aligned} S_t^+ &= \max(0, S_{t-1}^+ + X_t - k) \\ S_t^- &= \min(0, S_{t-1}^- + X_t - k) \quad , \\ S_t &= \max(S_t^+, |S_t^-|) \end{aligned} \tag{3}$$

with the initial value $S_0^+ = S_0^- = 0$. According to a specified IC ARL, we select a suitable control limit h, and then give a warning signal when $S_t > h$. It has been demonstrated is that a conventional CUSUM statistic with $k = \frac{\delta}{2}$ is optimal to detect a process which shift size is equal to δ.

2.2 The Adaptive Method and Nonparametric Ranks-based Method

We assume that the processes are expected to detect mean shifts. The CUSUM statistic can only be optimized if we can predetermine the exact sizes of the shift. Sparks (2000) [9] suggested an one-step-ahead formula as a forecasting tool to estimate the shift δ at each time t, which can be expressed as:

$$\hat{\delta}_t = (1-\lambda)\hat{\delta}_{t-1} + \lambda X_t , \tag{4}$$

where $0 < \lambda < 1$ is a smoothing parameter. The advantage of this formula is dynamically evaluating the shifts size step-by-step, so as to make the CUSUM chart obtain the optimal performance.

When the specific distribution form of sample observation $X_t , t = 1,2,3\cdots$ is unknown, the ranks-based method is good for addressing these online nonparametric problems. Suppose that observations $X_t , t = 1,2,3\cdots$ are i.i.d..We rank the sample observations by size from small to large. Then, we denote R_t as the sequential rank of X_t, which can be expressed as follows:

$$R_t = \sum_{i=1}^{t} I\{x_t \geq x_i\}. \tag{5}$$

It is clear that the distribution of R_t, which varies as t increases, is always a uniform distribution. Consequently, regardless of the distribution that X_t follows, we can monitor the process by R_t, which is the substitute for X_t.

Standardizing the sequential rank R_t:

$$R_t^* = \frac{R_t - E\,R_t}{\sqrt{Var\,R_t}} , t \geq 2 , \tag{6}$$

where $E\,R_t = (t+1)/2$ and $Var\,R_t = (t+1)(t-1)/12$, it can be easily confirmed in the IC situation. So, we can use the distribution is always uniformed standardized sequential ranks R_t^* in place of unknown distributions sample observations X_t.

2.3 Variable Sampling Interval Method

The design concept of VSI method, which is proposed by Reynolds et al.(1988) [21], is that the sampling intervals between current and next sample should be set to short if the samples exhibit some sign of a change and should be set to long if there is no sign of changes. In other words, when the (t-1)th statistic S_{t-1} is close to but in fact not out-of the control limit, we should sample the tth sample as soon as possible, that is, shorten the sampling interval and determine whether the production process is normal. When the(t-1)th statistic S_{t-1} is close to the target value, we have a reason to wait for a long period of time to sample the next sample, that is, increase the sampling interval to avoid unnecessary waste. Therefore, we choose a warning limit ω to segment the control area, and the sampling interval T_t is assumed to be:

$$T_t = \begin{cases} d_1 \ , & \omega \leq |S_{t-1}| \leq h, \\ d_2 \ , & |S_{t-1}| < \omega, \end{cases} \tag{7}$$

where h is the control limit and ω is a warning limit. We select the shorter sampling interval d_1 when the control chart statistics are above the warning limit and below the control limit, whereas we use the longer sampling interval d_2 when the control chart statistics are below the warning limit.

The time to give a warning signal of the VSI charts is related to sampling intervals, which is no longer a constant number, so we cannot simply use VSI charts compare with FSI charts by ARL. Here, we use ATS and AATS to evaluate the performance of the VSI control chart. ATS denotes the average run time from the process starting to giving a warning signal, when the process is IC. AATS denotes the average run time from shifts occurring to giving a warning signal.

