Predicting the Risk of Bankruptcy for ARO Stock

Yuexian Li[1], Jinguo Lian[2], Hongkun Zhang[3]

[1]Department of Mathematics and Statistics, Inner Mongolia Agricultural University, P.R.China
[2]Department of Mathematics and Statistics, University of Massachusetts Amherst, USA
[3]Department of Mathematics and Statistics, University of Massachusetts Amherst, USA

***Abstract*—** *In the last several decades, many researchers have been focused on finding effective experimental methods to predict stocks with tendency of bankrupt. Recent financial crisis has caused extensive world-wide economic damages, predicting bankruptcy before it happens could help investors avoid large losses. In this article, by observing the market dynamics of its stock price and trading volume, we estimate the risk of bankruptcy of Aeropostale (ARO). GARCH and EGARCH series models with normal distribution and t-student distribution are used to estimate the volatilities and value-at-risk (VaR) of ARO stock. By analyzing the VaR, we conclude that there is a high probability that the company will be facing bankruptcy in the near future. Moreover, our study shows that the asymmetric EGARCH model with t-student distribution eventually is a better choice to predict the behavior of this stock.*

Keywords*— *Asymmetric EGARCH model, GARCH model, normal distribution and t-student distribution, volatilities, Value-at-Risks.

I. INTRODUCTION

Bankruptcy forecasting has been an open challenging problem for financial analysts. The most recent financial crisis was caused by sub-prime mortgages written in 2006, which mainly contributed to Lehman Brother's bankruptcy in September 2008. The bankruptcies of many other corporations at that time also resulted in substantial losses to many investors and hedge funds. The general consensus is that if one could accurately predict bankruptcy, and identify a characteristic behavior exhibited by a stock before bankruptcy, it would help investors avoid certain large losses. Thus bankruptcy prediction is a topic of great interest, not only to investors and hedge funds, but also to researchers across a wide range of fields. In this paper, we investigate the risk of Aeropostale, Inc. (known as ARO), which is an American apparel retailer, principally targeting teenagers and young people. The company operates 773 Aeropostale stores in the U.S. and about 61 stores in Canada. The company's chief competitors Forever 21 Inc. and American Eagle offer just as fashionable clothing at much cheaper prices. Plus, the overall teenage fashion styles have changed, while Aeropostale's has not. The company continues to produce clothing that displays the brand's name, which has long gone out of style. Aeropostale has served up 11 straight quarters of losses, shrinking cash at the end of the second quarter to stand at $86 million, down from $151 million at the start of 2015. ARO stock has lost more than 60% in the last 12 months.

Many researchers and analysts have attempted to develop models for predicting corporate bankruptcy, but most of these models depend on the availability of detailed internal financial information or the financial statement about the corporation, such as Altman (1968) and Ohlson (1980). In general the financial statement has both balance sheet and income statement, which is often difficult to obtain for general investors, except possibly some large hedge funds. A commonly used financial tool is the Z-Score, which was developed in 1968 by Edward I. Altman, as a quantitative balance-sheet method of determining the financial health of a company. However as pointed by Altman (2002), the Z-Score was not intended to be used on non-manufacturing companies. Based on latest financial disclosure using the Z-Score calculation, Aeropostale Inc. only has probability of bankruptcy (Z-Score) of 34.58%. This is somewhat misleading for the investors. For more details, see, https://www.macroaxis.com/invest/ratio/ARO–Probability-Of-Bankruptcy.

Since all investors have access to historical data of daily stock prices and trading volumes, it would be more beneficial if one could predict a corporation's risk of bankruptcy by observing the market dynamics of its stock price. The goal of our study is to develop an early warning system to forecast the time of bankruptcy based on statistical analysis of the stock dynamics, rather than corporate internal financial information. Based on the daily closing share prices and trading volume of ARO, we use statistical properties to analyze ARO stocks to discover the tendency of stocks moving to bankruptcy.

One commonly used measurement of the stock risk is the so-called Value-at-Risk, VaR for short. It was made popular by US investment bank J.P. Morgan, who incorporated it in their risk management model RiskMetricsTM. The Value-at-Risk of a stock is mainly the maximum loss that may be suffered on that stock in a short period of time. More precisely, a VaR(a) is the a-th quantile of the distribution of the maximum loss, typically a is chosen in the range of 95% to 99.9%. Evidently, the higher the confidence level is, the larger the VaR. By varying the value of a, one should be able to explore a whole risk distribution of the maximum loss. Despite the extensive literature and empirical research of estimation of VaR in the financial markets, literature dealing with VaR calculation in bankrupt stocks is very scarce. In this paper, we estimate the VaR for the stock ARO. Throughout our detailed analysis, a holding period of one day will be used. The method we use is the so-called variance method, which related the VaR to the volatility of the underlying stock prices. Intuitively, the larger the volatility of the expected return, the more likely the occurrence of large swings in the stock price and the larger the Value-at-Risk.

Thus to estimate the risk of the stock, we must accurately predict the volatility. For volatility forecasts there are two major sources, volatility models based on time series and the volatility implied from option prices. In this paper, we use the GRACH and EGARCH series models to estimate the volatilities. The Generalized Autoregressive Conditional Heteroscedastic (GARCH) model was introduced by Bollerslov (1986), which has been extensively used in financial time series. Since stocks with bankrupt tendency usually are penny stocks, thus bad news has a greater impact on volatility. To improve the estimation on volatilities, we also use the Exponential GARCH (EGARCH) model, proposed by Nelson (1991). In this paper, we analyze the relative return series from statistical point of view. By comparing the VaR together with the relative returns, we discover that the VaR are strongly correlated with the stock price. We also notice that the bankrupt tendency is characterized by the rapid decreasing trend of the VaR. The statistical approach used in this paper may eventually help investors forecast stock bankruptcies weeks or months in advance.

