

# Two-Dimensional Numerical Simulation of Bed Level Variation around Vertical Wall Bridge Abutments

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**Abstract**—An accurate prediction of bed level variation and especially of the mechanism of local scour hole development around bridge abutments is of paramount importance, in river engineering, for a safe design of the construction. In the present research work, a two-dimensional, explicit, finite-volume numerical algorithm, which combines the hydrodynamic equations of viscous, unsteady, free-surface flow in rivers with the continuity equation for the conservation of sediment mass is used to simulate scour depth variation in the region of vertical wall abutments. The capabilities of the applied numerical model are demonstrated by comparing the computed results with available measurements of bed formation in the region of three orthogonal abutments, with different widths, normal to the flow direction. All the experimental results were conducted in a laboratory flume in Technological Educational Institute of Thessaly and scouring depths were obtained in the vicinity of each construction, for different inflow discharges and flow duration. Numerical simulation results of the maximum scour depth and of the developed scour whole area are satisfactorily compared with the experimental measurements. Comparisons show the accuracy and the validity of the applied two dimensional, movable bed numerical techniques.

**Keywords**—Abutment, Local scour, Two-dimensional sediment transport model.

## I. INTRODUCTION

The most common cause of bridge failures is from floods scouring bed material from around bridge foundations. Scour is the engineering term for the erosion caused by water of the soil surrounding a bridge foundation, piers and abutments. The basic mechanism causing local scour at piers or abutments is the formation of vortices at their base which removes bed material from around the base of the construction. Extensive research has been conducted to determine the depth and location of the scour hole that develops from the vortex that occurs at the abutment, and numerous abutment scour equations have been developed to predict this scour depth [5]. Numerous experimental investigations have been performed on the study of the flow, the bed level variation and mainly the scour mechanism in rivers and especially around bridge piers and abutments [9], [13], [7], [11], [2], [4] and others.

Besides experimental studies, several numerical investigations using Reynolds averaged Navier-Stokes equations of the flow have been developed to examine the flow structure in the hole of local scour and the development of local scour. Three-dimensional model provides the most realistic simulation of flow field under turbulence conditions adjacent to bridge piers and abutments. The development of the three-dimensional scour hole around a cylinder by solving simultaneously water flow field with sediment calculation was numerically simulated [12]. A 3-D time-accurate RANS solver with a nonlinear k- $\epsilon$  closure with wall functions was used to predict the scour evolution around an isolated vertical abutment (spur dike) in a channel [10]. Local scour depth around bridge pier and abutment, using the commercial solver Fluent with a user defined function for the calculation of channel bed elevation changes, was also numerically simulated [8].

The objective of this research work is to investigate bed formation in alluvial channels as well as in regions around vertical wall bridge abutments. For this purpose a two-dimensional, fully coupled hydrodynamic-sediment transport model was applied using an explicit finite-volume numerical technique [3]. The results of the model are verified by comparing them with available measurements of bed level variation around vertical wall abutments in uniform sediments under clear water scour conditions. The range of water discharge and width of the abutment was sufficient in order new and existing codes properly depict their capabilities against the satisfactorily compared results.

## II. TWO DIMENSIONAL HYDRODYNAMIC AND SEDIMENT TRANSPORT MODEL

A two-dimensional, subcritical, supercritical or mixed flow regime, fully coupled, free-surface flow and movable bed numerical model was developed to simulate flow-field and bed morphology variations in alluvial channels. Vertically averaged free-surface flow equations in conjunction with sediment transport equation were numerically solved using an

explicit, finite-volume scheme in integral form. Hydrostatic pressure distribution was assumed throughout the flow field and incompressible flow was simulated with wind and Coriolis forces neglected. The two-dimensional, unsteady, free-surface flow in channels with sediment transport and movable bed was described by a system of non-linear, parabolic, partial differential equations using the following equations [6]:

The water continuity equation:

$$-\frac{\partial h}{\partial t} = \frac{\partial(hu)}{\partial x} + \frac{\partial(hv)}{\partial y} \quad (1)$$

