Calendar Effects Analysis of Americas Indexes

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Abstract— We apply an approach for estimating Value-at-Risk (VaR) describing the tail of the conditional distribution of a heteroscedastic financial return series. The method combines quasi-maximum-likelihood fitting of AR(1)-GARCH(1,1) model to estimate the current mean as well as volatility, and extreme value theory (EVT) to estimate the tail of the adjusted standardized return series. We employ the approach to investigate the existence and significance of the calendar anomalies: seasonal effect and day-of- the-week effect in Americas Indexes VaR. We also examine the statistical properties and made a comprehensive set of diagnostic checks on the one decade of considered Americas Indexes returns. Our results suggest that the lowest VaR of considered Americas Indexes negative log returns occurs on the fourth season among all seasons. Moreover, comparatively low Wednesday VaR is captured among all weekdays during the test period.

Keywords— Risk Measures, Value-at-Risk, GARCH models, Extreme value theory, Generalized Pareto Distribution, Day-of-the-week effect, Seasonal effect.

I.   INTRODUCTION

In today's financial world, the large increase in the number of traded assets in the portfolio of most financial institutions has made the measurement of market risk a primary concern for regulators and for internal risk control. Following the Basle Accord on Market Risk (1996) most banks in more than 100 countries around the world have to calculate its risk exposure for every individual trading desk. Banks are also required to hold a certain amount of capital as a cushion against adverse market movements. Value-at-Risk has become the benchmark risk measure. From mathematics point of view, *VaR* is simply a quantile of the profit-and-Loss distribution of a given portfolio over a prescribed holding period. The importance of *VaR* is undoubted since regulators accept this model as a basis for setting capital requirements for market risk exposure.

In this paper, we discover the calendar anomalies in Americas equity market movements, which including the seasonal effect and the day-of-the-week effect on Americas Indexes returns. The calendar effect in stock market returns includes day-of-the-week effect, weekend effect, January effect, and holiday effect, etc. It has been widely studied and investigated in finance literature. Studies by Cross (1973), and Rogalski (1984) demonstrate that there are differences in distribution of stock returns for each day of the week. Studies by Baillie and DeGennaro (1990), Berument and Kiymaz (2001) posit that day-of-the-week effect has an impact on stock market volatility. In recent years, another stream of research has considered seasonality in stock returns and volatility, see Saunders (1993), Bouman and Jacobsen (2002), Hirshleifer and Shumway (2003), Kamstra, Kramer and Levi (2003), and Cao and Wei (2005), etc. These studies generally report that calendar anomalies are present in both returns and volatility equations in the stock market. None of these studies, however, test for the possible existence of day-of-the-week and seasonal variation in stock return *VaR*. Hence, the goal of this paper is to characterize the *VaR* of Americas Indexes returns. Based on investigations of the day-of-the-week effect and seasonal effect in extreme risk, we also provide valuable and applicable analysis for equity market investors. The major obstacle to this investigation is a viable measure of tail risk over time.

We are concerned with tail estimation for those considered financial return series. Our basic assumption, whose validation is examined in this paper, is that returns follow a stationary time series model with stochastic volatility structure. The presence of stochastic volatility implies that returns might dependent over time. Therefore, we consider to model the return distribution as the conditional return distribution where the conditioning is on the current volatility and mean. Although *VaR* only characterizes the extreme quantiles, disregarding the centre of the distribution, estimation of the extreme quantile is not an easy task. As one wants to make inferences about the extremal behavior of a portfolio, there is only a very small amount of data in the tail area of a sample set. Advanced methods and tools are needed to enable us to explore beyond the range of the limited data set. In this study, we use a semi-parametric method, based on extreme value theory (EVT), which is rather effective for obtaining reliable estimates.

EVT is widely used to study the distribution of extreme realizations of a given distribution function, or stochastic processes that satisfy suitable assumptions. The foundations of the theory were laid by Fisher and Tippett (1928) and Gnedenko (1943), who demonstrated that the distributions of the extreme values of an independent and identically distributed sample from a

cumulative distribution function, when adequately rescaled, can converge towards one out of only three possible distributions. Unfortunately, most financial time series are not independent, but exhibit some very delicate temporal dependence structure. In this study, we capture it by a fully parametric method, which is based on an econometric model for volatility dynamics and the assumption of conditional normality, AR-GARCH model. We use AR(1)-GARCH(1,1) model and quasi-maximum-likelihood estimation to obtain estimates of the conditional mean and the conditional volatility. Statistical tests and exploratory data analysis confirm that the standardized returns, i.e. mean and volatility adjusted returns, do form approximately i.i.d. series. If we only use GARCH model to estimate *VaR*, the assumption of conditional normality does not seem to hold for real data. Thereafter, we use threshold methods from EVT to estimate the distribution of the standardized returns. EVT is a well known technique in many fields of applied sciences including risk management, insurance and engineering. Numerous research studies surfaced recently which analyze the extremes in the financial markets due to currency crises, stock market turmoils and credit defaults. The behavior of financial series tail distributions has, among others, been discussed in Reiss and Thomas (1997), McNeil and Frey (2000), Longin (1999) and (2000), and Ameli and Malekifar (2014). An estimate of the conditional return distribution is now easily constructed from the estimates of the conditional mean and volatility as well as the estimated distribution of the standardized returns. We learned the central idea of the dynamic two stage extreme value process from McNeil and Frey (2000), to forecast daily *VaR* with historical data in a moving window. This approach reflects two stylized facts exhibited by most financial return series, stochastic volatility, and the non-normal behavior of conditional return distributions.

In terms of the organization of this paper, we first introduce the data set and present a comprehensive set of diagnostic checks on it in Section 2. We review certain aspects of *VaR* and introduce the extreme value based approach to it in Section 3. Section 4 contains the empirical tests and results on the Americas Indexes returns. The seasonal effect and day-of-the-week effect on Americas Indexes returns *VaR* and economic implications of the empirical results are examined in Section 4 as well. Conclusion is made in Section 5.

