Synthesis of Discrete State Observer by One Measurable State Variable and Identification of Immeasurable Disturbing Influence

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Abstract *– The entire research presents synthesis of discrete state observer by one measurable state variable and identification of immeasurable disturbing influence, aiming to suggest a procedure for reconstruction of the immeasurable disturbing influence. The suggested solution aims also optimization according to the time of completion of the reconstruction process.*

Keywords *–* ***discrete state observer, disturbing influence, immeasurable disturbing influence, measurable state variables, observer synthesis, state space, state space model, pole placement.***

I. INTRODUCTION

For the aim of analysis and synthesis in state space, an option for direct measurement or reconstruction of the state variables is needed. If such information is hard to be obtained even in case of availability of methods for direct measuring, it is often considered the use of information needed received through other channels. For the purpose of collecting the needed information for the state variables of the system a wide use of observers of full and reduced order, steady-state error free or with steady-state error, is natural. Beside the number of methods and approaches every practical realization is still a matter of interest, especially when if immeasurable disturbing influence is to be taken into consideration for analysis and synthesis.

II. FORMULATION OF THE PROBLEM

The object considered for analysis is continuous linear time-invariant system with structure presented in Fig.1 [1,3].

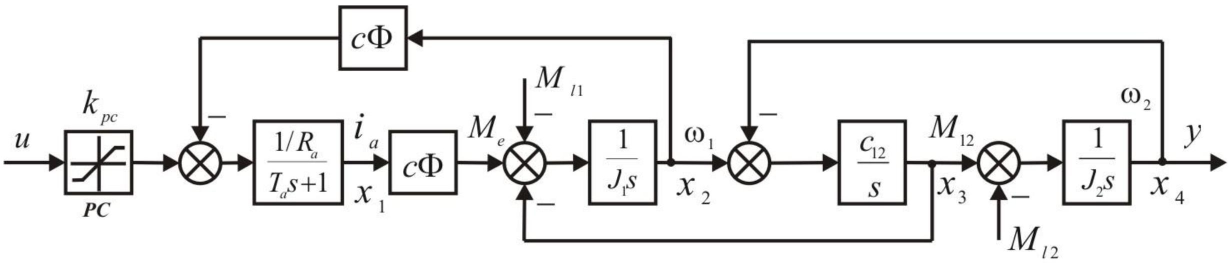

FIGURE 1. STRUCTURE OF THE PLANT UNDER CONSIDERATION

The continuous linear time-invariant system is considered in particular dual-mass DC electromechanical plant. For the purpose of the synthesis of state observer the plant will be described and analyzed in the state space. Considering eventual technical realization there is a set of state variables chosen, which elements corresponds to real physical variables as follows: x_1 - armature current i_a; x_2 - rotor speed ω_1; x_3 – engine elastic torque M_{12} and x_4 – working/actuating machine speed ω_2 (plant output). As a result and with accordance with the structure of the plant, presented above the following system of equations describes the state variables of the plant:

$$\left|\begin{array}{l} \dot{x}_1 = \frac{1}{T_a R_a}\left(k_{pc}u - c\Phi x_2\right) - \frac{1}{T_a}x_1 \\ \dot{x}_2 = \frac{1}{J_1}\left(c\Phi x_1 - x_3 - M_{l1}\right) \\ \dot{x}_3 = c_{12}\left(x_2 - x_4\right) \\ \dot{x}_4 = \frac{1}{J_2}\left(x_3 - M_{l2}\right) \end{array}\right. \tag{1}$$

In regards to the basis considered the plant can be described in the state space with the following equations:

$$\begin{array}{l} \dot{\boldsymbol{x}} = \boldsymbol{A}x + \boldsymbol{b}u + \boldsymbol{v} \\ y = \boldsymbol{c}^{\boldsymbol{T}}x + \boldsymbol{d}u \end{array} \tag{2}$$

where x is an n-dimensional state vector, u is an r-dimensional input vector, and y is an m-dimensional output vector, $\boldsymbol{A}$, $\boldsymbol{b}$, $\boldsymbol{c}$, $\boldsymbol{d}$ are the system matrices of appropriate sizes and $\boldsymbol{v}$ is the disturbance input matrix.