The flow momentum equation in the longitudinal direction:

$$\begin{aligned} -\frac{\partial(hu)}{\partial t} &= \frac{\partial(gh^2/2 + hu^2)}{\partial x} + \frac{\partial(huv)}{\partial y} + gh(S_{0x} + S_{fx}) - \\ &\frac{\partial}{\partial x} \left[ v_t \left( \frac{\partial(hu)}{\partial x} + \frac{\partial(hu)}{\partial x} \right) \right] - \frac{\partial}{\partial y} \left[ v_t \left( \frac{\partial(hu)}{\partial y} + \frac{\partial(hv)}{\partial x} \right) \right] \end{aligned} \quad (2)$$

The flow momentum equation in the transverse direction:

$$\begin{aligned} -\frac{\partial(hv)}{\partial t} &= \frac{\partial(gh^2/2 + hv^2)}{\partial y} + \frac{\partial(huv)}{\partial x} + gh(S_{0y} + S_{fy}) - \\ &\frac{\partial}{\partial x} \left[ v_t \left( \frac{\partial(hu)}{\partial y} + \frac{\partial(hv)}{\partial x} \right) \right] - \frac{\partial}{\partial y} \left[ v_t \left( \frac{\partial(hv)}{\partial y} + \frac{\partial(hv)}{\partial y} \right) \right] \end{aligned} \quad (3)$$

The continuity equation of sediments :

$$-\frac{\partial[(1-p)z_b]}{\partial t} = \frac{\partial q_{sx}}{\partial x} + \frac{\partial q_{sy}}{\partial y} \quad (4)$$

where x and y represent the Cartesian co-ordinate positions in the longitudinal and transverse directions, respectively,

t is the time,

u and v are the average velocity components in the x and y directions,

h is the water depth,

g is the gravity acceleration,

$v_t$  is the kinematic viscosity (summation of molecular and eddy kinematic viscosity),

$z_b$  is the bed elevation,

p is the sediment porosity,

$q_{sx}$  and  $q_{sy}$  are the sediment discharges per unit width in the x and y directions,

$S_{0x} = \frac{-\partial z_b}{\partial x}$  and  $S_{0y} = \frac{-\partial z_b}{\partial y}$  are the channel slopes and

$S_{fx}$  and  $S_{fy}$  are the friction slopes which were defined as [6]:

$$S_{fx} = \frac{n^2 u \sqrt{u^2 + v^2}}{h^{4/3}} \text{ and } S_{fy} = \frac{n^2 v \sqrt{u^2 + v^2}}{h^{4/3}} \quad (5)$$

where n is the Manning's flow friction coefficient. In the two-dimensional numerical model the frictional resistance was expressed as function only of the bottom friction. The kinematic viscosity,  $v_t$ , was defined in a similar way to the work of [14] as:

$$v_t = \frac{g}{C^2} h \sqrt{u^2 + v^2} \quad (6)$$

where  $C = \frac{h^{1/6}}{n}$  is the Chezy's, friction coefficient for free surface flow. The sediment discharge may be predicted by different knowing from the bibliography empirical bed-load formulae [3].

In the current numerical method the equations (1)-(4) were solved in an integral form and applied to a series of finite volumes with adjacent volumes sharing a common face. At the end of each time step  $\Delta t$ , the net flux into each elemental volume is zero, so that overall water mass flow and sediment transport are conserved, and the changes in momentum are equal to the forces imposed by the boundaries of the system. The flow equations were solved along with the sediment transport equation simultaneously using current values of flow and sediment variables, which have mutual interaction. Two convergence criteria were used and if they are not satisfied, then the iterations within the same time step continue. The above criteria require that: (a) the averaged over the flow field relative error based on the axial velocity drops below  $10^{-6}$  and (b) the relative averaged change in sediment transport drops below  $10^{-6}$ . As with all marching in time methods, the theoretical maximum stable time step  $\Delta t$  was specified according to the Courant–Friedrichs–Lewy (CFL) criterion[3].