This paper is organized as follows. In Section 2, we analyze fundamental statistical properties of the time series of the relative returns for ARO, including the normality and autocorrelations. The GARCH and EGARCH models are fitted and estimation of VaR is performed in Section 3.

II. DATA AND DESCRIPTIVE STATISTICS

2.1 Data Description

We denote P_t as the daily closing price of a stock, for integer $t \in Z$. The stochastic properties of the price time series $\{P_t\}$ is characterized by the relative returns process, which are defined as:

$$R_t = 100(\log P_t - \log P_{t-1}) \tag{1}$$

In this paper, we mainly concentrate on the daily ARO stock price time series over the thirteen-year period. There were 3,272 daily data points from Oct. 29, 2002 to Oct. 27, 2015. We collect ARO daily closing price from Yahoo Finance.

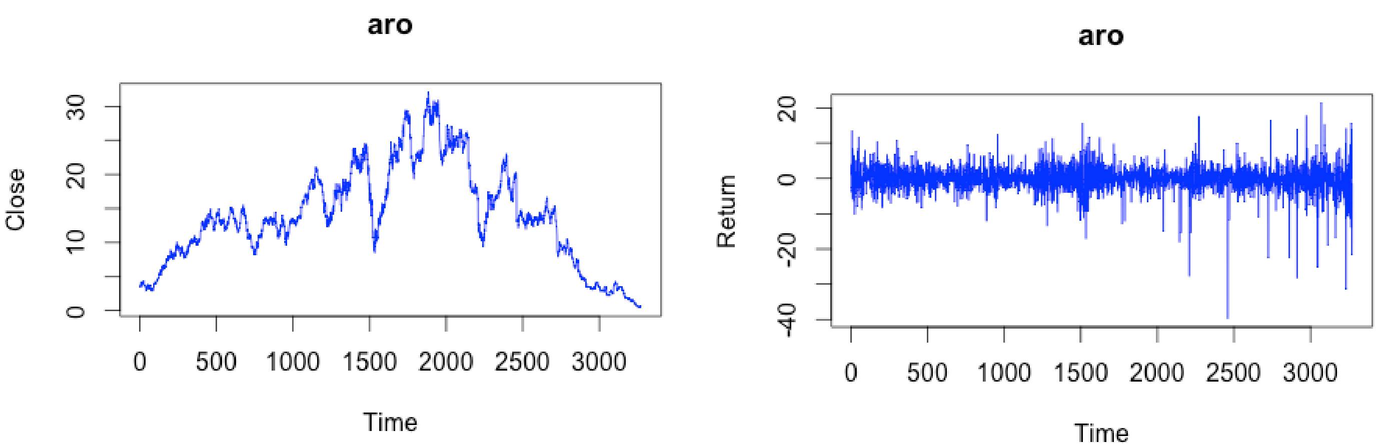

FIGURE 1: THE UPPER PLOT IS FOR ADJUSTED CLOSING PRICE, AND THE LOWER PLOT IS FOR DAILY RETURNS

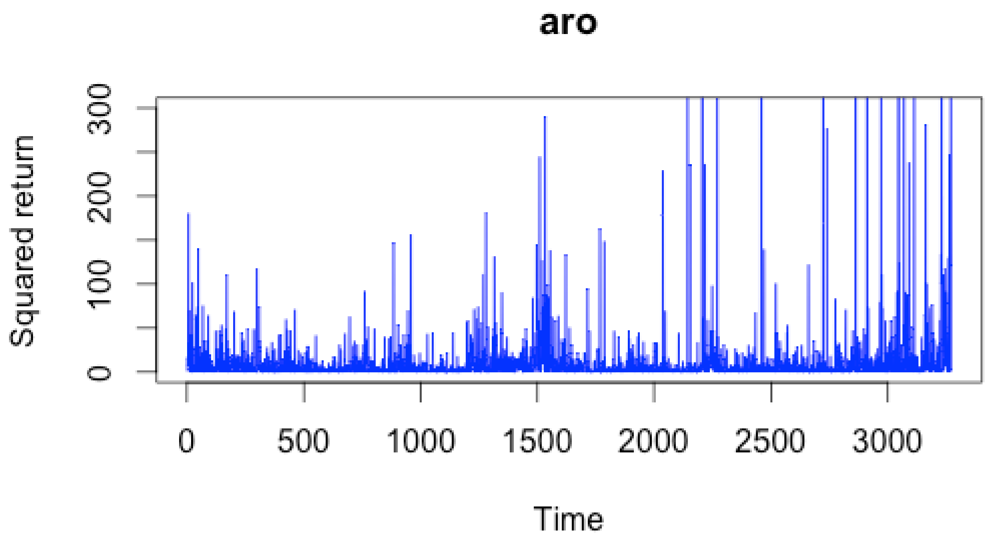

FIGURE 2: DAILY SQUARED RETURNS FOR ARO

Fig.2 exhibits the time series plot of daily squared returns about ARO stock. Note that there are growing signs that the process is more volatile than that demonstrated in Fig.1. There are extensive literatures that have documented the evidence that high stock volume is closely related to volatile returns; see for example Gallant, Rossi, and Tauchen (1992), Harris (1987); Jain and Joh (1988); Jones, Kaul, and Lipson (1991); and the survey in Karpoff (1987). Numerous papers have noted that volume tends to be higher when stock prices are increasing than when prices are falling. However, our Fig.3 clearly contradicts to these results. For example, from 2010 to 2015, the stock price has a significant decreasing trend; however the trading volume tends to be higher and with more peaks in this period. Weak high relative volume is an indicator on underlying activities such as stock news, analyst downgrade, insider selling, or that hedge funds and stock traders are piling out of the stock ahead of a catalyst.