Therefore matrices $\boldsymbol{A}$, $\boldsymbol{b}$, $\boldsymbol{c}$, $\boldsymbol{d}$ and $\boldsymbol{v}$ of equation (2) are:

$$\boldsymbol{A}=\begin{bmatrix}-\frac{1}{T_a} & -\frac{c\Phi}{T_aR_a} & 0 & 0\\ \frac{c\Phi}{J_1} & 0 & -\frac{1}{J_1} & 0\\ 0 & c_{12} & 0 & -c_{12}\\ 0 & 0 & \frac{1}{J_2} & 0\end{bmatrix};\boldsymbol{b}=\begin{bmatrix}\frac{k_{pc}}{R_aT_a}\\ 0\\ 0\\ 0\end{bmatrix};\boldsymbol{c}^{\boldsymbol{T}}=[0\quad 0\quad 0\quad 1];\boldsymbol{d}=0;\boldsymbol{v}=\begin{bmatrix}0 & 0\\ -\frac{1}{J_1} & 0\\ 0 & 0\\ 0 & -\frac{1}{J_2}\end{bmatrix}\begin{bmatrix}M_{l1}\\ M_{l2}\end{bmatrix} \quad (3)$$

Once the linear time-invariant system, presented by equations (2), is transferred to discrete model the below equations describe the discrete system in the state space:

$$\begin{aligned} x(k+1)&=\boldsymbol{A}^*x(k)+\boldsymbol{b}^*u(k)+\boldsymbol{v}^*(k)\\ y(k)&=\boldsymbol{c}^{*\boldsymbol{T}}x(k)+\boldsymbol{d}^*u(k)\end{aligned} \quad (4)$$

The following conditions should be noted for the plant under analysis:

(i) *Real measurable state variables to be considered are the armature current i_a and the rotor speed ω_1;*

(ii) *The load torque M_{l2} in the common case is considered as constant or unknown variable with small fluctuations;*

(iii) *State variables M_{12} (engine elastic torque) and ω_2 (working/actuating machine speed) are to be considered immeasurable.*

Even if a certain devices are being used for measurement of ω_1, it is preferable such devices not to be implemented. For observation and monitoring purposes is usually natural the implementation of a current measuring device (for the state variable i_a), thus in this case there is no need of engine speed measurement.

One of the premises for granting correct and error free estimations of state variables is to ensure that all of the variables influencing the system are measured. In this case in particular appears an element, for which direct measurement is not possible – the load torque M_{l2}. There is a need that this unknown and immeasurable element is measured or recognized – one of the options is by identifying it (observe it) as a new state variable. In this regard the entire research is aiming synthesis of discrete state observer by one measurable state variable and identification of immeasurable disturbing influence.

III. SOLVING THE PROBLEM

The first step in finding a solution of the problem defined is check performance for the state variables observability, which is only available in case of measurable armature current $i_a=x_1$. If $\boldsymbol{c}^{*\boldsymbol{T}}=[1\quad 0\quad 0\quad 0]$ and $y=x_1$, the observability matrix of the plant should be one of full rank: $rank(\boldsymbol{Q}_{\boldsymbol{ob}}^*)=n$, where n is the plant order.

As it was clarified above there is occurrence of disturbing influence for which direct information is not available and it is also considered immeasurable – load torque M_{l2}. Under those considerations the estimation of the state variables through the so defined observer will only be possible through feedback based on integral of the difference between the measurable state variables (in this case the measurable state variable is the armature current only $x_1=i_a$) and their estimated values ($\hat{x}_1$) [4]. The feedback defined as stated above will be implemented for reconstruction of $M_{l2}\rightarrow M_{l2}(k+1)=M_{l2}(k)+k_{ob}^*T_q\big(y_{ob}(k)-c_{ob}^{*T}\hat{x}(k)\big)$, where k_{ob}^* is coefficient of the feedback based on integral of the difference between the measurable state variable x_1 and its estimated value $\hat{x}_1$, and T_q is discretization period, used for the discrete model of the linear time-invariant system under investigation.

The observer equations then might be written as follows:

$$\begin{aligned}\hat{\boldsymbol{x}}(k+1)&=\boldsymbol{A}^*\hat{\boldsymbol{x}}(k)+\boldsymbol{B}_{\boldsymbol{ob}}^*\boldsymbol{u}_{\boldsymbol{ob}}(k)+\hat{\boldsymbol{f}}^*\boldsymbol{c}_{\boldsymbol{ob}}^{*\boldsymbol{T}}\big(\boldsymbol{x}(k)-\hat{\boldsymbol{x}}(k)\big)+\hat{\boldsymbol{v}}_{ob}(k)\\ y_{ob}(k)&=\boldsymbol{c}_{\boldsymbol{ob}}^{*\boldsymbol{T}}\boldsymbol{x}(k)\end{aligned} \quad (5)$$