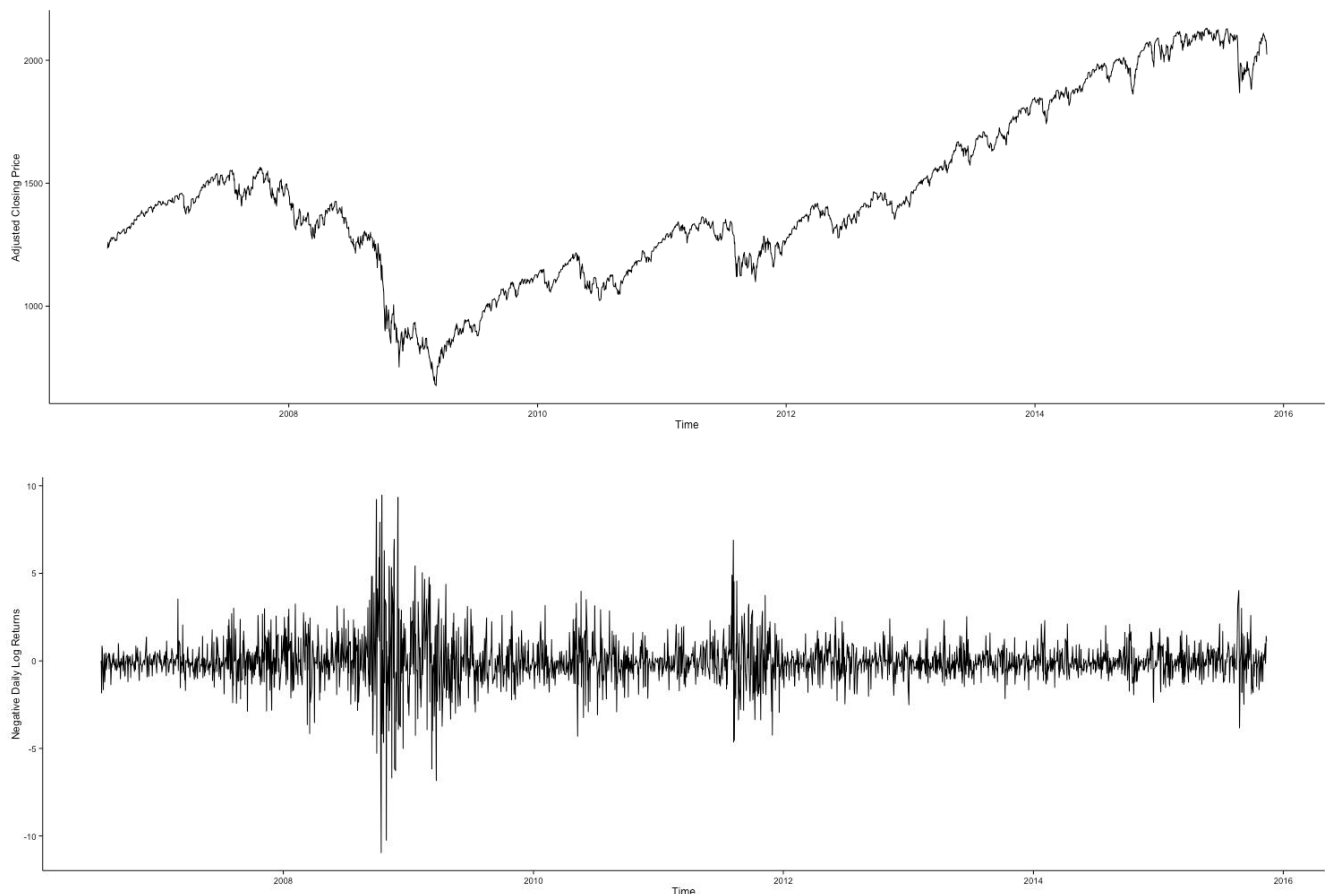## II.   DATA EXPLORATION AND STATISTICAL ANALYSIS

2.1   Data Description

Our sample covers the period from July 17, 2006 to November 13, 2015. Five different Americas Indexes, namely, the S&P 500, Financial Select Sector SPDR ETF, NASDAQ-100 Technology Sector, Dow Jones Utility Average, and Dow Jones Transportation Average, are used to characterize the performance of specific sectors of the market. The S&P 500 is an American national index composed of large capitalization stocks. It represents the overall performance of the stock market. The Financial Select Sector SPDR ETF tracks the overall S&P Financial Select Sector Index. The NASDAQ-100 Technology Sector is an equally weighted index based on the securities of the NASDAQ-100 Index that are classified as Technology according to the Industry Classification Benchmark classification system. The Dow Jones Utility Average is a stock index from Dow Jones Indexes that keeps track of the performance of 15 prominent utility companies. The Dow Jones Transportation Average is a U.S. stock market index from S&P Dow Jones Indices of the transportation sector, and is the most widely recognized gauge of the American transportation sector. The collections of those indices' daily adjusted closing price were from *Yahoo Finance*. The adjusted closing price is used to develop an accurate track record of the stock's performance.

Furthermore, we use the daily negative log return to examine extreme losses of the stock. Let P_i denotes the adjusted closing price of a stock on day *t*, then the daily percentage change on the day is defined by

$$r_i = -100 log p_i = 100 log \frac{p_{i-1}}{p_i}.$$

(1)

The reason for using the negative returns is that we are mainly interested in the possibility of large losses rather than large gains.

FIGURE 1: TIME PLOTS OF THE STANDARDS AND POORS INDEX FROM 2006-7-17 TO 2015-11-13. THERE ARE 2351 DAYS EXCERPT FROM THE SERIES OF ADJUSTED CLOSING PRICE (UPPER) AND THE SERIES OF NEGATIVE DAILY LOG RETURNS (LOWER).

Fig.1 shows the time plots of adjusted closing price and negative daily log returns of S&P 500 from July 17, 2006 to November 13, 2015. From the lower plot, we observe that daily log returns of the Index show clear evidence of volatility clustering. That is, periods of large returns are clustered and distinct from periods of small returns, which are also clustered. If we measure such volatility in terms of variance, then it is nature to think that variance changes with time, reflecting the clusters of large and small returns. We also observe that there are more pronounced peaks than one would expect from Gaussian data. Since the possibility of time-varying variance and non-normal behavior are noticed in Fig.1, we provide formal tests to check the stationarity, normality, and independency of those log return series.

2.2 Test for Stationary Property

The invariance of statistical properties of the return time series corresponds to the stationarity hypothesis that the joint probability distribution of the returns does not change when shifted in time. Here we use the KPSS test to verify the hypothesis of weak stationarity, i.e. time invariance of the mean value and the autocorrelation function of America Indexes return series.

Proceeding in the spirit of Kwiatkowski, Phillips, Schmidt and Shin (1992), we assume that the series $\{r_t\}_{t=1}^{T}$ can be decomposed into the sum of a random walk and a stationary error1. We express this symbolically by writing

$$r_t = \alpha_t + \varepsilon_t,$$

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1 In general, the assumption of KPSS test is that the series {rt} can be decomposed into the sum of a deterministic trend, a random walk and a stationary error. In this study, we only consider to test series stationary with no trend.

where ε_t is assumed to be stationary; αt is a random walk, i.e. $\alpha_t = \alpha_{t-1} + u_t$. Here $\{u_t\}$ is a white noise series with zero mean and variance σ_u^2. The hypothesis for the KPSS test is

$$H_0 : \sigma_u^2 = 0 ,$$

i.e. the series is stationary vs $H_1 : \sigma_u^2 \neq 0$, i.e. the series is not stationary

Assume $e_t = r_t - \hat{\alpha}_t$ as residuals of the regression of r_t on an intercept, $S_t = \sum_{i=1}^{t} e_i, t = 1, 2, \cdots, T,$ as the partial sum process of the residuals. The KPSS test statistics is

$$KPSS = \frac{T^{-2} \sum_{t=1}^{T} S_t^2}{s^2} ,$$

(2)

where s^2 is a consistent estimator of the long-run variance of α_t.

The rejection rule is that if the value of the KPSS statistic in Eq.(2) exceeds the critical values estimated in [19], or the p-value is less than or equal to the significance level α, we reject H_0.