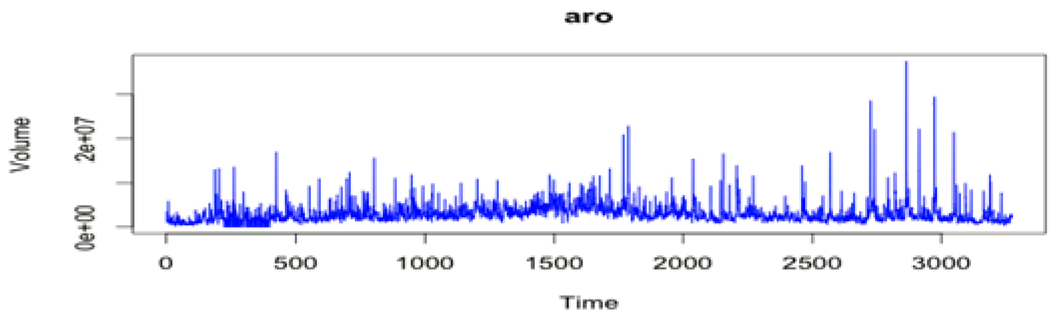

FIGURE 3: DAILY VOLUME OF ARO

To get a better description of the statistics of the relative returns, we have the following table on estimations of its mean and higher moments. The table shows a clear negative drift of the returns, which is indicates a strong sign of bankruptcy.

TABLE 1
SUMMARY STATISTICS OF THE RETURNS

Mean	Range	Standard Deviation	Skewness	Kurtosis	Observations
-0.0528	(-39.6763,21.4255)	3.5328	-1.2271	16.5943	3272

In order to accurately predict the VaR, we need to examine some statistical properties for the return series, including the test for normality as well as the autocorrelations.

2.2 Test for Normality

We first use the Q-Q plot to test the fitting for normal distribution, as well as the student t-distribution

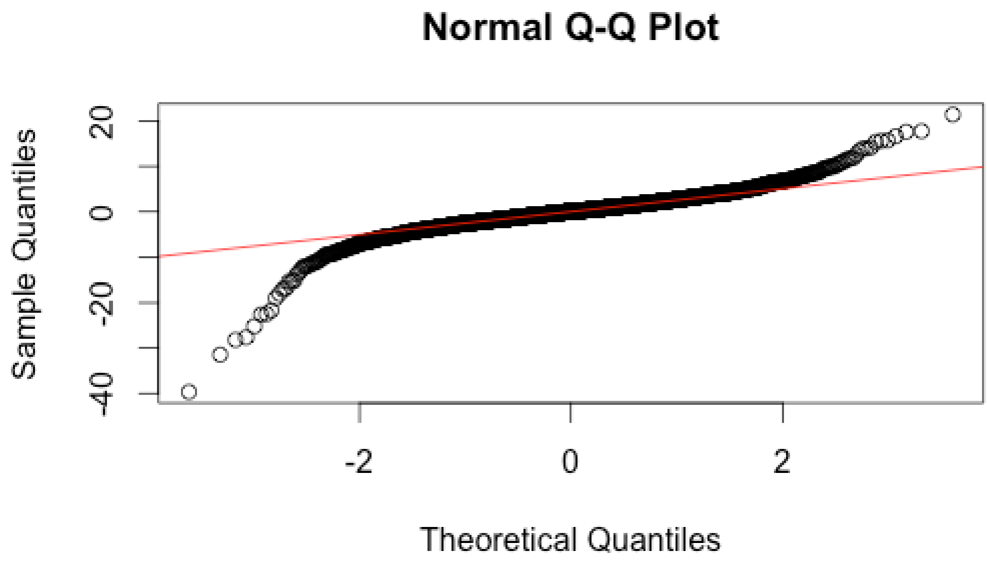

FIGURE 4: QUANTILE-QUANTILE PLOT OF RETURNS AGAINST THE NORMAL DISTRIBUTION

The Fig.4 is the Q-Q plot of the empirical distribution of the daily returns (y-axis) against the normal distribution (x-axis). It can be observed from plot, the empirical distribution of the daily returns exhibits heavier tails than the normal distribution, so the normal distribution is not an ideal fit for the return process. To support our observation, Jarque-Bera test can be used as a goodness-of-fit test to examine if the sample data have kurtosis and skewness similar to a normal distribution. The test statistics is denoted as JB, which is defined by

$$\text{JB} = \frac{n}{6}\left(S^2 + \frac{1}{4}(K-3)^2\right) \tag{2}$$

Where n is the sample size, S is the sample skewness and K is the sample kurtosis. If the sample data comes from a normal distribution, the statistic JB should follow asymptotically a chi-squared distribution with two degrees of freedom. The null hypothesis is that the sample data have a skewness of zero and an excess kurtosis of 3 which is what the normal distribution has. Our results show that the returns time series of ARO fails the null hypothesis with 95% confidence