where $\boldsymbol{c}_{\boldsymbol{ob}}^{*\boldsymbol{T}} = [1 \quad 0 \quad 0 \quad 0]$; $\hat{\boldsymbol{v}}_{ob}(k) = \bar{\hat{\boldsymbol{f}}}^{*}\widehat{M}_{l2}(k)$; $\boldsymbol{u}_{\boldsymbol{ob}}(k) = \begin{bmatrix} u(k) \\ M_{l1}(k) \end{bmatrix}$ and $\hat{\boldsymbol{f}}^{*}$ denotes feedback of the state variables; $\bar{\hat{\boldsymbol{f}}}^{*}$ is feedback based on integral of the difference between the measurable state variable x_1 and its estimated value $\hat{x}_1$; $\widehat{M}_{l2}(k)$ - estimation of the disturbance influence $M_{l2}(k)$.

If it is considered $\widehat{M}_{l2}(k+1) = \widehat{M}_{l2}(k) - k_{ob}^{*}T_q\tilde{x}_1(k)$, where $\tilde{x}_1(k)$ is an error of the estimation of state variable x_1, then the synthesized observer can be presented with the following equivalent matrix $\boldsymbol{A}_{obequiv}^{*} = \boldsymbol{A}^{*} - \hat{\boldsymbol{f}}^{*}\boldsymbol{c}_{\boldsymbol{ob}}^{*\boldsymbol{T}}$. Defined in this manner the synthesis of the observer will be formulated as synthesis of the feedback matrix $\hat{\boldsymbol{f}}^{*}$ elements and k_{ob}^{*}.

There is a requirement for the discrete observer to have zeroes poles, which will grant the shortest possible time for reconstruction of the state variables and identification of $\widehat{M}_{l2}(k)$. The procedure for pole placement in predefined area is explained and simulated in details in [1].The same procedure is applied for synthesis of the above described observer.

In this case the procedure is significantly simplified as the desired polynomial is the following: $h_d^{*}(z) = z^5$, e.g. the coefficients before grades of z which are less than 5, are zeroes. The elements of $\hat{\boldsymbol{f}}^{*}$ and k_{ob}^{*} could be obtained, considering that all the coefficients before the polynomial elements of the polynomial $h(z) = \left[z\boldsymbol{E} - \boldsymbol{A}_{obequiv}^{*}\right]$ (where $\boldsymbol{E}$ is an identity matrix) are required to be zeroes, with exception the one which is before the highest grade, which is equal to 1.

IV. EXPERIMENTAL RESULTS

Dual-mass DC electromechanical plant shown in fig.1 is operative through DC motor with the following parameters [1,3]:

$$U_{nom} = 220V;\ P_{nom} = 0{,}3kW;\ n_{nom} = 1000\ tr/min; I_{anom} = 2A; u \to 0 \div 10V;$$
$$R_a = 20{,}8828\Omega;\ T_a = 0{,}0126s; c\Phi = 1{,}702Vs;\ c_{12} = 1{,}702Nm;$$
$$k_{pc} = 22;\ J_1 = 0{,}042kgm^2;\ J_2 = 0{,}021kgm^2;\ M_{l1} = 0{,}3404Nm; M_{l2} = 3{,}0636Nm$$

For continuous-time state space system model we calculate:

$$\boldsymbol{A} = \begin{bmatrix} -\frac{1}{T_a} & -\frac{c\Phi}{T_aR_a} & 0 & 0 \\ \frac{c\Phi}{J_1} & 0 & -\frac{1}{J_1} & 0 \\ 0 & c_{12} & 0 & -c_{12} \\ 0 & 0 & \frac{1}{J_2} & 0 \end{bmatrix} = \begin{bmatrix} -79{,}3651 & -6{,}468451 & 0 & 0 \\ 40{,}52381 & 0 & -23{,}8095 & 0 \\ 0 & 1{,}702 & 0 & -1{,}702 \\ 0 & 0 & 47{,}619 & 0 \end{bmatrix}, \boldsymbol{b} = \begin{bmatrix} \frac{k_{pc}}{R_aT_a} \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 83{,}611 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\boldsymbol{c}^{\boldsymbol{T}} = [0 \quad 0 \quad 0 \quad 1], \boldsymbol{d} = 0, \boldsymbol{v} = \begin{bmatrix} 0 & 0 \\ -\frac{1}{J_1} & 0 \\ 0 & 0 \\ 0 & -\frac{1}{J_2} \end{bmatrix} \begin{bmatrix} M_{l1} \\ M_{l2} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ -23{,}8095 & 0 \\ 0 & 0 \\ 0 & -47{,}619 \end{bmatrix} \begin{bmatrix} 0{,}3403 \\ 3{,}0636 \end{bmatrix}$$