TABLE 1
STATIONARY TEST AND NORMALITY TEST RESULTS ON THE FIVE AMERICAS INDEXES NEGATIVE DAILY LOG RETURNS FROM 2006-7-17 TO 2015-11-13
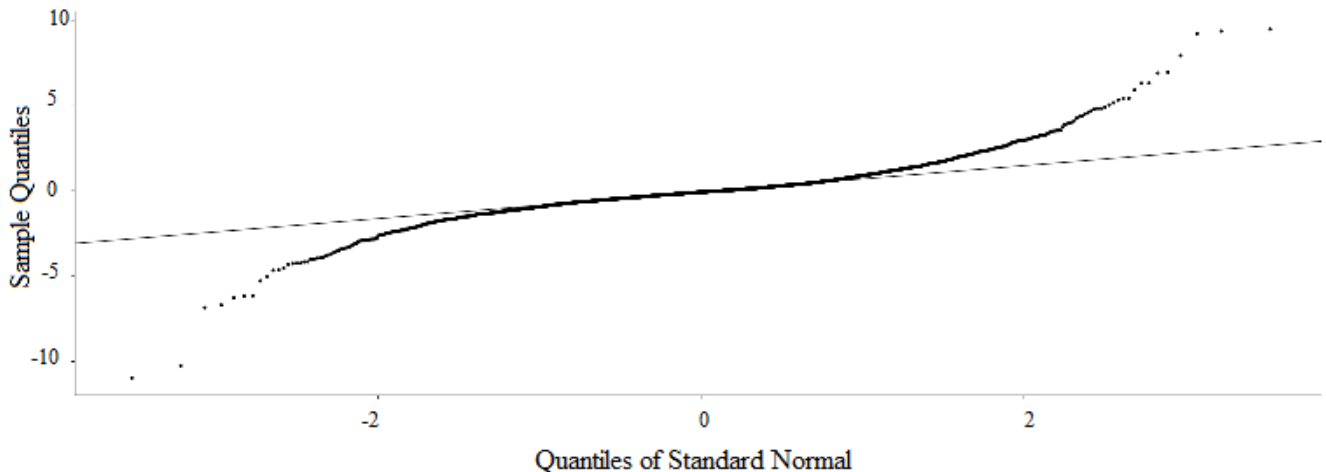
Data	S&P	XLF	NDXT	DJU	DJT
Observations	(2350)	(2350)	(2398)	(2351)	(2350)
KPSS Test for time series level stationarity					
$KPSS$ statistics	0.17128	0.28586	0.072364	0.081087	0.1073
p-value	0.1	0.1	0.1	0.1	0.1
Shapiro-Wilk Test for time series normality					
W statistics	0.87934	0.82416	0.94139	0.88601	0.9479
p-Value	<2.2e-16	<2.2e-16	<2.2e-16	<2.2e-16	<2.2e-16

H_0. In this study, we choose the significant level α as 5% for KPSS test.

The KPSS rest results on the five America Indexes negative daily log returns from July 17, 2006 to November 13, 2015 are shown in Table 1, all p-values are greater than the significant level 5%. Therefore, we accept the null hypothesis and conclude that the five return series are stationary during the test period.

2.3   Tests for Normality

In studying the financial time series, people usually assume that the log return follow a normal distribution. However, it is unrealistic to make the normality assumption on America Indexes returns, according to our QQ-plot on the S&P 500 negative daily log returns against the normal distribution, see Fig.2.

FIGURE 2: QUANTILE-QUANTILE PLOT OF THE S&P 500 NEGATIVE DAILY LOG RETURNS FROM 2006-7-17 TO 2015-11-13 AGAINST THE NORMAL DISTRIBUTION

To prove that the returns do not follow normal distribution, we use one of the most powerful formal normality tests (as Razali and Wah (2011) demonstrate) – the Shapiro- Wilk test, to verify an empirical fact that the five America Indexes negative daily log return series do not have the normality property.

The Shapiro-Wilk test utilizes the null hypothesis principle to check whether the series $\{r_i\}_{i=1}^{T}$ come from a normally distributed population. The Shapiro-Wilk test statistic is defined as

$$W = \frac{(\sum_{i=1}^{T} a_i r_i)^2}{\sum_{i=1}^{T} (r_i - \bar{r}_i)^2},$$

(3)

where r_i is the i-th order statistic; $\bar{r}$ is the sample mean; $(a_1, a_2, \ldots a_T)$ are the weights.

The value of W lies between zero and one. Small values of W lead to the rejection of normality whereas a value of 1 indicates the normality of data. Consequently, we reject the null hypothesis if the p-value of the test is less than the predetermined significance level, which is 5% in this study.

Applying the Shapiro-Wilk test on the five America Indexes negative daily log returns from July 17, 2006 to November 13, 2015, we show the test result in Table 1. Because all p-values are less than 2.2e-16, we reject the null hypothesis and conclude that all the five return series are not normally distributed during the test time period.
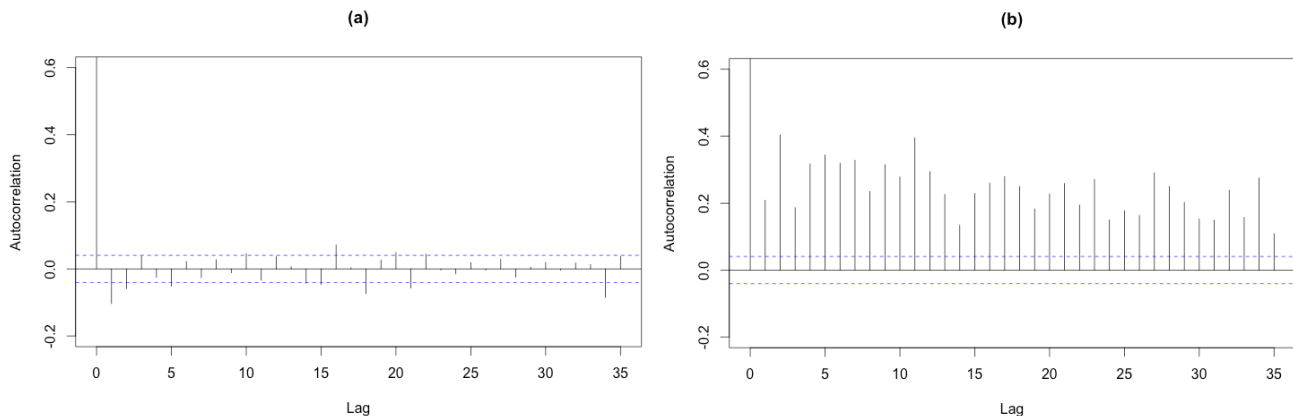
2.4　Test for Correlations

Besides the verified stylized fact of the fat tail distribution, we further explore the autocorrelations correlations for the returns series as well as their squared values. We begin by considering the autocorrelation function of a time series $\{r_i\}$. The correlation between r_i and its past values r_{i-l} is called the lag-l autocorrelation of $\{r_i\}$ and is commonly denoted by ρ_l, for l=0,1,…t. Under the weakly stationary assumption, we assume ρ_l is a function of l only.

$$\rho_l = \frac{Cov(r_i, r_{i-l})}{\sqrt{Var(r_i)Var(r_{i-l})}} = \frac{Cov(r_i, r_{i-l})}{Var(r_i)} = \frac{\gamma_l}{\gamma_0},$$

(4)

where the property $Var(r_i) = Var(r_1) = \gamma_0$ for a weakly stationary series is used.

For a given sample of returns $\{r_i\}_{i=1}^{T}$, let $\bar{r} = (\sum_{i=1}^{T} r_i)/T$ be the sample mean. The lag-l sample autocorrelation of $\{r_i\}_{i=1}^{T}$ can be represented as:

$$\hat{\rho}_l = \frac{\sum_{t=l+1}^{T}(r_t - \bar{r})(r_{t-l} - \bar{r})/(T - l + 1)}{\sum_{t=1}^{T}(r_t - \bar{r})^2/(T-1)}, 0 \le l \le T - 1.$$

(5)

FIGURE 3: SAMPLE AUTOCORRELATIONS OF (A) RETURNS AND (B) SQUARED RETURNS OF THE S&P 500 (2006-7-17 TO 2015-11-13)

If a time series is not autocorrelated, the estimate of $\hat{\rho}_l$ will not be significantly different from 0.