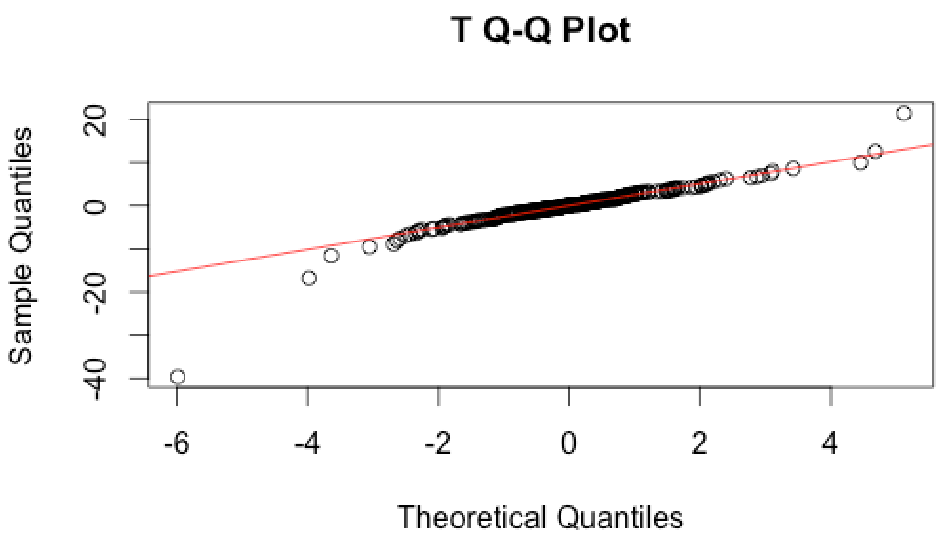

FIGURE 5: QUANTILE-QUANTILE PLOT OF RETURNS AGAINST THE T DISTRIBUTION

We also test the empirical distribution of the daily returns (y-axis) against the t distribution using Q-Q plot. From the Fig.5, one can see clearly that the t distribution is a much better fit and the empirical distribution of the daily returns has lighter tails than the t distribution.

2.3 Tests for Autocovariance

Analyzing the second-order structure of the processes is an important step to understand a classical time series. The Autocorrelation Coefficients (ACF) and Partial Autocorrelation Coefficients (PACF) are extremely useful as they help us identify the correct specification for an ARMA model that describes the stochastic process. In particular, if the process is white noise, all autocorrelation and partial autocorrelation coefficients equal zero. If the process is an AR(p), the PACF will equal zero for all lags $k > p$, while if the process is a MA(q) the ACF will equal zero for all lags $k > q$