To obtain the possibility of comparing our VSI-ANC chart with other FSI charts, we record the average sampling interval from the process starting to give a warning signal as:

$$E(T_t) = P(\omega < |S_{t-1}| < h)d_1 + P(|S_{t-1}| < \omega)d_2, \tag{8}$$

Without loss of generality, we can choose a pair of suitable ω and h to obtain $E(T_t) = 1$ when the process is IC, the ATS of a VSI chart can be defined as:

$$\text{ATS} = (\text{IC ARL}) \times E(T_t) = (\text{IC ARL}) \times 1 = \text{IC ARL}. \tag{9}$$

Similarly, when the process is OC, we record the average sampling interval from shifts occurring to giving a warning signal as $E(T_t)$; simultaneously, we select suitable h, ω, d_1, and d_2 to obtain $E(T_t) = 1$. Then, the AATS can be defined as:

$$\text{AATS} = (\text{OC ARL}) \times E(T_t) = (\text{OC ARL}) \times 1 = \text{OC ARL}. \tag{10}$$

Consequently, we do not distinguish between ATS and IC ARL as well as AATS and OC ARL in this paper. This mean that, for all the control charts, a chart with smaller AATS is performance better than the others in the same ATS.

2.4 The VSI-ANC Chart

The VSI-ANC chart combined with adaptive, nonparametric and VSI features can efficiently detect various sizes of shifts in different distributions. Inspired by Liu et al. (2014) [18], we set our statistics as follows:

$$\begin{aligned} S_t^+ &= \max\left(0,\ S_{t-1}^+ + \left(R_t^* - \frac{\delta_t^+}{2}\right) \Big/ h\left(\frac{\delta_t^+}{2}\right)\right), \\ S_t^- &= \min\left(0,\ S_{t-1}^- + \left(R_t^* - \frac{\delta_t^-}{2}\right) \Big/ h\left(-\frac{\delta_t^-}{2}\right)\right), \\ S_t &= \max\left(S_t^+, |\ S_t^-\ |\right). \end{aligned} \tag{11}$$

where R_t^* is the standardized sequential rank of observation X_t, which is i.i.d. but the distribution form is unknown. δ_t^+, δ_t^- are the upward and downward evaluated value of shifts size respectively, which can be defined as follows:

$$\begin{aligned} \hat{\delta}_t^+ &= \max(\delta_0, \hat{\delta}_t) \\ \hat{\delta}_t^- &= \min(-\delta_0, \hat{\delta}_t) \end{aligned} \tag{12}$$

Here, we are not interested in too small shifts, so we add a threshold $\delta_0 > 0$ to improve the ability of the chart to detect $\delta > \delta_0$ and $\delta < -\delta_0$. $\hat{\delta}_t$ should be a small value when the process is IC or when a tiny shift occurs, and should increase rapidly when there are some large shifts occur, and it should be an one-step-ahead estimate of δ_{R^*} (δ_{R^*} is related to δ_X, but not a linear relationship with δ_X). Therefore, we choose the moving average of recent m R_t^* as the expression of $\hat{\delta}_t$,

$$\hat{\delta}_t = \sum_{i=1}^{m} R_{t-i+1}^* / m. \tag{13}$$

Obtained by simulation, we also suggest setting m=2 and $\delta_0 = 0.7$ to get the best performance for both small and large shifts in IC ARL=400.

$h(*)$ is an operating function which denotes the control limit. In the adaptive procedure, the control limit h changes with the evaluated values of shifts under fixed IC ARL. So the statistical equation should be divided by control limit function $h(*)$ in both sides. Liu et al. (2014) [18] used a polynomial model of order 8 to acquire the control limit function $h(*)$ with fixed IC ARL, their results are presents in Table 1. When we design this chart, we can select the appropriate control limit function in the light of the IC ARL in column 1 of Table 1.