Fig.3(a) shows that the sample autocorrelation coefficient $\hat{\rho}_l$ plotted against different lags l (measured in days), along with the classical 95% significance bands around zero for S&P 500 negative daily log returns. The dashed lines represent the upper and lower 95% confidence bands $\pm\frac{1.96}{\sqrt{T}}$, where the time length for our S&P 500 returns is $T = 2350$ days. A stylized fact that absence of autocorrelation for the daily price variations is illustrated in Fig.3(a). The series of S&P 500 returns displays small autocorrelations, making it close to a white noise. However, the S&P 500 squared returns are strongly autocorrelated, see Fig.3(b). This property is not incompatible with the white noise assumption for America Indexes returns, but shows that the white noise is not strong.

Based on the statistical analysis for the five America Indexes negative daily log return series, we discovered that those America Indexes returns are stationary, and uncorrelated time series, yet are not normally distributed and the squared returns are strongly correlated. Those properties illustrate the difficulty of daily price returns modeling. Any satisfactory statistical model for daily returns must be able to capture the main stylized facts, including the leptokurticity, the unpredictability of returns, the existence of positive autocorrelations in the squared returns, and the conditional heteroscedasticity. Some computations of VaR are based on the assumption that the series $\{r_i\}$ is normally distributed, or has t-distribution, see reference [27][1][3][18][15]. That is the main reason why these study can use volatility to estimate VaR. However, the real time series $\{r_i\}$ may not follow any known distributions. To overcome the difficulty of a return series $\{r_i\}$ with an unknown distribution, we compute the VaR of America Indexes returns by the Extreme Value Theory, which avoid making assumption about the distribution of $\{r_i\}$.

III.   Value-at-Risk Methodology

Exposure to risk can be summarized as a single number by estimating the VaR, which is defined by Jorion as "the worst expected loss over a great horizon within a given confidence level", it is crucial to have an accurate estimate on VaR. Before we go into our VaR methodology in more detail, we introduce the probability theory behind VaR.

3.1   VaR Introduction

VaR is the amount that might be lost in a portfolio of assets over a specified time period with a specified small failure probability α, usually set as 0.01 or 0.05. In this paper, we choose such short period as one day. Suppose a random variable X characterizes the distribution of negative daily returns of a portfolio, the right-tail α-quantile of the portfolio is then defined to be the VaR_α such that

$$\Pr(X \leq VaR_\alpha) = 1 - \alpha. \tag{6}$$

The VaR_α is the largest value for X such that the probability of a loss over a day is no more than $1 - \alpha$. Consequently, the VaR can be viewed as a point estimate of potential financial loss.

Although the parameter α is arbitrarily chosen, the analysis in this study does not refer to the process of choosing the parameter of VaR which were considered to be $\alpha \in \{0.01, 0.05, 0.1\}$. Hence the crux of being able to provide an accurate estimate for the VaR is in estimating the cutoff return VaR_α.

Following the approach by Longin (1999a,b), and McNeil and Frey (2000), we use a two-stage approach to estimate the VaR of the five America Indexes negative daily log return time series.

(1) Fit a AR(1)-GARCH(1,1) model to the returns and use a quasi-maximum-likelihood approach to estimate parameters. Use the fitted model to standardize the raw returns to a strict white noise process, i.e. independent, identically distributed process with zero mean and unit variance.

(2) Use EVT to model the tail of the marginal distribution of the standardized returns, and use this EVT model to estimate VaR.

3.2   Standardization – Estimating σ_t and μ_t by QML

Since stock returns have heavy-tailed and/or outlier-prone probability distributions, we use GARCH models to deal with both the conditional heteroskedasticity and the heavy-tailed distributions of American Indexes returns. we consider the reason for outliers may be that the conditional variance is not constant, and the outliers occur when the variance is large. According to our test, the returns process is not Gaussian. Therefore, we assume the standardized, i.e. mean and variance adjusted American Index returns series is an i.i.d. white noise process with a generalized Pareto distribution.

Let $\{r_t\}_{t=T-n+1}^{T}$ be a strictly stationary time series representing the negative daily log return on a financial asset price. We fix a constant memory n so that at the end of day T our data consist of the last n negative daily log returns $\{r_{T-n+1}, \ldots, r_{T-1}, r_T\}$.

Let F_t be the information about the return process available up to time t. We assume that the dynamics of $\{r_t\}_{t=T-n+1}^{T}$ to be a realization from a AR(1)-GARCH(1,1) process, which are given by
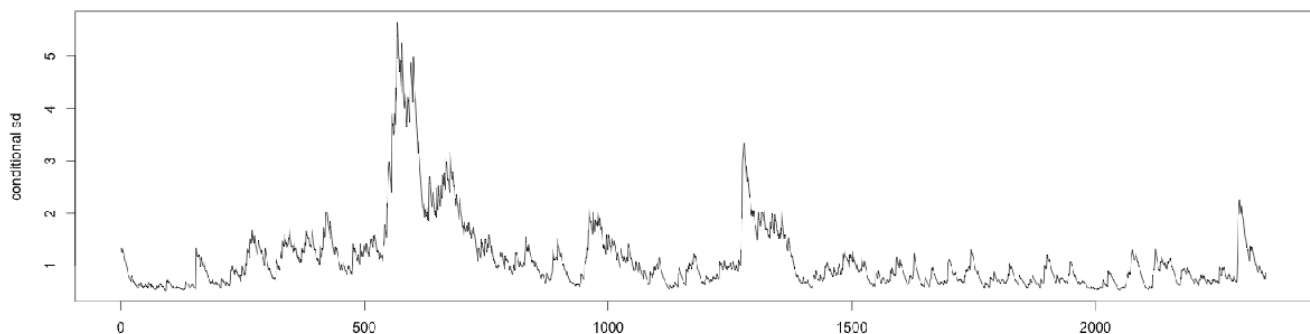
$$\begin{cases} r_t = \mu_t + \sigma_t z_t, \\ \mu_t = \phi_0 + \phi r_{t-1}, \\ \sigma_t^2 = \omega + \alpha(r_{t-1} - \mu_{t-1})^2 + \beta\sigma_{t-1}^2, \end{cases} \tag{7}$$

where the innovations z_i are white noise process with zero mean, unit variance, and marginal distribution function F; $w > 0, \alpha > 0,$ and $\beta > 0$; the conditional mean $\mu_i \mid \mathcal{F}_{i-1}$, and the conditional volatility $\sigma_i \mid \mathcal{F}_{i-1}$ are measurable, $\mathcal{F}_{i-1}$ is the information about the return process available up to time $t-1$.