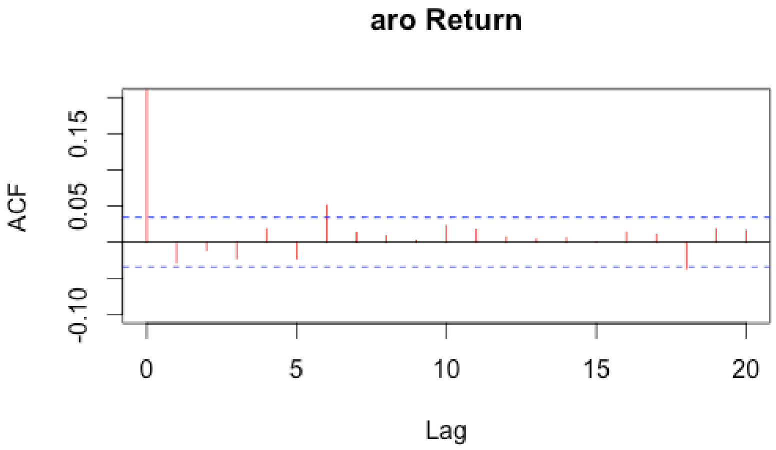

FIGURE 6: SAMPLE AUTOCORRELATION COEFFICIENTS UP TO 20 LAGS FOR RETURNS

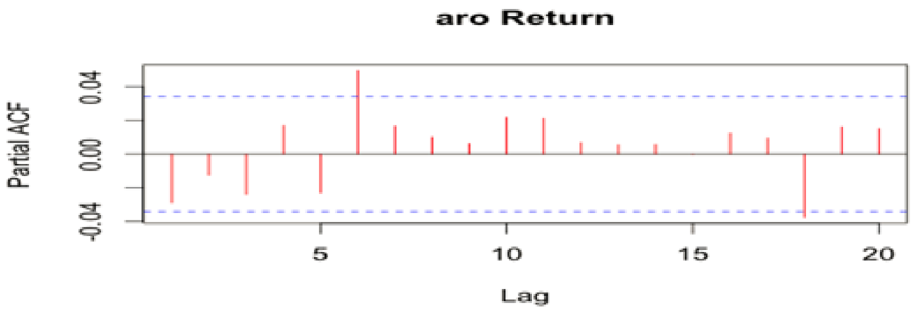

FIGURE 7: SAMPLE PARTIAL AUTOCORRELATION COEFFICIENTS UP TO 20 LAGS FOR RETURNS

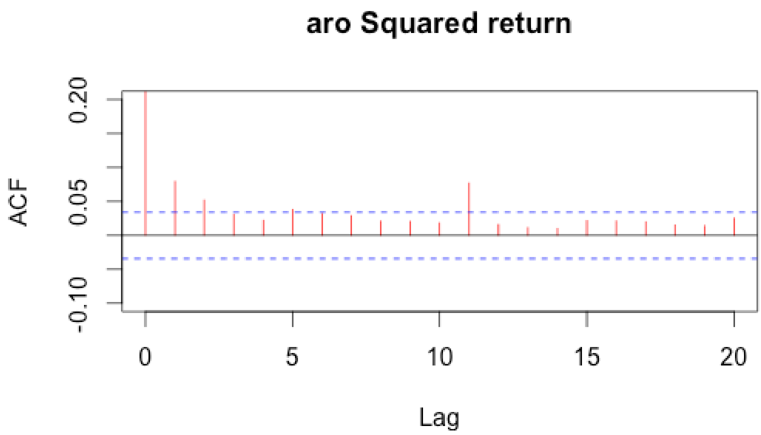

FIGURE 8: SAMPLE AUTOCORRELATION COEFFICIENTS UP TO 20 LAGS FOR SQUARED RETURNS

To confirm our observations, we apply two formal statistic tests, including the Ljung-Box test by Ljung and Box (1978) and Lagrange multiplier test (Engle, 1982), which test to check serial correlation of returns and squared returns. In both tests, the null and alternative hypothesis is defined respectively as:

$$H_0: \widehat{\rho}(1) = \widehat{\rho}(2) = \cdots = \widehat{\rho}(m) = 0 \text{ vs } H_1 : \widehat{\rho}(i) \neq \text{ for some } i \in \{1, 2, \cdots, m\}$$

Where n is the sample size, $\widehat{\rho}(l)$ is the sample autocorrelation at lag l , and m is the number of lags being tested. The Ljung-Box Q test statistic is

$$Q(m) = n(n+2)\sum_{l=1}^{m} \frac{\widehat{\rho}(l)^2}{n-l} \tag{3}$$

As Ljung & Box (1978) proposed, under the assumption that $\{R_t\}_{t=1}^{n}$ is i.i.d, the distribution of $Q(m)$ can be approximated as chi-squared with m degrees of freedom. A too large value of Q would suggest that the sample autocorrelations are too high for the data to be observations from an iid sequence.

Our test results are shown in the following table: where the p-value is the quantile that H_0 fails. Thus the Ljung-Box-Q-test null hypothesis is rejected for lags 6, 8 and 10 at a 95% confidence level.

TABLE 2
LJUNG-BOX Q TEST

χ-squared	m	p-value
16.397	6	0.01177
17.269	8	0.02743
19.008	10	0.04016

The Lagrange multiplier test is used for the squared data series. We can see that the null hypothesis H_0 is rejected for lags 6, 8 and 10 at a 99%confidence level.

TABLE 3
LAGRANGE MULTIPLIER TEST

χ-squared	m	p-value
35.897	6	2.886e-06
38.154	8	7.051e-06
39.276	10	2.271e-05

III. MODEL FITTING AND ESTIMATION OF VAR

3.1 Methodology

Let $F_t = \sigma(R_s, s \leq t)$ be the σ-algebra generated by all historical information (based on the time series) up to time t. Consequently, we obtain a filtration $\{F_t, t \leq T\}$ generated by the random process $\{R_t\}$. Wold's decomposition theorem (c.f. Fuller (1996) pg. 96) states that any covariance stationary time series $\{R_t\}$ has a representation of the form:

$$R_t = \mu + \sum_{k=0}^{\infty} c_k \eta_{t-k} \tag{4}$$

where $\mu = E(R_t)$ is the expected return; $\eta_t = R_t - E(R_t | F_{t-1})$ is an uncorrelated process with zero mean and adapted to the filtration $\{F_t\}$. The coefficients c_k called the moving average weights or impulse responses. Moreover $c_0 = 1$ and $\sum_{k=0}^{\infty} c_k^2 < \infty$. The covariance can be calculated as:

$$\gamma(j) = \sum_{k=1}^{\infty} c_k c_{k+j} \tag{5}$$

The usefulness of the Wold Theorem is that it allows the dynamic evolution of a variable R_t to be approximated by a linear model. If the innovations $\{\eta_t\}$ are independent, then the linear model is the only possible representation relating the observed value of r_t to its past evolution. We denote the conditional mean as:

$$\mu_t = E(R_t | F_{t-1}) \tag{6}$$

And the conditional variance is denoted as:

$$\sigma_t^2 = Var(R_t | F_{t-1}) = E((R_t - E(R_t | F_{t-1}))^2 | F_{t-1}) = E(\eta_t^2 | F_{t-1}) \tag{7}$$

The random variable σ_t is called the volatility of R_t. Moreover, one can see that $\{\sigma_t^2\}$ is a predictable process. By the Pythagoras Theorem,

$$\sigma^2 = E(\sigma_t^2) + \sum_{k=1}^{\infty} c_k^2 E(\sigma_{t-k}^2) \tag{8}$$

where $\sigma^2 = Var(R_t)$ is the variance of the expected returns.