TABLE 1

POLYNOMIAL MODELS $\mathrm{h}(k)=\sum_{i=0}^{8}(-1)^i a_i k^i$ **FOR CONVENTIONAL TWO-SIDED NONPARAMETRIC CUSUM CHART**

	Coefficients of $h(k)$									
IC-ARL	a_8	a_7	a_6	a_5	a_4	a_3	a_2	a_1	a_0	MSE
200	315.248943	1451.51618	2916.82195	3368.70868	2478.04108	1216.95083	409.073791	98.2896235	17.8433751	0.0044
300	1174.61785	5039.68062	9288.90317	9650.93878	6248.17159	2640.56468	747.856101	148.101804	22.1700620	0.0053
400	1659.02624	7029.95996	12786.5656	13087.2063	8318.78097	3432.20738	941.036988	177.995350	25.0063301	0.0055
500	3120.61219	13089.5939	23377.2065	23238.6203	14136.9620	5473.92542	1373.99793	231.176036	28.5205274	0.0052
800	2798.25633	11966.0937	21906.4647	22483.5212	14248.6545	5803.34129	1541.36179	271.319072	33.2343174	0.0049
1000	3036.18784	13028.2646	23922.2336	24612.3576	15626.1061	6371.01358	1691.62849	296.413611	35.6796918	0.0057

Here, the sampling interval of X_t should be as in (7), which is determined by the previous control chart statistic. If $\omega \le |\mathrm{S}_{t-1}| \le \mathrm{h}$ we should select the shorter sampling interval d_1; If $|\mathrm{S}_{t-1}| < \omega$,we should use the longer sampling interval d_2; If $|\mathrm{S}_{t-1}| > h$, a warning signal is given and the process should be stopped.

III. THE DESIGN DETAILS OF THE VSI-ANC CHART

The monitoring efficiency of a control chart with VSI technology is considerably faster than the conventional control chart with FSI. But there are many difficulties in selecting VSI chart parameters; we discuss some properties of the VSI-ANC chart to select parameters in this section. Some simulation results and performance comparisons are also given in this section.

3.1 Selecting the Sampling Intervals d_1 and d_2

Using the VSI-ANC chart to monitor processes, we need to select a set of sampling intervals d_1 and d_2. There is no doubt that a smaller d_1 spends less time for the VSI-ANC chart on detecting various shifts when the sampling interval d_2 is fixed. Stoumbos et al. (2000) [25] investigated VSI-CUSUM charts with various sampling d_1 and d_2;their results show that a chart with small values of d_2 is more efficient for large shifts in δ and large values of d_2 makes it more efficient for small shifts in δ. From the above, we can find that the length of short sample interval d_1 should be as short as possible, and the long sample interval d_2 should depends on both δ and d_1. Thus, we choose $\mathrm{d}_1 = 0.1$ and $\mathrm{d}_2 = 1.6$, 1.9, 2.2,2.5, 2.8, 3, 4, 5, 15, 25, 30 to monitor, and the performances of our chart with diverse sampling intervals are presebted in Table 2. We set $\tau = 50$, ATS=400, and assume that the IC underlying distribution is the standard normal N(0,1) while OC shift range is (0.25~4), and those simulation results are estimated by 50,000 times.

TABLE 2
THE AATS VALUES OF THE VSI-ANC CHART ACCORDING TO VARIOUS d_1 AND d_2 UNDER N(0,1) WHEN $\tau = 50$, ATS=400 AND $d_1 = 0.1$.