Discrete-time state space model (4) could be obtained from the continuous one (2), considering (1) and (3), using MATLAB function for continuous to discrete model transition – c2d(.). Discretization period is defined $T_q = 0{,}1s$ which is chosen based on requirement $T_q \leq (0{,}05 \div 0{,}1)t_c$. In this case t_c (forecasted control time) is set to be equal to the expectation for the continuous-time closed-loop system [2]. Considering these assumptions for the discrete-time state-space model (4) is calculated:

$$\boldsymbol{A}^{*} = \begin{bmatrix} -0{,}0291 & -0{,}05289 & 0{,}127 & -0{,}01104 \\ 0{,}3313 & 0{,}5864 & -1{,}672 & 0{,}1689 \\ 0{,}05688 & 0{,}1195 & 0{,}4648 & -0{,}1383 \\ 0{,}1383 & 0{,}3378 & 3{,}869 & 0{,}6337 \end{bmatrix}, \boldsymbol{b}^{*} = \begin{bmatrix} 0{,}0376 \\ 0{,}1438 \\ 0{,}0110 \\ 0{,}0168 \end{bmatrix}, \boldsymbol{c}^{*\boldsymbol{T}} = [0 \quad 0 \quad 0 \quad 1]; \boldsymbol{d}^{*} = 0$$

$$\boldsymbol{v}^{*}(k) = \begin{bmatrix} 0{,}1438 & 0{,}01675 \\ -1{,}959 & -0{,}2868 \\ -0{,}1689 & 0{,}3663 \\ -0{,}2868 & -4{,}156 \end{bmatrix} \begin{bmatrix} M_{l1}(k) \\ M_{l2}(k) \end{bmatrix}$$

For the matrix of observability according to the chosen discretization period ($T_q = 0{,}1s$) is calculated:

$$\boldsymbol{Q}_{ob}^{*} = \begin{bmatrix} 1 & -0{,}0291 & -0{,}0110 & -0{,}0045 \\ 0 & 0{,}3313 & 0{,}1129 & 0{,}0571 \\ 0 & 0{,}0569 & 0{,}0453 & -0{,}0236 \\ 0 & 0{,}1383 & 0{,}4157 & 0{,}4751 \end{bmatrix}$$

It is of full rank - $rank(\boldsymbol{Q}_{ob}^{*}) = 4$. Thus we can conclude that the discrete object is fully observable by one measurable state variable.

For the observer equations (5) the following expression will be valid:

$$\boldsymbol{A}_{obequiv}^{*} = \begin{bmatrix} -0{,}0291 - \hat{f}_1^{*} & -0{,}0529 & 0{,}127 & -0{,}011 & 0{,}01675 \\ 0{,}3313 - \hat{f}_2^{*} & 0{,}5864 & -1{,}672 & 0{,}1689 & -0{,}2868 \\ 0{,}0569 - \hat{f}_3^{*} & 0{,}1195 & 0{,}4648 & -0{,}1383 & 0{,}3663 \\ 0{,}1383 - \hat{f}_4^{*} & 0{,}3378 & 3{,}869 & 0{,}6337 & -4{,}156 \\ -k_{ob}^{*} T_q & 0 & 0 & 0 & 1 \end{bmatrix}, \boldsymbol{B}_{ob}^{*} = \begin{bmatrix} 0{,}8264 & 0{,}1438 \\ 3{,}163 & -1{,}959 \\ 0{,}2429 & -0{,}1689 \\ 0{,}3686 & -0{,}2868 \end{bmatrix}$$

$$\boldsymbol{c}_{ob}^{*T} = [1 \quad 0 \quad 0 \quad 0], \widehat{\boldsymbol{v}}_{ob}(k) = \bar{\bar{\boldsymbol{f}}}^{*} \widehat{M}_{l2}(k) = [0{,}01675 \quad -0{,}2868 \quad 0{,}3663 \quad -4{,}156]^{T} \widehat{M}_{l2}(k)$$

In this case in particular if we have to meet the zero poles observer requirement the procedure for pole placement [1] within elliptical area will produce the following:

- elongation of the center of the ellipse from the origin → $\mu = 0$;

- major semi-axis → $r_1 = 0$;

- minor semi-axis → $r_2 = 0$.