This model is fitted using the quasi-maximum-likelihood estimation (QML) method, which assumes normal distribution and uses robust standard errors for inference. It means that the likelihood for a GARCH(1,1) model with normal innovations is maximized to obtain parameter estimates $\{\hat{w}, \hat{\alpha}, \hat{\beta}\}$. Although this amounts to fitting a model using a distributional assumption we do not necessarily believe, the QML method delivers reasonable parameter estimates. Bollerslev and Wooldridge (1992) proved that if the mean and the volatility equations are correctly specified, the QML estimates are consistent and asymptotically normally distributed. From Eq.(7), we get estimates of the conditional mean $\{\hat{u}_{T-n+1}, \cdots, \hat{u}_{T-1}, \hat{u}_{T}\}$ and the conditional

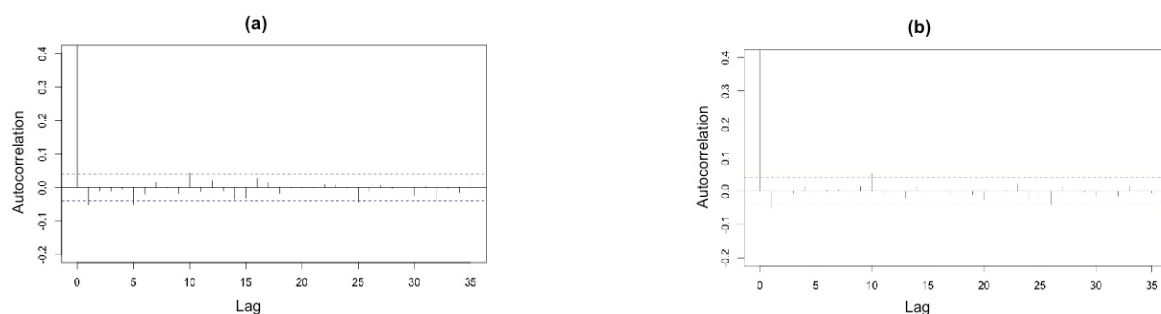volatility $\{\hat{\sigma}_{T-n+1}, \cdots, \hat{\sigma}_{T-1}, \hat{\sigma}_{T}\}$ of $\{r_i\}_{i=T-n+1}^{T}$. The estimated conditional volatility of the S&P 500

returns derived from the GARCH fit2 is shown in Fig.4.

FIGURE 4: ESTIMATE OF THE CONDITIONAL STANDARD DEVIATION DERIVED FROM QML FITTING OF AR(1)-GARCH(1,1) MODEL OF THE S&P 500 NEGATIVE DAILY LOG RETURNS (2006-7-17 TO 2015-11-13).

To check the adequacy of the model and to use in next stage of the approach, we calculate the standardized returns

$$\{z_{T-n+1}, z_{T-n+2}, \cdots, z_{T}\} = \{\frac{\hat{r}_{T-n+1} - \hat{\mu}_{T-n+1}}{\hat{\sigma}_{T-n+1}}, \frac{\hat{r}_{T-n+2} - \hat{\mu}_{T-n+2}}{\hat{\sigma}_{T-n+2}}, \cdots, \frac{\hat{r}_{T} - \hat{\mu}_{T}}{\hat{\sigma}_{T}}\}.$$

(8)

FIGURE 5: SAMPLE AUTOCORRELATIONS OF (A) STANDARDIZED RETURNS AND (B) SQUARED STANDARDIZED RETURNS OF THE S&P 500 (2006-7-17 TO 2015-11-13).

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[2]In this research, the AR(1)-GARCH(1,1) model is fit to the American Indexes negative daily log returns using R's garchFit function in the fGarch package[32].

The standardized returns should be i.i.d. if the fitted model is tenable. In Fig. 5, we plot the sample autocorrelation of the standardized S&P 500 returns as well as the squared standardized S&P 500 returns. As shown in Fig.3, while the raw returns are clearly not i.i.d., this assumption may be tenable for the standardized returns.

We end the standardization stage by calculating estimates of the conditional mean and variance for day $T+1$, which are the 1-step forecasts

$$\hat{\mu}_{T+1} = \hat{\phi}_0 + \hat{\phi} r_T,$$
$$\hat{\sigma}_{T+1}^2 = \hat{\omega} + \hat{\alpha}(r_T - \hat{\mu}_T)^2 + \hat{\beta}\hat{\sigma}_T^2.$$

(9)

3.3 VaR Estimation – Apply Extreme Value Approach

In the second stage, we estimate the upper tail behavior of the cumulative distribution function of the standardized returns F by using the Extreme Value Theory (EVT). EVT is experiencing a boom in the financial field, especially with respect to the application to the market risk measure VaR. Its appearance as a popular instrument for estimating VaR can be explained as a consequence of two factors. On the one hand, the assumption of normality of financial markets does not reflect the reality of the situation. As a consequence, the VaR estimation methods which based on the normality assumption may provide inaccurate estimates. Historical or Monte Carlo simulation methods arise as alternative methods. But given the difficulties and "slowness" of these methods, EVT has been used as a powerful solution for estimation of VaR. On the other hand, although VaR can be calculated with simulation methods, it still has limitations, so this measure needs to be complemented with others. We present it as a way of solving the problem of fat tails when calculating VaR.

The mathematical foundation of EVT is the class of extreme value limit theories, originally stated by Fisher and Tippett (1928) and later derived rigorously by Gnedenko (1943). The central result in EVT is that the extreme tail of a wide range of distributions can approximately be described by the Generalized Pareto distribution (GPD), which is derived by Smith(1989), Davison and Smith (1990).

For a random variable X, we first fix some high threshold μ and consider the distribution of excess values $Y = X - \mu$ as

$$F_\mu(y) = \Pr(X - \mu \le y \mid X > \mu) = \frac{F(\mu + y) - F(\mu)}{1 - F(\mu)},$$

(10)
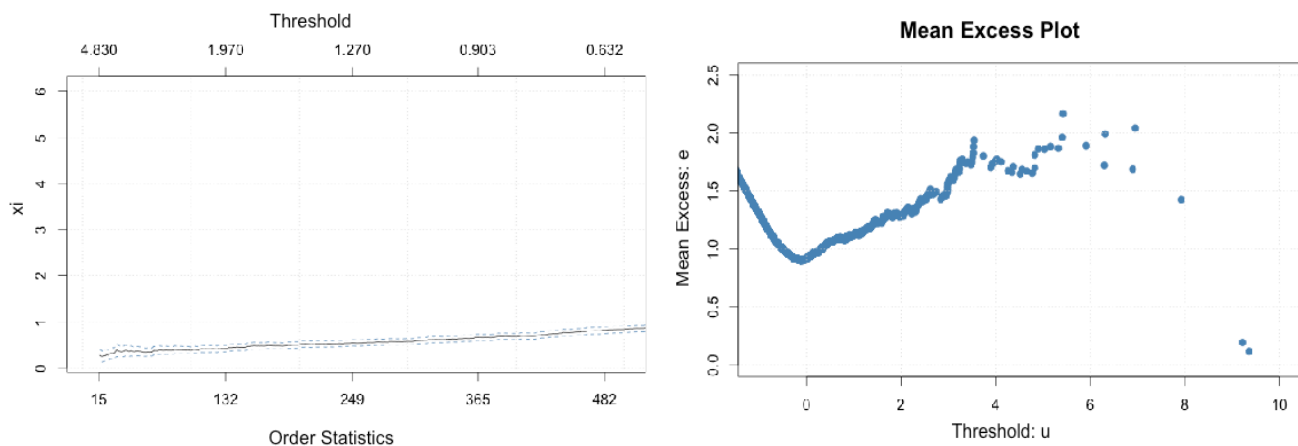
where F is the underlying distribution of X, F_μ is the conditional excess distribution function. Pickands (1975) introduced the GPD as a two parameter family of distributions for exceedances over a threshold. More precisely, it was proved that for a large class of underlying distribution functions F, the conditional excess distribution function $F_\mu(y)$, as $\mu \to \omega_F = \sup\{x : F(x) < 1\}$, is well approximated by $F_\mu(y) = H_{\sigma,\xi}(y)$, where $H_{\sigma,\xi}(y)$ is called GPD, specified as

$$H_{\sigma,\xi}(y) = 1 - (1 + \xi \frac{y}{\sigma_\mu})^{-1/\xi}.$$

(11)

The parameters of GPD are the scale parameter σ_μ and the shape parameter ξ.