				$N(0,1)$ versus $N(\delta,1)$					$d_1 = 0.1$			
τ	δ	$d_2 = 1.6$	$d_2 = 1.9$	$d_2 = 2.2$	$d_2 = 2.5$	$d_2 = 2.8$	$d_2 = 3$	$d_2 = 4$	$d_2 = 5$	$d_2 = 15$	$d_2 = 25$	$d_2 = 30$
	0.25	293.6322	293.2343	290.4238	292.4231	292.3064	290.4786	287.1712	287.2184	282.9491	275.0755	270.2508
	0.50	116.3234	117.7490	116.5712	116.4151	115.5929	115.5673	114.9832	114.4641	110.8350	106.4886	106.4818
	0.75	23.77849	23.39469	22.53957	22.85839	22.32731	22.52922	22.37632	22.20203	20.51002	20.53424	19.57384
	1.00	6.401600	2.925397	5.733919	5.593566	5.525046	5.231744	5.134259	5.098971	4.674330	4.592139	4.509405
	1.50	3.101549	2.918578	2.801075	2.588115	2.470241	2.446488	2.365092	2.315996	2.157717	2.077112	1.964916
50	2.00	2.379214	2.274195	2.191544	1.904352	1.776809	1.777059	1.740821	1.687505	1.600883	1.529495	1.438835
	2.50	2.093262	2.019125	1.947492	1.579566	1.493244	1.501915	1.451562	1.443748	1.359325	1.357713	1.246328
	3.00	1.978547	1.912710	1.861233	1.427282	1.362116	1.367835	1.342660	1.345849	1.319013	1.285946	1.231648
	4.00	1.921884	1.877005	1.796287	1.311658	1.280831	1.302431	1.291591	1.277203	1.310330	1.293316	1.144665
	RMI	4.0103551	3.5988073	3.1841671	1.7963588	1.4481442	1.4049694	1.2208945	1.1211732	0.6893424	0.4687583	0.0000000
	IRMI	-	1.4384927	1.3151673	4.6260277	1.1607153	0.215872	0.184075	0.0997213	0.0431831	0.0220547	0.0937517

Note that because of the independent observation X_t is nonparametric in our chart, we should use the standardized sequential rank R_t^* to address this case. R_t^* is distributed as a uniform random variable, and it can be confirmed that for any time t the scope of R_t^* can be expressed as follows:

$$[-\sqrt{3(t-1)/(t+1)}, \sqrt{3(t-1)/(t+1)}] \tag{14}$$

Therefore, when $t \to \infty$, the asymptotic distribution of R_t^* is $U(-\sqrt{3}, \sqrt{3})$. In other words, regardless of how large the shifts size of X_t is, the largest shift size of R_t^* is equal to $\sqrt{3}$ at most. It is a relatively small shift, and thus, a larger d_2 is better. This conclusion can also be drawn from Table 2: the AATS always decreases with the increase of d_2 when $d_1 = 0.1$. However, note that when the value of d_2 is too large, it may cause the value of statistics beyond the control limit without providing an OC signal, because the waiting time for the next sample is too long. Simultaneously, when the value of d_2 is too large, to obtain $E(T_t) = 1$, the value of the warning limit ω will decrease continuously, and it decreased to0.02 when $d_2 = 30$. An infinitesimally small value of ω will result in the statistics falling in the area between ω and h before the shifts occur, which requires the chart to use the shorter sampling interval d_1 and makes the number of sampling times excessive. In conclusion, we should choose a moderate value of d_2.

The simulation results of the chart with various sampling intervals d_1 and d_2 are close. To comprehensively evaluate the performances of our chart with diverse sampling intervals, we introduce an evaluation criterion known as relative mean index (RMI), which is defined by Han and Tsung (2006) [26] as:

$$RMI = \frac{1}{N}\sum_{l=1}^{N} \frac{AATS_{\delta_l} - AATS_{\delta_l}^{min}}{AATS_{\delta_l}^{min}}, \tag{15}$$

where N is the total number of shifts monitored, AATS_{δ_l} is the AATS of the given chart when sample shift is δ_l, and $\text{AATS}_{\delta_l}^{\min}$ is the minimum value among all AATS values when sample shift is δ_l. A chart with a smaller RMI value means that the chart performance better overall.