With the defined elliptical area is ensured reconstruction of state variables and identification of disturbance influence $M_{l2}(k)$ for $t = nT_q = 0{,}5s$.

As result of described synthesis procedure are calculated the following:

$$k_{ob}^{*} = 79{,}7643; \ \hat{\boldsymbol{f}}^{*} = [2{,}6558 \quad -35{,}0648 \quad 12{,}416 \quad -29{,}9285]^{T}$$

In fig.2 and Fig.3 are shown immeasurable disturbance $M_{l2} = 3{,}0636Nm$ (applied at the 8^{th} second of the experiment) and the start processes of the state variables under signal value $u = 9V$, as well as their estimations. All the estimated values in the MATLAB environment and the plotted graphics are indexed – “est”.

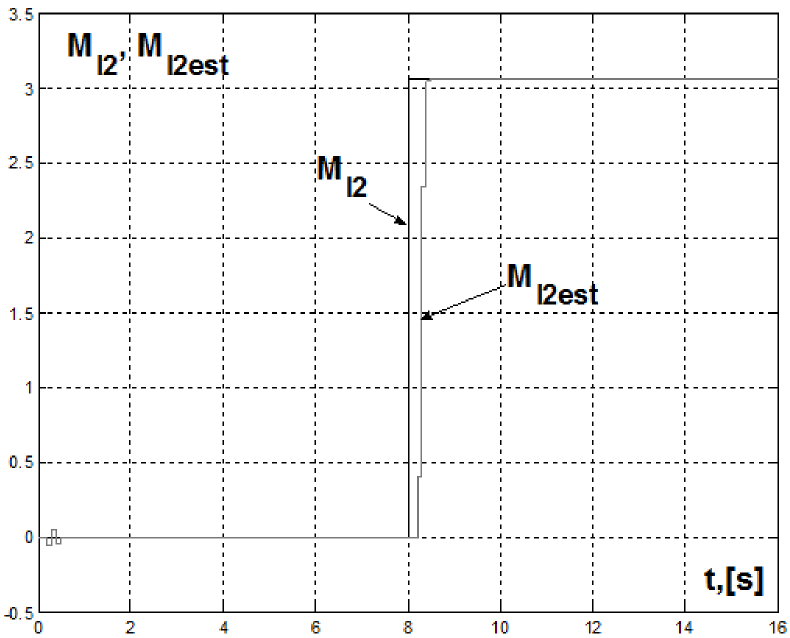

FIGURE 2. IMMEASURABLE DISTURBANCE $\mathbf{M_{l2}}$ AND ITS ESTIMATED VALUE $\widehat{\mathbf{M}}_{\mathbf{l2}}$