### III. BED LEVEL VARIATION AROUND VERTICAL WALL ABUTMENTS

Bridge abutments projecting into a river system produce bed level variation and deep scour holes in the region of the construction. Intense erosion develops upstream of the abutment and deposition of bed material occurs downstream of the obstruction. Local scour around abutments results from flow disturbances introduced by the presence of the structure. Experimental measurement tests were carried out in the Hydraulics Laboratory of Civil Engineering T.E. Department at the Technological Educational Institute of Thessaly [4]. An experimental flume of 6.0 m long and 0.078m wide was utilized. Three different geometries of orthogonal abutments with vertical walls were placed at the one side of the flume. The streamwise length of each abutment was equal to 0.10m and the lengths of the abutments transverse to the flow, abutment width, B, were constructed equal to 0.036m, 0.048m and 0.051m. The bottom of the tested experimental area was carefully covered with material, consisted of sand, producing a uniform layer of sediment of 0.15 m thickness. The used bed material had a mean diameter  $D_{50}$  of 2.0 mm, a specific weight  $S_g$  of 1.60 and was assumed to be uniform as the geometric standard deviation  $\sigma_g$ , computed by  $(d_{84}/d_{16})^{0.5}$  was equal to 1.26. The experimental inflow discharges were equal to 0.0004 m<sup>3</sup>/s, 0.0005 m<sup>3</sup>/s, 0.0006 m<sup>3</sup>/s, 0.0007 m<sup>3</sup>/s, 0.0008 m<sup>3</sup>/s and 0.00095 m<sup>3</sup>/s.

In the two dimensional numerical simulation model, after a sensitivity analysis, the Manning's roughness coefficient  $n$  was estimated at 0.022 and the porosity  $p$  of the sediment bed layer was set equal to 0.4. The channel was carrying an initial uniform flow discharge,  $Q$ , equal to the experimental discharges with a uniform flow depth,  $h_0$ , presented in Table 1. The slope of the flume bed was initially set equal to zero. There was no sediment discharge entering the channel at the upstream end while at the downstream end, the bed elevation was free to change. The flow and geometry conditions resulted into subcritical throughout the flow field as the Froude number,  $Fr < 1.0$  (Table 1). A constant value of water depth at the downstream end was set for all  $t \geq 0$ . The time step  $\Delta t$  was computed according to the CFL criterion for stability.

**TABLE 1**  
**HYDRAULIC CONDITIONS**

a/a	Q (m <sup>3</sup> /s)	Fr	h <sub>0</sub> (m)
1	0.0004	0.28	0.0320
2	0.0005	0.33	0.0335
3	0.0006	0.37	0.0350
4	0.0007	0.38	0.0385
5	0.0008	0.41	0.0400
6	0.00095	0.47	0.0430

The unit sediment discharges  $q_{sx}$  and  $q_{sy}$  were numerically predicted using the empirical relation developed by Engelund and Hansen [1]:

$$q_{sx} = \frac{uC_{bx}h}{pS_g}$$

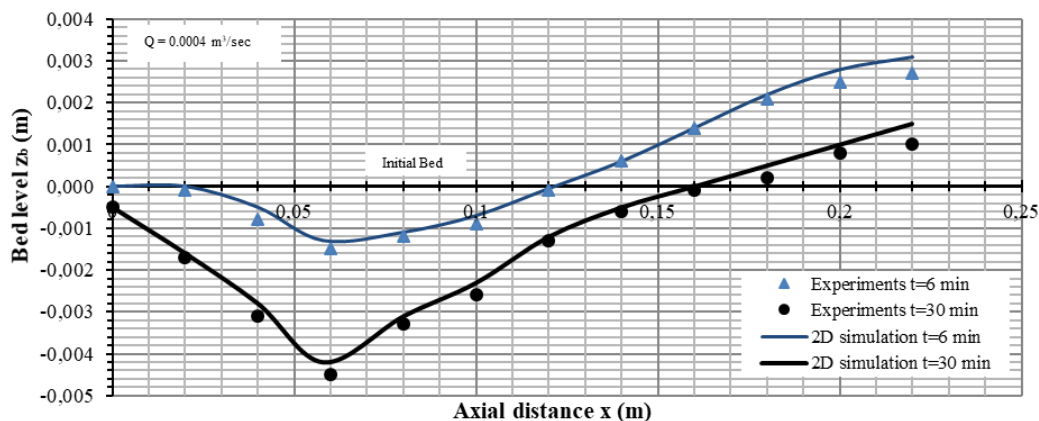
$$q_{sy} = q_{sx} \frac{u}{v}$$

where  $C_{bx}$  is the sediment concentration in the x-direction as:

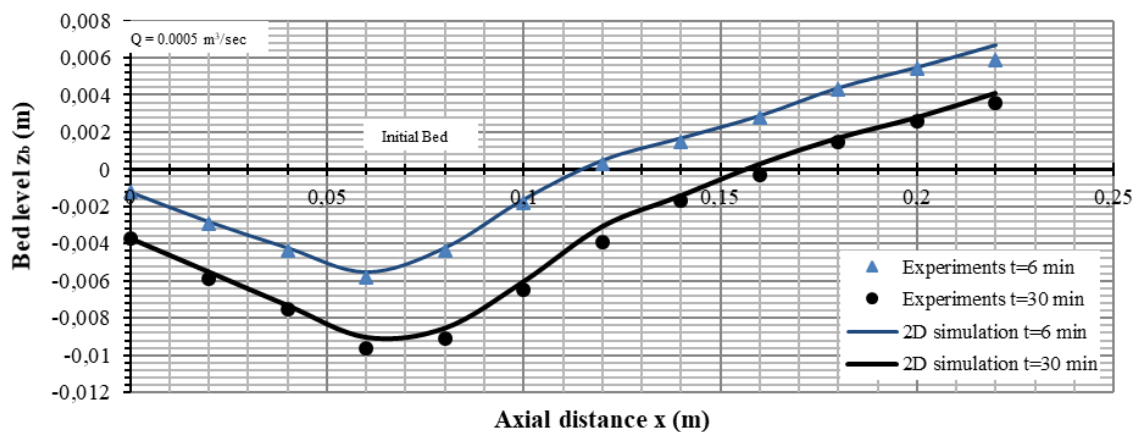
$$C_{bx} = \frac{50 S_g u_*^2}{hg^2 D_{50} (S_g - 1)^2}$$