From Table 2, the monitoring efficiency of the VSI-ANC chart increases very slowly with the increase of d_2 when $d_2 > 2.5$. To make the results more visual, we define the increment of RMI (IRMI) as follows:

$$\text{IRMI}\{t\} = \frac{\text{RMI}\{t-1\} - \text{RMI}\{t\}}{d_2\{t\} - d_2\{t-1\}}. \tag{16}$$

We can see from Table 2 that the maximum IRMI appears when $d_2 = 2.2 \sim 2.5$, and the IRMI are becoming smaller when $d_2 > 2.5$. Thus, considering the IRMI and the number of sampling times, were commend selecting $d_1 = 0.1$ and $d_2 = 2.5$ for the VSI-ANC chart when the size of shifts is not clear.

3.2 Selecting the 'Warning Limit' ω

We record the average sampling interval from the process starting to giving a warning signal as (8), and we confirmed that ATS is equivalent to IC ARL when $E(T_t) = 1$. According to (8), we know that the warning limit ω actually depends on d_1, d_2 and $E(T_t)$. Some values of the control limit h (h is decided by zero-state ATS) and warning limit ω with various d_1, d_2 are presented in Table 3. These simulation values are obtained in the IC process where underlying distribution is assumed to be N(0,1). Note that the VSI-ANC chart is distribution-free. From Chakraborti et al. (2001) [27] we known that distribution-free charts with other underlying distributions have the same results.

TABLE 3

THE VALUES OF H AND ω ACCORDING TO VARIOUS d_1, d_2 AND ZERO-STATE ATS UNDER N(0,1).

ATS	200		300		400		500		800		1000	
h	1.228		1.251		1.266		1.278		1.299		1.308	
d_1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
d_2	1.9	2.5	1.9	2.5	1.9	2.5	1.9	2.5	1.9	2.5	1.9	2.5
ω	0.275	0.217	0.265	0.205	0.255	0.196	0.245	0.188	0.234	0.177	0.226	0.170

3.3 The steps for designing a VSI-ANC chart

To design the VSI-ANC chart, we can follow these steps.

1) Specify the value of ATS (or IC ARL) according to the actual requirements.

2) Select an appropriate control limit function h(k) in accordance with the ATS in column 1 of Table 1.

3) Calculate $\hat{\delta}_t^+$ and $\hat{\delta}_t^-$. Set $\hat{\delta}_t = (R_t^* + R_{t-1}^*)/2$ and $\delta_0 = 0.7$ as the evaluated size of shifts; meanwhile, for the initial observation, let $R_0^* = R_1^* = 0$.

4) Find the control limit h in accordance with the selected ATS. Some reference values are available in Table 3.

5) Select the sampling intervals values of d_1 and d_2. In the situation that scant knowledge about the size of mean shifts, we always set $d_1 = 0.1$ and $d_2 = 2.5$.

6) Find the warning limit ω in accordance with d_1 and d_2 and control limit h. Table 3 also provides some reference values.

7) For the subsequent observations, calculate S_t. If $|S_t| > h$, a warning signal is given and the process should be stopped. If $\omega \le |S_t| \le h$, monitoring the (t+1)th observation with the smaller interval d_1 sequentially; otherwise, monitoring the (t+1)th observation with the larger interval d_2.

3.4 Performance comparisons

We compare the performance of the VSI-ANC chart with the ANC chart in this subsection. The ANC chart is performance better than other nonparametric charts, see Liu et al. (2014) [18] for details. Let $\text{ATS(VSI - ANC)} = \text{IC ARL(ANC)} = 400$, the ANC chart is equivalent to the VSI-ANC chart with $d_1 = d_2 = 1$. Set the change point τ as 50, 100 and 300, where the change point τ represents the shifts occur point. We choose N(0,1), t(4), $\chi^2(4)$ and $\Gamma(3,1)$ for representative illustrations, in which are two symmetrical distributions and two asymmetrical distributions. To investigate the overall performance of the two chart across a series of shifts, we also compute their RMI values of AATS.