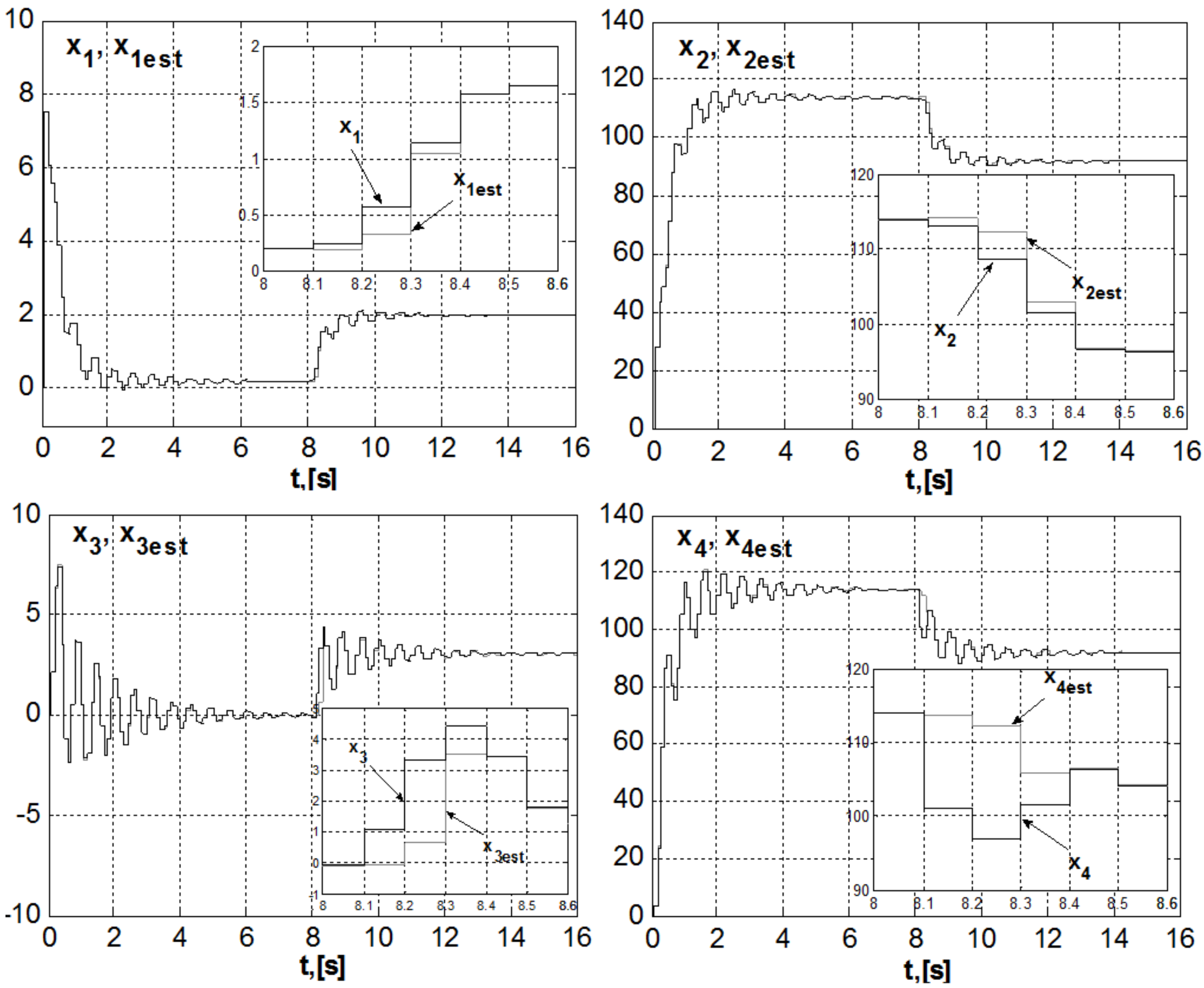

FIGURE 3. STATE VARIABLES AND THEIR ESTIMATIONS DURING THE START PROCESSES

In fig.4 is presented the block-diagram of the system "plant-observer", used as a basis for the simulating model built in MATLAB/Simulink for the purpose of experiment.

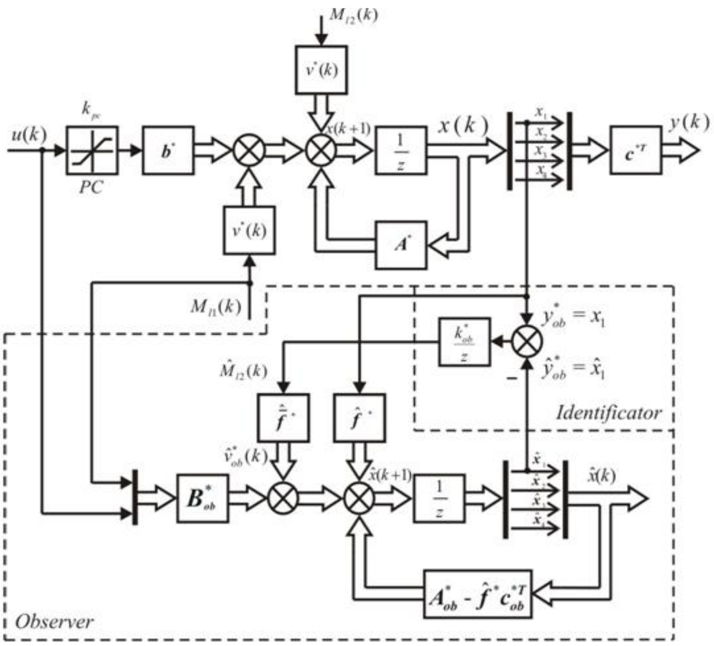

FIGURE 4. BLOCK DIAGRAM OF THE SYSTEM "PLANT-OBSERVER"

V. CONCLUSIONS

Analysis of the start processes shows that in $0{,}5s$ time are reconstructed the state space variables and identification of disturbance is performed. The steady state error is zero – practically no steady state error occurs. The synthesized discrete observer reconstructs the state variables and identifies the disturbance M_{l2} in terms of shortest period of time possible.

FUTURE DEVELOPMENTS

There are number of methods applicable for reconstruction of immeasurable disturbing influence and further research will be focused on factorial design with further implementation in modal control algorithm.

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