EVT describes specifically at the distribution of the standardized returns in the tails. The tail fatness of the distribution is reflected by the shape parameter: the case when $\xi < 0$ means thin tails, $\xi = 0$ means the kurtosis is 3 as for the standard normal distribution; while $\xi > 0$ implies fat tails. Therefore, the shape parameter measures the speed with which the distribution's tail approaches zero. The fatter the tail, the slower the speed and the higher the shape parameter is. Since almost all returns in EVT assume that the returns are i.i.d., the analysis was developed on the standardized returns which, in many cases, could be reasonably assumed to be i.i.d.. Because we are interested in extreme negative returns, we use EVT to model the right tail of the distribution, i.e. the standardized returns in excess of a high threshold.

It is necessary to choose a specific threshold to confine the estimation to those observations that are above the given threshold. However, it is difficult to apply threshold based methods because of the lack of a clear-cut criterion for choosing the threshold. If the threshold is chosen too low, the GPD may not be a good fit to the excesses over the threshold, and consequently there will be a bias in the estimates. Conversely, if the threshold is too high, then there are not enough exceedances over the threshold to obtain reliable estimates of the extreme value parameters, and consequently, the variances of the estimators will be high. In this paper, an optimal threshold is selected by employing graphical methods, known as the

Hill plot and the mean excess plot. The Hill plot displays the estimated values of shape parameter ξ as a function of the cut-off threshold in order to find some interval of candidate cut-off points that yields stable estimates of the shape parameter ξ. Technical details about Hill plot can be found in Hill (1975). The mean excess function is the mean of exceedances over a threshold. If the underlying distribution of those exceedances follows a GPD, then the corresponding mean excess must be linear in the threshold. Details about the mean excess plot are described in Davison and Smith (1990). Fig.6 shows the Hill plot and the mean excess plot of the negative daily S&P 500 log returns. A threshold 1.968748, with 132 exceedances, seems to be reasonable for the S&P 500 returns.

FIGURE 6: HILL PLOT (LEFT) AND MEAN EXCESS PLOT (RIGHT) OF THE S&P 500 NEGATIVE DAILY LOG RETURNS (2006-7-17 TO 2015-11-13)

We have seen that the GPD contains two parameters, shape parameter ξ and location parameter σ_μ. They can be estimated by using either parametric or non-parametric methods. From the research of Hosking and Wallis (1987), for the tail index $\xi > 0.5$, it can be shown that maximum likelihood regularity conditions are fulfilled and that maximum likelihood estimates $\{\sigma_\mu, \xi\}$ based on a sample of n excesses are asymptotically normally distributed. Therefore, we use the parametric approach, maximum likelihood method (MLE), to estimate parameters in GPD.

Next, we make explicit the relationship between excess value and the standardized return series, denoted as $\{z_i\}$. We may use the following relationship to estimate the VaR of the standardized asset returns $\{z_i\}$.

Assume that $\{z_i\}$ are i.i.d. random variable with CDF F, and a high enough threshold μ is given. Define $N_\mu = card\{i : z_i > \mu, i = 1, \cdots, n\}$. Then

$$F_\mu(y) = \Pr(z_i - \mu \le y \mid z_i > \mu) = \frac{F(\mu + y) - F(\mu)}{1 - F(\mu)},$$

$$\bar{F}_\mu(y) = \Pr(z_i - \mu > y \mid z_i > \mu) = \frac{\bar{F}(\mu + y)}{\bar{F}(\mu)},$$

i.e.

which is equivalently to

$$\overline{F}(\mu + y) = \overline{F}(\mu)\overline{F}_\mu(y).$$

Then, the estimators of $\overline{F}(\mu)$ and $\overline{F}_\mu(y)$ can be written as:

$$\hat{\overline{F}}(\mu) = \frac{1}{n}\sum_{i=1}^{n} I(X_i > \mu) = \frac{N_\mu}{n},$$

$$\hat{\overline{F}}(y) = 1 - H_{\hat{\sigma}_\mu, \hat{\xi}}(y) = (1 + \hat{\xi}\frac{y}{\hat{\sigma}_\mu})_+^{-1/\hat{\xi}},$$

(12)

where $\hat{\xi}$ and $\hat{\sigma}_\mu$ are maximum likelihood estimators of the shape parameter ξ and the location parameter σ_μ. Therefore the tail estimator can be written as:

$$\hat{\overline{F}}(\mu + y) = \frac{N_\mu}{n}(1 + \hat{\xi}\frac{y}{\hat{\sigma}_\mu})_+^{-1/\hat{\xi}}.$$

(13)

This relationship between probabilities allows us to obtain VaR for the original asset return series $\{r_i\}$. More precisely, for a specified small probability α such that the $\alpha - th$ upper tail quantile VaR of $\{r_i\}$ is v. Consequently, for a given small probability α, one can check that the VaR of holding a long position in the asset underlying return $\{r_i\}$ is

$$\alpha = \Pr(r_{T+1} > v) = \Pr(z_{T+1} > \frac{v - \mu_{T+1}}{\sigma_{T+1}}) = \Pr(z_{T+1} - \mu > \frac{v - \mu_{T+1}}{\sigma_{T+1}} - \mu \mid z_{T+1} > \mu) = \overline{F}_\mu(\frac{v - \mu_{T+1}}{\sigma_{T+1}} - \mu) = \frac{\overline{F}(\frac{v - \mu_{T+1}}{\sigma_{T+1}})}{\overline{F}(\mu)},$$

$$VaR_\alpha = \begin{cases} \left(\mu + \frac{\hat{\sigma}_\mu}{\hat{\xi}}\left((\frac{n}{N_\mu}\alpha)^{-\hat{\xi}} - 1\right)\right)\sigma_{T+1} + \mu_{T+1}, \hat{\xi} \neq 0, \\ \left(\mu + \hat{\xi}\ln(\frac{n\alpha}{N_\mu})\right)\sigma_{T+1} + \mu_{T+1}, \hat{\xi} = 0. \end{cases}$$

(14)

We favor the extreme value approach, or the GPD approach in this study to tail estimation mainly for three reasons. One is that in finite samples of the order of points from typical return distributions, EVT quantile estimators are more efficient than the historical simulation method. Moreover, considering the fact that most financial returns series are asymmetric, the EVT approach is advantageous over models which assume symmetric distributions such as t-distributions, GARCH distribution family. In addition, comparing with Hill method which is designed specifically for the heavy tail ($\xi > 0$) data, the EVT approach to VaR has larger applicability since it also applicable to light tail ($\xi = 0$) cases or even short tail ($\xi < 0$) cases.

IV. EMPIRICAL RESULTS AND CALENDAR EFFECT ANALYSIS

We backtest the approach on the five Americas Indexes historical series of negative daily log returns: the the Standard and Poors index S&P 500, the financial index SPDR ETF, the technology index NASDAQ-100, the utility index Dow Jones Utility Average, and the transportation index Dow Jones Transportation Average. As introduced in Section 2, we excerpt all five indexes' adjusted closing price from July 17, 2006 to November 13, 2015.