The performances of the two charts with different distributions and different shifts are shown in Table 4. Up to time τ the underlying distributions are assumed to be the N(0,1), t(4), $\chi^2(4)$ and $\Gamma(3,1)$ distributions, and after time τ are $N(\delta_X,1)$, $t(4)+\delta_X\cdot\sigma$, $\chi^2(4)+\delta_X\cdot\sigma$ and $\Gamma(3,1)+\delta_X\cdot\sigma$. Where σ is the respective standard deviation of each distribution.

As shown in table 4, both the ANC chart and the VSI-ANC chart become more sensitive to change as τ increases. Moreover, at the same τ, the value of AATS decreases as δ_X increases. This table also shows that the RMI of the VSI-ANC chart is almost equal to zero, which indicates that the VSI-ANC chart performs better than the ANC chart regardless of whether the shift is small or large and regardless of distributions.

TABLE 4
THE AATS VALUES OF THE VSI-ANC AND ANC CHARTS UNDER VARIOUS DISTRIBUTIONS.

		$N(0,1)$		$t(4)$		$\chi^2(4)$		$\Gamma(4,1)$	
τ	δ_X	VSI-ANC	ANC	VSI-ANC	ANC	VSI-ANC	ANC	VSI-ANC	ANC
	0.25	**290.19**	300.09	**262.64**	273.98	337.06	**330.18**	273.85	**273.61**
	0.50	**116.20**	132.47	**72.657**	89.260	**128.35**	135.96	**136.61**	145.45
	0.75	**23.317**	36.463	**9.9956**	20.490	**22.577**	30.859	**27.418**	39.305
	1.00	**5.4755**	13.733	**3.7699**	10.396	**4.8774**	12.999	**5.3228**	13.689
50	1.50	**2.5999**	7.2663	**2.2641**	6.3446	**2.3939**	7.1647	**2.4668**	7.2928
	2.00	**1.8863**	5.3938	**1.7824**	5.0433	**1.9348**	5.4855	**1.9326**	5.5074
	2.50	**1.5833**	4.5938	**1.5455**	4.5183	**1.6959**	4.7316	**1.6676**	4.7095
	3.00	**1.4179**	4.2453	**1.4087**	4.2644	**1.5207**	4.3689	**1.4912**	4.3299
	RMI	**0.0000**	1.2245	**0.0000**	1.3327	**0.0026**	1.1978	**0.0001**	1.2005
	0.25	**233.58**	247.02	**190.56**	207.05	275.92	**271.06**	269.64	**269.35**
	0.50	**48.273**	65.449	**23.057**	38.448	**46.716**	60.254	**51.926**	66.016
	0.75	**8.6308**	19.166	**5.3748**	13.752	**6.7332**	16.803	**7.9281**	18.406
	1.00	**4.4806**	11.351	**3.4036**	9.1109	**3.6426**	10.472	**3.9294**	10.965
100	1.50	**2.4858**	6.7139	**2.1568**	5.8946	**2.2868**	6.5898	**2.4722**	7.2875
	2.00	**1.8561**	5.0895	**1.7010**	4.7610	**1.8711**	5.1282	**1.8728**	5.1564
	2.50	**1.5349**	4.3271	**1.4902**	4.2625	**1.6464**	4.4636	**1.6222**	4.4434
	3.00	**1.3697**	3.9663	**1.3589**	4.0074	**1.4659**	4.1481	**1.4352**	4.1128
	RMI	**0.0000**	1.2906	**0.0000**	1.4164	**0.0022**	1.3529	**0.0001**	1.3362
	0.25	**135.30**	154.39	**94.228**	116.03	**157.45**	161.66	**165.48**	172.69
	0.50	**19.093**	34.099	**11.417**	23.984	**14.567**	28.441	**17.409**	31.593
	0.75	**7.0957**	16.097	**4.9238**	12.418	**5.4115**	14.028	**5.9903**	14.980
	1.00	**4.2452**	10.479	**3.2434**	8.4969	**3.4391**	9.6388	**3.6682**	10.029
300	1.50	**2.4253**	6.3982	**2.0949**	5.6250	**2.2462**	6.2710	**2.2526**	6.3591
	2.00	**1.7932**	4.8828	**1.6469**	4.5799	**1.8347**	4.9299	**1.8158**	4.9438
	2.50	**1.5054**	4.1777	**1.4215**	4.0872	**1.5901**	4.3110	**1.5819**	4.2952
	3.00	**1.3585**	3.8196	**1.3432**	3.8196	**1.3866**	3.9936	**1.3952**	3.9583
	RMI	**0.0000**	1.3265	**0.0000**	1.4574	**0.0000**	1.4305	**0.0000**	1.3989