The fact that large absolute returns tend to be followed by large absolute returns (whatever the sign of the price variations) is hardly compatible with the assumption of constant conditional variance. This phenomenon is called conditional heteroscedasticity, i.e. σ_t^2 is not a constant. Note that σ_t^2 is measurable with respect to F_{t-1}, so it can be represented as a function of $\{R_s, s < t\}$, i.e. the conditional volatility should satisfy:

$$\sigma_t^2 = g(\cdots, R_0, \cdots, R_{t-1}) \tag{9}$$

To account for the very specific nature of financial series (price variations or log-returns, interest rates, etc.), one usually denote

$$\eta_t = \sigma_t \varepsilon_t \tag{10}$$

where $\{\varepsilon_t\}$ is a white noise process with zero mean, unit variance and they are uncorrelated. Different classes of models can be distinguished depending on the specification adopted for σ_t, such as the Conditionally heteroscedastic (GARCH) processes and the Exponential GARCH model.

In this paper, AR(1) model is used to simulate the conditional mean. More precisely, the AR(1) model is defined by

$$\mu_t = \phi_0 + \phi_1 r_{t-1} \tag{11}$$

where ϕ_0 and ϕ_1 are two constants, here we use lower case r to refer to the realization data series for R. To estimate the conditional variance, we use the GARCH model as well as the EGARCH model. More precisely, we assume

$$r_t = \mu_t + \eta_t \text{ and } \eta_t = \sigma_t \varepsilon_t \tag{12}$$

where σ_t is estimated using the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model (Bollerslev, 1986) as :

$$\sigma_t^2 = \alpha_0 + \sum_{j=1}^{p} \alpha_j \eta_{t-j}^2 + \sum_{i=1}^{q} \beta_i \sigma_{t-i}^2 \tag{13}$$

where $p > 0, q > 0$, and α_j, β_i are constants, for $i = 1, \cdots, p$ and $j = 1, \cdots, q$. Here $\{\varepsilon_t\}$ is the weak white noise, with zero mean, unit variance and they are uncorrelated.

The GARCH model is the most commonly used model in financial time series analysis by far, but it requires the parameters to be nonnegative. The models assume that positive and negative shocks have the same impact on volatility. This model can account for persistence of financial time-series data, but it is well known that financial asset volatility has an asymmetric impact. Typically, the bad news has a greater impact on volatility. For example, declining stock prices are more likely to give rise to massive portfolio rebalancing (and thus volatility) than increasing stock prices. This asymmetry arises naturally from the existence of thresholds below which positions must be cut unconditionally for regulatory reasons.

To be able to model this behavior and relax the limitation of parameters, Nelson (1991) proposed the Exponential GARCH (EGARCH) model. For $p, q > 0$, the EGARCH (p,q) model is given by

$$\log \sigma_t^2 = \alpha_0 + \sum_{j=1}^{p} [\alpha_j \eta_{t-j} + \gamma_j (|\eta_j| - E|\eta_{t-j}|)] + \sum_{i=1}^{q} \beta_i \log \sigma_{t-i}^2 \qquad (14)$$

Indeed studies have found that the predictive effect of higher order model is not necessarily better than the low order model, see Hansen and Hansen, P. R., Lunde, A.(2005) and Bollerslev, T., Chou, R.Y., Kroner, K.F (1992). Because of the computational complications, we use the GARCH(1,1) and the EGARCH(1,1) model in this paper. GARCH and EGARCH parameters are estimated by maximum likelihood, see Table 4. Although a Gaussian assumption is common, the distribution is often fat tailed, which has prompted the use of the Student-t distribution (Bollerslev 1987). We analyze the empirical distribution of the daily returns by taking ε_t to be the normal distribution and the t -distribution, respectively. Although our empirical results show that the t -distribution is a more ideal fit, interestingly the normal distribution also passed the Jarque-Bera test with 95% confidence.

TABLE 4
ESTIMATED GARCH MODELS AND EGARCH MODEL FOR THE DAILY RETURNS OF ARO

distribution of ε_t	normal	t	normal	T
Model	Garch	Garch	Egarch	Egarch
ϕ_0	0.9596442	0.967713	0.996800	0.993598
ϕ_1	0.1818314	0.080000	0.009424	0.013742
α_0	0.0267214	0.025209	-0.041655	-0.026558
α_1	0.0064066	0.041331	-0.033586	0.038978
β_1	-0.0242822	-0.007945	-0.022967	-0.008206
γ_1			0.016691	0.075905

3.2 Estimation of VaR

Value at risk (VaR) has become very popular in risk management because it is an easily understood and obviously relevant concept. Despite its conceptual simplicity and popularity as an industrial standard, estimating the value of VaR of a stock is highly non-trivial. In statistical terms, the task is to provide a given quantile for the unknown distribution of the relative returns of the stock. Moreover no consensus has been reached as to the best method for estimating VaR.

As introduced in Section 1, VaR(a) is the a-quantile of the distribution of the maximum loss, with a chosen as either 95% or 99%. Consequently, the VaR(a) for the negative return is defined as

$$\mathrm{P}(R_t > VaR(a)) = a \qquad (15)$$

i.e., VaR(a) is the a-th quantile of the distribution of the negative relative return $-R_t$.