$$\text{and the shear velocity } u_* = \sqrt{gh \sqrt{(S_{fx}^2 + S_{fy}^2)}}.$$

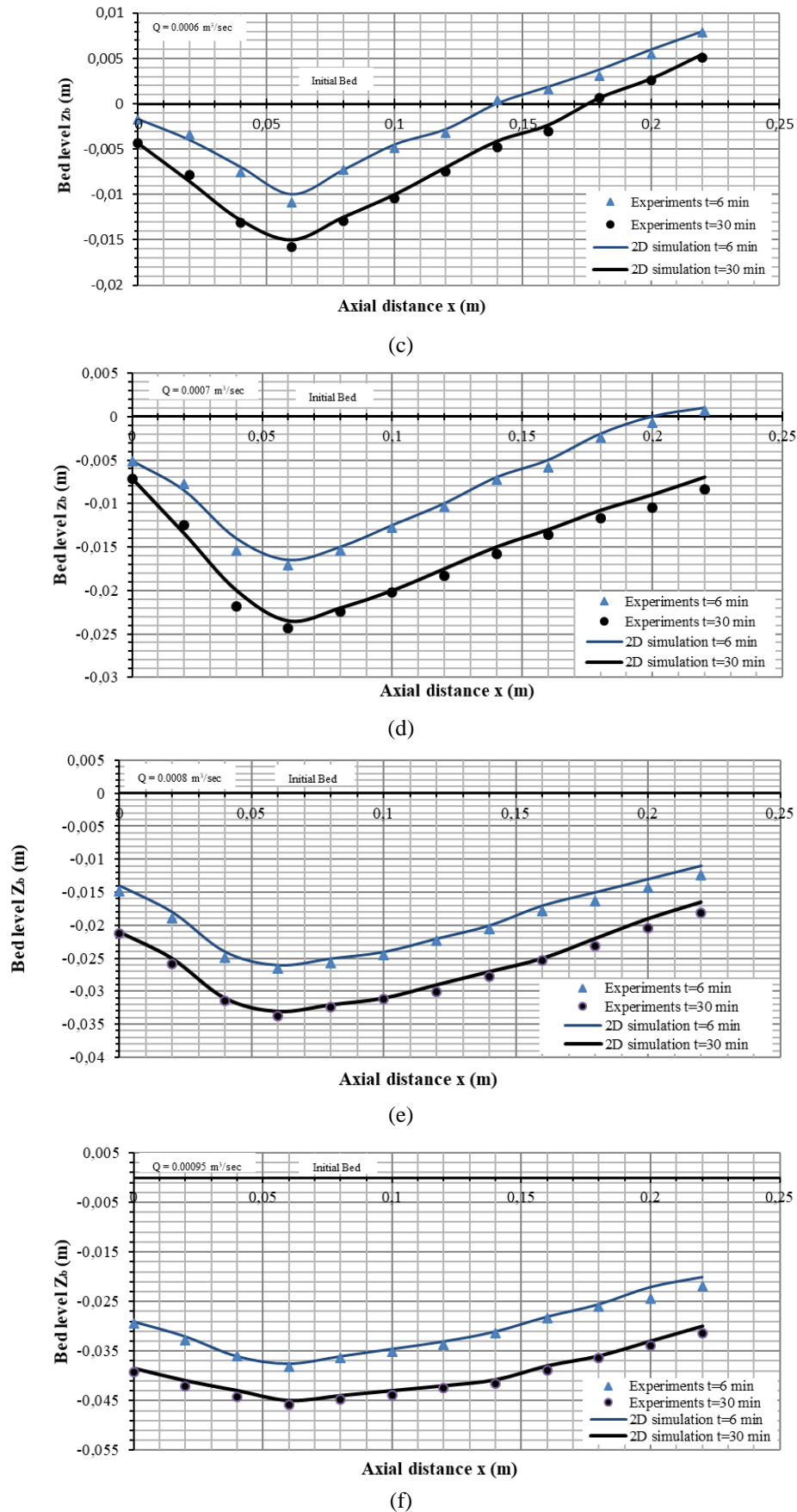
The validity of several empirical bed load relations, available in the model, was tested and comparisons between experimental measurements show that the Engelund and Hansen formula gives accurate results. Direct comparisons between two-dimensional numerical simulation predictions and experimental measurements of scour depth variation are shown in Fig. 1(a)-(f), along the flow line located at a constant distance of 0.005m to the streamwise face of the abutment parallel to the flow direction, in an area extending 0.06 m upstream to the abutment to 0.06 m downstream to it, for abutment width  $B=0.036$  m, for different inflow discharges and after flow duration  $t=6$  min and  $t=30$  min. The abutment construction starts at an axial distance  $x=0.06$  m to  $x=0.16$  m. Fig. 2(a)-(f) and 3(a)-(f) present comparisons of scour depth variation, in the region of the abutment, for different inflow discharges and for abutment widths  $B=0.048$  m and  $B=0.051$  m, respectively. The scour hole development and the maximum scour depth evolution in the region of each abutment are adequately predicted by the numerical technique for all discharges. High viscous effects in the regions immediately upstream and downstream to the abutment give rise to substantial differences between predictions and measurements. It is in these regions that high viscous effects combined with three-dimensional flow restrict a two dimensional model to predict exactly flow behavior and the maximum scour depth value. Maximum measured scour depths along the abutment are under predicted by the current method and this is the rule for all results. Erosion upstream of the abutment and deposition of the sediments at the downstream region, along the streamwise face of the abutment, which is the area of the maximum scour depths, are satisfactorily numerically simulated. The impact of flow duration, flow discharge and width of the abutment, normal to the flow direction, on scour depth variation was accurately predicted by the two-dimensional numerical model.