IV. CHEMICAL DATA EXAMPLE

In this section, the use of our proposed VSI-ANC chart can be demonstrated by applying it to a chemical dataset. There are 149 readings of the triglyceride concentrations in this dataset. The triglyceride concentrations of a chemical products should be set in a certain range. More detailed introduction can be found in Chapter 3 of Hawkins and Olwell (1998) [28]. According to Hawkins and Olwell (1998) [28], the 149 observations are mutually independent and stabilized, and the shift sizes is unknown. Li et al. (2014) [18] also used this chemical dataset in their article and demonstrated it is significantly different from a normal distribution. Similarly, we use the R function shapiro.test to inspect the first 75 observations, which provides the statistics W = 0.96548 and p-value = 0.03852. The inspection result shows that the dataset distribution is clearly different from a normal distribution; it can be identified as nonparametric. Therefore, our VSI-ANC chart can address these issues with ease.

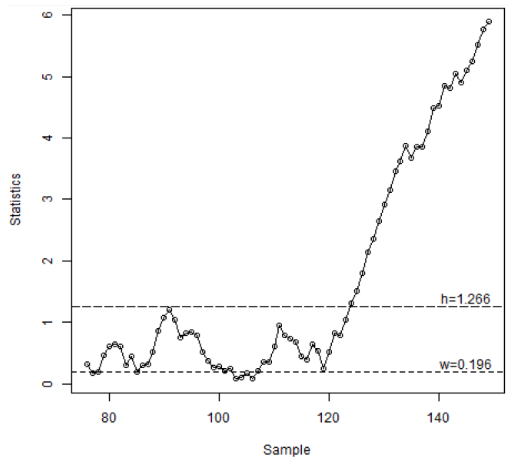

FIG. 1 THE STATISTICS OF THE VSI-ANC CHART ALONG WITH THE WARNING LIMIT AND CONTROL LIMIT FOR MONITORING THE TRIGLYCERIDE CHEMICAL PROCESS.

TABLE 5
THE VSI-ANC AND ANC CHART APPLIED TO THE TRIGLYCERIDE DATASET.