It follows from (15), we have

$$\varepsilon_t = \frac{R_t - \mu_t}{\sigma_t} \tag{16}$$

Assume ε_t follows distribution P_ε, and denote u_a as the a-th quantile of ε_t, i.e. $P_\varepsilon(\varepsilon_t < u_a) = a$, then the a-th quantile of $-R_t$ can be calculated as

$$VaR(a) = \mu_t - \sigma_t u_a \tag{17}$$

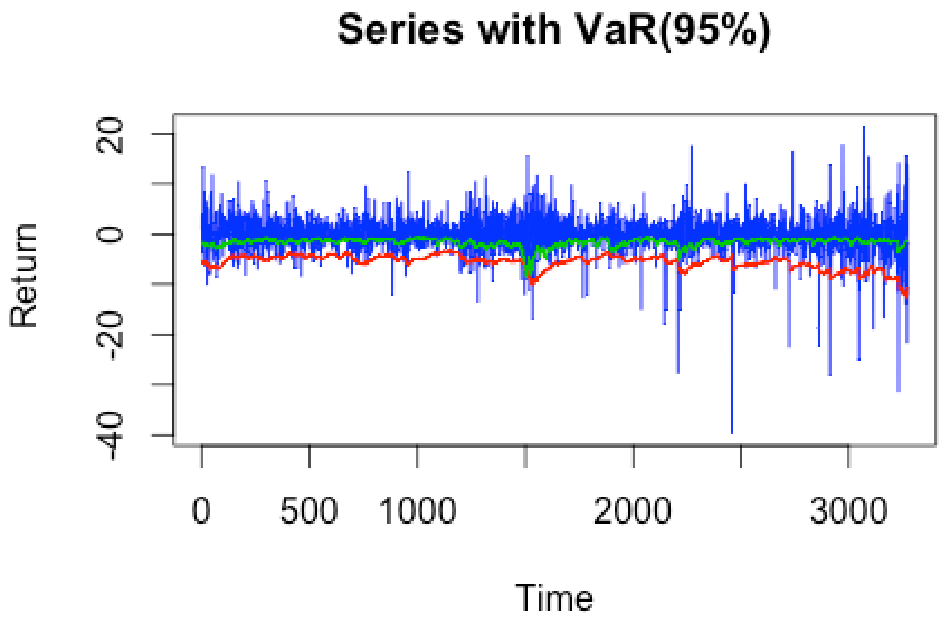

FIGURE 9: THE SERIES PLOT OF DAILY RETURNS WITH VAR(95%) OF ARO IN RED AND VAR(95%) OF SPY IN GREEN FOR NORMAL DISTRIBUTION

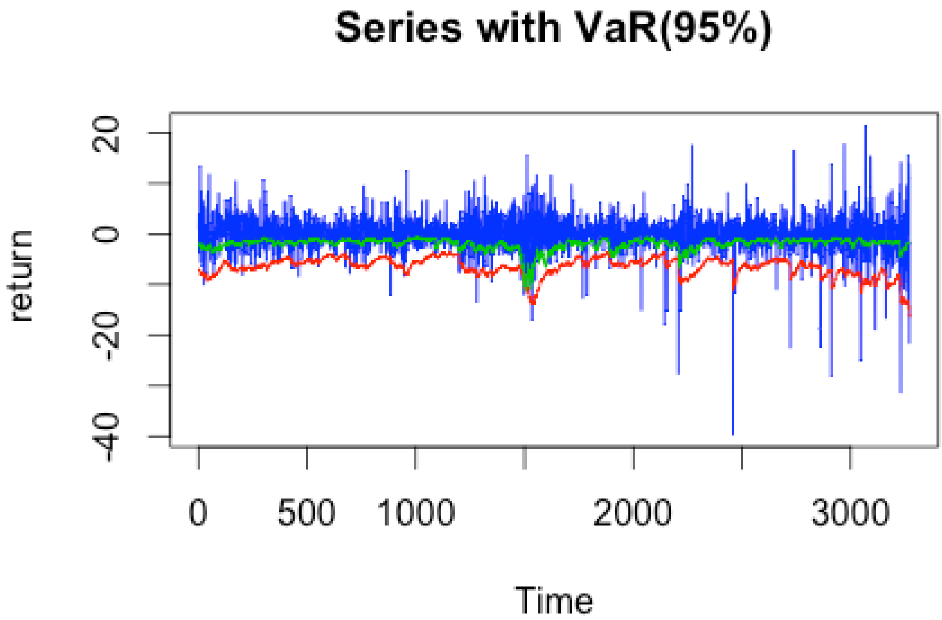

FIGURE 10: THE SERIES PLOT OF DAILY RETURNS WITH VAR(95%) OF ARO IN RED AND VAR(95%) OF SPY IN GREEN FOR T DISTRIBUTION

Fig.9 demonstrates our simulation of VaR(95%) for stock ARO as well as that of SPY by assuming ε_t following the standard normal distribution. Fig.10 is plotted similarly assuming ε_t following the student t distribution.

TABLE 5
ESTIMATION OF THE VAR FOR THREE-DAY-AHEAD PERIOD

Model	Garch-normal	Garch-t	Egarch-normal	Egarch-t
σ_{T+1}	7.363	7.525	7.26	7.795
VaR(0.05)(T+1)	-12.064	-15.11	-11.938	-15.654
VaR(0.01)(T+1)	-17.081	-25.267	-16.885	-26.176
σ_{T+2}	7.325	7.504	7.249	7.746
VaR(0.05)(T+2)	-12.044	-15.116	-11.958	-15.57
VaR(0.01)(T+2)	-17.035	-25.244	-16.897	-26.026
σ_{T+3}	7.287	7.483	7.237	7.698
VaR(0.05)(T+3)	-11.981	-15.072	-11.937	-15.473
VaR(0.01)(T+3)	-16.946	-25.172	-16.868	-25.864

IV. CONCLUSION

Our statistical studies discover that the ARO stock has a high probability of approaching bankruptcy. According to recent work by Li etc.(2011), one of the most significant features in the distribution of returns: pre-bankrupt stocks are more likely to have larger daily returns (both positive and negative) than stocks that do not become bankrupt. In other words, pre-bankrupt stocks have larger daily price fluctuations. According to the statistical quantities given in Table 1, we know that the difference is bigger for negative returns than positive returns, indicating the falling stock price preceding a bankruptcy. Indeed the closer the day of bankruptcy approaches, the greater the possibility for these dramatic price changes.