To backtest the two stage approach, we first estimate σ_t and μ_t and use it to standardize the daily negative log returns $\{r_i\}$. The reason we use the negative returns is that loss occurs when the returns are negative for a long financial position. We show the estimation results of AR(1)-GARCH(1,1) model for Americas Indexes negative daily log returns in Table 2. After getting the standardized returns $\{z_i\}$, we apply the second stage to test the calendar effect on Americas Indexes returns.

TABLE 2
ESTIMATION RESULTS OF AR(1)-GARCH(1,1) MODEL FOR AMERICAS INDEXES NEGATIVE DAILY LOG RETURNS FROM 2006-7-17 TO 2015-11-13

Parameter	S&P	XLF	NDXT	DJU	DJT
Mean equation AR(1)					
ϕ_0	-0.069493	-0.037231	-0.082928	-0.042878	-0.060223
ϕ	-0.058278	-0.079456	-0.015441	-0.029874	-0.016125
Variance equation GARCH(1,1)					
ω	0.025183	0.027637	0.026652	0.018436	0.028343
α	0.114753	0.128632	0.078668	0.097303	0.077237
β	0.867333	0.867966	0.908884	0.889307	0.910656
LogLikelihood	-3342.299	-4189.893	-4062.424	-3268.324	-4117.66
p-Value of Standardised Residuals Tests					
Shapiro-Wilk Test	0	1.92972e-15	6.597162e-11	9.209965e-13	7.049808e-10
Ljung-Box Test Q(10)	0.1749702	0.02211829	0.4162687	0.6369799	0.3831675
Ljung-Box Test Q(15)	0.1254892	0.03006419	0.4009677	0.8425757	0.2452127
Ljung-Box Test Q(20)	0.2380792	0.06046306	0.4091626	0.9131188	0.1832375
Ljung-Box Test Q(10) for squared residuals	0.02556951	0.4572453	0.7371797	0.06534186	0.08447315
Ljung-Box Test Q(15) for squared residuals	0.05896544	0.4860042	0.8815682	0.1705194	0.2434914
Ljung-Box Test Q(20) for squared residuals	0.1596955	0.5549062	0.7223037	0.3516701	0.2387464

4.1　Seasonal Effect on Americas Indexes VaR

Because a three-month period on a financial calendar acts as a basis for the reporting of stock earnings and the paying of dividends, the seasonal effect is a vital factor in determining stock performance. To identify the existence of seasonal effect on Americas Indexes returns, we divide each Americas Index's standardized returns $\{z_i\}$ into the following four subsets:

$$\{z_i \mid t \in Q_i\}, i = 1,2,3,4,$$

which are referred to the four quarters Index returns. A quarter refers to one-fourth of a year and is typically expressed as Q. Table 3 provides a summary of descriptive statistics for those considered return series.

TABLE 3
DESCRIPTIVE SUMMARY STATISTICS OF AMERICAS INDEXES SEASONAL STANDARDIZED RETURNS

Summary Statistics	S&P	XLF	NDXT	DJU	DJT
Observations n					
Q1	550	550	565	550	550
Q2	569	569	579	569	569
Q3	626	626	639	626	626
Q4	604	604	614	605	604
Skewness					
Q1	0.81	0.6	0.45	0.42	0.32
Q2	0.56	0.34	0.16	0.45	0.4
Q3	0.53	0.27	0.35	0.35	0.21
Q4	0.23	0.03	0.16	0.19	0.24
Kurtosis					
Q1	3.15	2.97	1.19	1.3	1.23
Q2	0.56	0.68	0.33	0.54	0.47
Q3	1.32	1.5	0.95	1.7	0.59
Q4	0.73	0.87	0.39	0.85	0.49
Shapiro-Wilk normality test p-Value					
Q1	1.021e-10	1.959e-09	2.88e-06	3.952e-06	3.669e-05
Q2	1.37e-07	0.0004483	0.001367	1.454e-05	0.0001729
Q3	3.42e-09	1.923e-07	3.879e-05	2.109e-06	0.003301
Q4	0.0001126	0.005276	0.06336	0.006695	0.02605

Table 3 reports skewness and kurtosis for the standardized return series of each quarter. In statistics, skewness and kurtosis, which are normalized third and fourth central moments of a process, are often used to summarize the extent of asymmetry and tail thickness. For the normal distribution, kurtosis is 3. We observe that distributions of all four seasons' standardized returns are positively skewed, indicating that they are nonsymmetric. Further, except for the S&P 500 $Q1$ standardized returns, all kurtosis are less than 3, indicating that those series have distributions with tails that are thinner than those of the normal distribution. This indication of non-normality is also supported by the Shapiro-Wilk test results, which reject the null hypothesis of a normal distribution at 5% significance level. Based on the Ljung-Box test results from Table 2 and the Shapiro-Wilk test results from Table 3, we consider all five Americas Indexes seasonal standardized returns are i.i.d. and non-normally distributed processes.

TABLE 4

RESULTS FROM FITTED GPD FOR STANDARDIZED RETURNS & ESTIMATES FOR VARS FOR NEGATIVE DAILY LOG RETURNS

Standardized Return (Threshold μ)	S&P (1.651907)	XLF (1.658118)	NDXT (1.595358)	DJU (1.630336)	DJT (1.590404)
Shape Parameter ξ					
Q1	0.01263226	0.08324439	0.2507826	-0.2261632	-0.05614722
Q2	-0.4155444	0.04520885	-0.8514776	-0.2794260	-0.06843322
Q3	-0.4418723	-0.1682089	-0.0387090	0.1329915	-0.2223172
Q4	-0.1534931	0.1375718	-0.1974187	0.06310129	-0.1744764
Scale Parameter σ_μ					
Q1	0.77437179	0.77968703	0.4372119	0.9268999	0.68909844
Q2	0.9013567	0.53571577	0.9237556	0.7847136	0.61659084
Q3	1.2207484	0.8877799	0.6161069	0.6354024	0.7442377
Q4	0.6340782	0.4532491	0.6423728	0.49605948	0.6551974
Exceedances N_μ					
Q1	37	36	44	31	39
Q2	44	46	45	44	43
Q3	41	44	48	35	46
Q4	34	33	35	33	32
95% quantile VaR (VaR$_{0.05}$) of original negative daily log returns					
Q1	1.61934	1.947586	2.00777	1.996732	1.908087
Q2	1.739842	2.000061	2.166308	2.252225	1.921836
Q3	1.694324	2.036937	2.059414	1.951028	1.947265
Q4	1.474158	1.759062	1.864537	1.917086	1.691029
99% quantile VaR (VaR$_{0.01}$) of original negative daily log returns					
Q1	2.797445	3.453345	3.147227	3.459343	3.032853
Q2	2.562514	3.002238	2.820178	3.334415	2.909932
Q3	2.857624	3.333057	3.172462	3.341999	2.947104
Q4	2.301884	2.665381	2.879703	2.932556	2.675514

Thereafter, we apply the extreme value approach to the considered seasonal standardized return series. Table 4 summarizes estimation results of the shape parameter ξ, the scale parameter σ_μ from fitted GPD model for the standardized Americas Indexes returns as well as 0.95 quantile VaR and 0.99 quantile VaR for the original considered negative daily log returns. To better investigate systematic seasonal differences for Americas Indexes VaR, we first find a proper threshold for each seasonal series by Hill plot and then choose the highest one as the common threshold for all four seasonal series. Using the same threshold allows a better comparison of the quarterly VaRs.