t	X_t	R_t^*	S	$AATS^{VSI-ANC}$	$AATS^{ANC}$	t	X_t	R_t^*	S	$AATS^{VSI-ANC}$	$AATS^{ANC}$
76	133	1.7094	0.3226	0	0	113	118	-0.0153	0.7358	15.7	37
77	114	-1.4397	0.1735	2.5	1	114	118	-0.0152	0.6825	15.8	38
78	117	-0.6218	0.1977	5	2	115	121	1.1296	0.4470	15.9	39
79	113	-1.6226	0.4619	5.1	3	116	118	-0.0299	0.3960	16	40
80	115	-1.1692	0.6152	5.2	4	117	109	-1.7173	0.6435	16.1	41
81	117	-0.5560	0.6390	5.3	5	118	119	0.3083	0.5378	16.2	42
82	118	-0.1901	0.6136	5.4	6	119	126	1.6448	0.2487	16.3	43
83	124	1.5652	0.3087	5.5	7	120	123	1.4867	0.5194	16.4	44
84	115	-1.1754	0.4401	5.6	8	121	125	1.6033	0.8323	16.5	45
85	124	1.5488	0.1908	5.7	9	122	119	0.2556	0.7895	16.6	46
86	114	-1.4099	0.3066	8.2	10	123	127	1.6617	1.0392	16.7	47
87	117	-0.5376	0.3174	8.3	11	124	124	1.5086	1.3200	16.8	48
88	114	-1.3975	0.5122	8.4	12	125	122	1.2748	1.5076	*	*
89	111	-1.6932	0.8590	8.5	13	126	125	1.5671	1.7931	*	*
90	114	-1.3472	1.0747	8.6	14	127	127	1.6503	2.1331	*	*
91	115	-1.0469	1.1949	8.7	15	128	123	1.3532	2.3503	*	*
92	120	0.6213	1.0403	8.8	16	129	126	1.5844	2.6472	*	*
93	123	1.4714	0.7503	8.9	17	130	125	1.5056	2.9233	*	*
94	116	-0.7924	0.8208	9	18	131	124	1.4015	3.1554	*	*
95	117	-0.4376	0.8347	9.1	19	132	126	1.5746	3.4510	*	*
96	118	-0.0722	0.7905	9.2	20	133	123	1.2502	3.6308	*	*
97	122	1.3571	0.5187	9.3	21	134	125	1.4736	3.8803	*	*
98	122	1.3433	0.3683	9.4	22	135	116	-0.9110	3.6796	*	*
99	117	-0.4724	0.2687	9.5	23	136	125	1.4646	3.8570	*	*
100	120	0.5889	0.2754	9.6	24	137	120	0.4299	3.8479	*	*
101	118	-0.1029	0.2033	9.7	25	138	128	1.6693	4.1129	*	*
102	120	0.5944	0.2422	9.8	26	139	133	1.7072	4.4889	*	*
103	116	-0.8577	0.0808	9.9	27	140	121	0.7794	4.5332	*	*
104	117	-0.4830	0.1020	12.4	28	141	136	1.7198	4.8422	*	*
105	116	-0.8578	0.1828	12.5	29	142	120	0.3659	4.8064	*	*
106	119	0.2288	0.0907	12.6	30	143	129	1.6352	5.0566	*	*
107	115	-1.1332	0.2154	15.1	31	144	117	-0.6375	4.8994	*	*
108	115	-1.1227	0.3552	15.2	32	145	129	1.6246	5.1023	*	*
109	117	-0.4291	0.3624	15.3	33	146	123	1.1152	5.2387	*	*
110	113	-1.6061	0.6085	15.4	34	147	128	1.5671	5.5153	*	*
111	112	-1.6541	0.9531	15.5	35	148	126	1.4395	5.7638	*	*
112	120	0.6805	0.7891	15.6	36	149	123	1.0695	5.8892	*	*

We use the VSI-ANC chart and ANC chart for comparing analysis and monitor the 76th to 149th observations as same as Li et al. (2014) [18]. We set the zero-state ATS as 400, $\hat{\delta}_t = \left(R_t^* + R_{t-1}^*\right)/2$, $\delta_0 = 0.7$, $d_1 = 0.1$ and $d_2 = 2.5$. Consequently, the control limit h=1.266, and the warning limit $\omega = 0.196$. Meanwhile, we set the FSI of the ANC chart as $d = 1$. Table 5 presents the statistics values of the ANC chart and the VSI-ANC chart from the 76th observation on and calculates their AATS. Fig. 1 shows the results of our chart along with its warning limit and control limit. From Fig. 1 and Table 5, we can see that the two charts will present a warning signal after reading the 124th observation. The ANC chart requires 48 units of time to provide the warning signal, and the VSI-ANC chart requires only 16.8 units of time.

V. CONCLUSION

In this paper, we propose a CUSUM chart with VSI technology, which is based on the adaptive and nonparametric methods. This chart is distribution free, performs well for both small and large shifts and is not too computationally demanding. Moreover, our chart detects the process shift faster than other charts in standard with ATS; the supporting example is the triglyceride chemical process.

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