A second major feature pointed out in Li etc.(2011) is that the pre-bankrupt stocks experience a stronger correlation between volatility and volume. Previous research has shown that volatility and volume exhibit a positive correlation, meaning that large changes in stock price are often accompanied by large changes in trading volume. This is confirmed by comparing Figure 1 and Figure 3, as we can see that both the log returns and the trading volume have large fluctuations in 2015. This is pretty odd situation which is obviously distinguishes ARO from healthy stocks. For healthy stocks, if the mean of return is negative, the trading volume should decrease. This phenomenon is also evidence that many insiders are selling a large amount of shares and short sales have increased significantly, which is mainly due to some inter-information on the potential bankruptcy of the company. We also find the asymmetric EGARCH Model with t-student distribution should be a suitable tool to predict the volatilities of ARO, which indicates that the market dynamics of the stock is mainly impacted by some "bad news" or certain negative information.

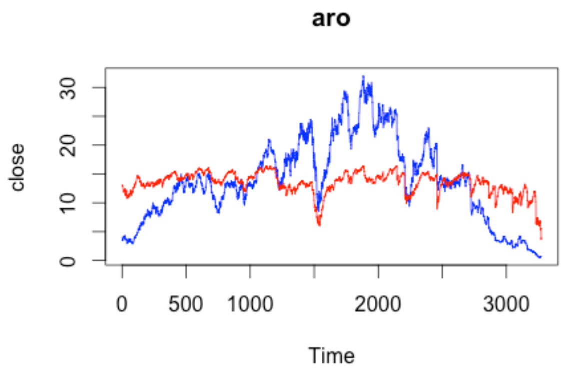

FIGURE 11: THE SERIES PLOT OF DAILY CLOSE PRICE IN BLUE WITH VAR(95%) OF ARO IN RED

Our empirical study also shows that the VaR and the stock price are strongly correlated. In Fig.11, we plot the time series plot of daily close price in blue with VaR(95%) of ARO in red which are shifted by 20 units up. One can see that these two curves have similar up/downward trends in last few years.

Consequently our statistical investigation of the historical data of ARO indicates that this stock has a strong tendency approaching bankruptcy. Therefore we suggest investors should avoid investing in ARO stocks

REFERENCES

[1] Vesna Bucevska.(2013) An Empirical Evaluation of GARCH Models in Value-at-Risk Esti- mation. Evidence from the Macedonian Stock Exchange, Business Systems Re- search, Vol. 4 No. 1 (March 2013), 49-64.

[2] Xiao Li, Mike Lipkin, Richard B. Sowers.(2011) Dynamics of Bankrupt Stocks. PhD Dissertation, url: http://hdl.handle.net/2142/26066.

[3] Engle, R. F.(1982) Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of United Kingdom Inflation. Econometrica, 50:987-1007

[4] Bollerslev, T.(1986) Generalized Autoregressive Conditional Heteroskedasticity. Journal of Econometrics, 31:307-327.

[5] Nelson, D. B.(1991) Conditional Heteroskedasticity in Asset Returns: A New Approach. Econometrica, 59:347-370

[6] Campbell, J. Y., Lo, A. W., MacKinlay, A.C.(1997) The Econometrics of Financial Markets. Princeton University Press: New Jersey.

[7] G. M. Ljung & G. E. P. Box (1978). On a Measure of a Lack of Fit in Time Series Models. Biometrika 65 (2): 297-303.

[8] Robert F Engle.(1982) A General Approach to Lagrange Multiplier Model Diagnostics. Journal of Econometrics. 20(1),83-104.

[9] Hansen, P. R., Lunde, A.(2005) A Forecast Comparison of Volatility Models: Does Anything Beat a GARCH(1,1). Journal of Applied Econometrics, 20:873-889.

[10] Bollerslev, T., Chou, R.Y., Kroner, K.F. (1992) ARCH Modeling in Finance: a Review of Theory and Empirical Evidence. Journal of Econometrics 52:5-59.

[11] Andersen, T.G., Bollerslev, T.(1997) Heterogeneous Information Arrivals and Return Volatility Dynamics: Uncovering the Long-Run in High Rrequency Returns. Journal of Finance, 52: 975-1005.

[12] Qian Li etc. Statistical analysis of bankrupting and non-bankrupting stocks. EPL (Europhysics Letters), Volume 98, Number 2, 98 (2012) 28005. DOI: 10.1209/0295- 5075/98/28005

[13] https://en.wikipedia.org/wiki/A

[14] Gholamreza Jandaghi, Reza Tehrani,Parvaneh Pirani, Ali Mokhles, British Journal of Economics,Finance and Management Sciences 37, October 2011, Vol. 2 (1)

[15] Jeffrey Lui, Estimating the Probability of Bankruptcy:A Statistical Approach

[16] Warren Miller Comparing Models of Corporate Bankruptcy Prediction: Distance to Default vs. Z-Score, 2009 wiki-1 http://www.stockopedia.com/content/the-altman-z-score-is-it-possible-to-predict-corporate-bankruptcy-using-a-formula-55725/sthash.7uLYpGW5.dpuf