From Table 4, we find that for S&P 500, at a quantile level of 95%, the smallest estimated VaR among all seasons is 1.474158 for the $Q4$ returns; at a quantile level of 99%, the smallest estimated VaR is 2.301884 for the $Q4$ returns as well. This implies that, in the fourth season, with the AR(1)-GARCH(1,1)-GPD model, we are 95% confident that the expected overall Americas equity market value would not lose more than 1.474158% for the worst case scenario; we are 99% confident that the expected market value of the S&P 500 would not lose more than 2.301884%. Similar interpretations can be made for the other Americas Indexes.

In comparison of all four seasonal returns, it is also interesting to note that our model produced the smallest VaR in the fourth season, at the 95% quantile level for all five America Indexes. While at the 99% quantile level, except for the NASDAQ-100 technology index, the four seasonal VaRs exhibits analogous characteristics as observed from 95% quantile VaRs under different Americas Indexes seasonal returns. Moreover, given the quantile levels, the corresponding VaR estimates for S&P 500 seasonal returns are less than the rest Indexes seasonal returns. It indicates that the trading risk of S&P 500 is the smallest among all five Americas Indexes.

Our findings have important implications for investors and financial institutions. For example, for conservative investors who would prefer lower risk, they can choose to trade during the lower VaR period or trade lower risk stocks to avoid potential high loss.

4.2 Day-of-the-Week Effect on Americas Indexes VaR

To formally test the timing and existence of weekly patterns, we divide the whole standardized returns $\{z_t\}$ to five subsets by day-of-the-week, which is written as:

$$\{z_t \mid d_{it} = 1\}, i = 1,2,3,4,5,$$

where d_{it} are dummy variables such that if day t is a Monday $d_{1t} = 1$, remove the data if $d_{1t} = 0$; if day t is a Tuesday $d_{2t} = 1$, otherwise remove the data, etc. The five subsets are the considered Indexes weekly returns for Monday through Friday respectively. The basic statistical characteristics of the five return series are calculated and shown in Table 5.

TABLE 5
DESCRIPTIVE SUMMERY STATISTICS OF AMERICAS INDEXES WEEKLY STANDARDIZED RETURNES

Summary Statistics	S&P	XLF	NDXT	DJU	DJT
Observations n					
Monday	442	442	468	442	442
Tuesday	480	480	483	481	480
Wednesday	483	483	485	483	483
Thursday	473	473	479	473	473
Friday	471	471	482	471	471
Skewness					
Monday	0.41	0.24	0.22	0.17	0.41
Tuesday	0.77	0.68	0.31	0.47	0.12
Wednesday	0.04	0.27	0.08	-0.1	0.08
Thursday	0.18	0.03	0.38	0.47	0.47
Friday	0.65	0.43	0.49	0.83	0.53
Kurtosis					
Monday	1.22	1.15	1.28	2.04	0.8
Tuesday	3.1	3.14	0.88	0.62	0.3
Wednesday	0.47	1.01	0.61	0.41	0.57
Thursday	0.94	0.78	0.44	0.77	0.52
Friday	1.04	1.1	0.41	1.82	1.69
Shapiro-Wilk normality test p-Value					
Monday	2.07e-06	0.0008976	0.0004619	1.911e-05	0.0007413
Tuesday	2.242e-09	1.94e-09	0.001296	9.745e-05	0.5958
Wednesday	0.002344	0.001305	0.005957	0.4103	0.00485
Thursday	3.495e-08	0.006136	0.0004881	8.066e-05	0.0001246
Friday	8.661e-08	8.494e-05	6.554e-06	2.965e-09	4.802e-06

Except the Dow Jones Utility Average Wednesday standardized returns, the distribution of the rest weekday's standardized returns are slightly positively skewed, indicating that they are nonsymmetric. The kurtosis of the S&P 500 and the SPDR ETF Tuesday standardized returns show Gaussian property while the rest of the week returns are substantially departure from

normal distribution. The Shapiro-Wilk test results also indicates that normal distribution is not a realistic assumption for the weekly standardized returns for considered Indexes.

Next, we apply the VaR estimation approach to capture the day-of-the-week effect on Americas Indexes VaR and show the result in Table 6. Given Americas Indexes and the 99% quantile level, we captured the comparatively low risk in Wednesday. Among the five Americas Indexes, the number of exceedances is comparatively small in Monday and Friday.

V. CONCLUSION

TABLE 6
RESULTS FROM FITTED GPD FOR STANDARDIZED RETURNS & ESTIMATES FOR VARS OF NEGATIVE DAILY LOG RETURNS

Standardized Return (Threshold μ)	S&P (1.685563)	XLF (1.597324)	NDXT (1.616329)	DJU (1.498996)	DJT (1.654929)
Shape Parameter ξ					
Monday	-0.490609	0.08324439	0.3182840	0.2423261	-0.4065323
Tuesday	0.09756212	0.1224898	0.0999698	-0.1381534	-0.1377236
Wednesday	-0.1117621	0.1075196	-0.2249917	-0.1726082	-0.2138769
Thursday	-0.3322698	-0.1846136	-0.3958165	0.06310129	-0.1077049
Friday	-0.5091703	-0.1816384	-0.1592645	-0.174639	-0.09100887
Scale Parameter σ_μ					
Monday	1.239719	0.77968703	0.3634457	0.4779636	1.0962016
Tuesday	0.65606741	0.7608726	0.5040591	0.6698328	0.6560630
Wednesday	0.5055943	0.4589546	0.6706803	0.5467823	0.6069613
Thursday	-0.3322698	0.7758208	0.9214631	0.49605948	0.6291783
Friday	1.1695172	0.8252605	0.5823002	1.037172	0.87551725
Exceedances N_μ					
Monday	25	30	29	26	22
Tuesday	33	36	33	37	27
Wednesday	30	35	27	44	40
Thursday	38	39	39	33	35
Friday	24	28	39	32	17
95% quantile VaR (VaR$_{0.05}$) of original negative daily log returns					
Monday	1.574378	1.9127	1.886357	1.844478	1.714618
Tuesday	1.63393	1.994884	1.976933	2.085929	1.802911
Wednesday	1.537115	1.838576	1.875154	2.123752	2.034037
Thursday	1.827274	2.054521	2.272032	1.949974	1.981222
Friday	1.456647	1.802248	2.10879	2.121393	1.406727
99% quantile VaR (VaR$_{0.01}$) of original negative daily log returns					
Monday	2.784977	3.423013	2.848478	3.020582	3.117782
Tuesday	2.733362	3.554699	3.044567	3.180491	2.810845
Wednesday	2.214732	2.758804	2.914875	2.956329	2.836568
Thursday	2.776619	3.134498	3.335698	2.981338	2.945308
Friday	2.642806	3.025973	3.009405	3.77907	2.867233

With the empirical analysis of this paper we demonstrate how we can use a GARCH based dynamic EVT approach to model VaR for short term forecasting. The dynamic EVT method has the advantage of dynamically reacting to changing market conditions which is useful in getting better VaR forecasts. We apply the two stage approach on five Americas Indexes negative daily log return series. Empirical findings in this paper show that both seasonal effect and day-of-the-week effect are present in Americas Indexes returns. We captured the comparatively low VaR in $Q4$ and Wednesday for considered returns during the test period.

Overall, our findings have implications for investors, financial institutions, and futures exchanges. For example, for conservative investors who would prefer lower risk, they can choose to trade during the lower VaR period to avoid potential high loss. The two stage approach to VaR can also be used in other stock or asset returns. Finally, it has significant value for investors and regulators in terms of an in depth analysis of the equity market.

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