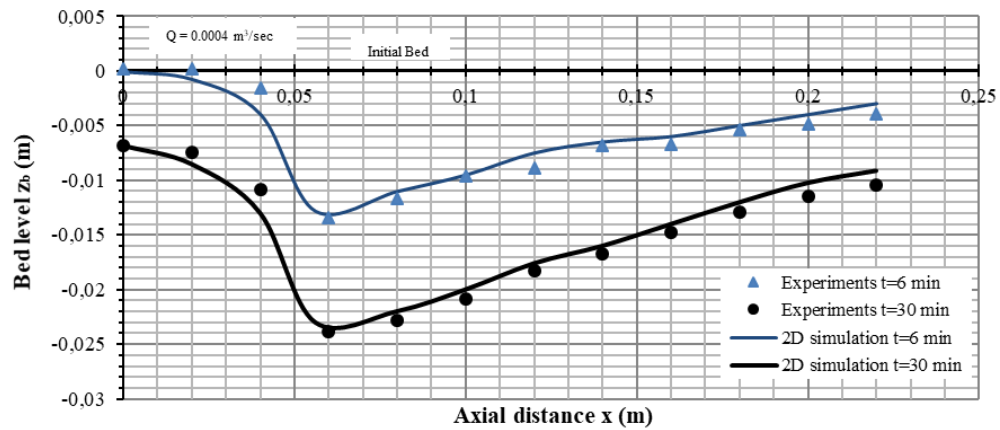
(a)



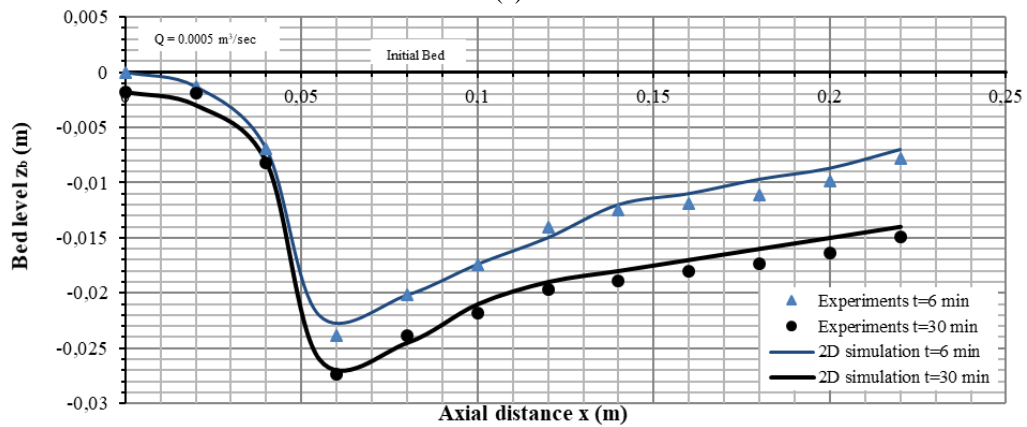
(b)



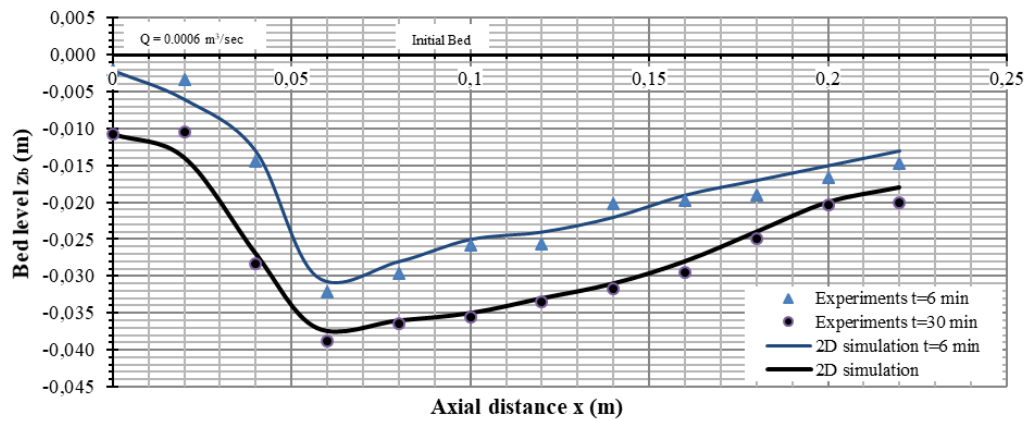
**FIG. 1. COMPARISON BETWEEN COMPUTED AND MEASURED BED LEVEL VARIATION ALONG THE STREAMWISE FACE OF THE ABUTMENT, FOR  $B=0.036$  M AND DIFFERENT INFLOW DISCHARGES**



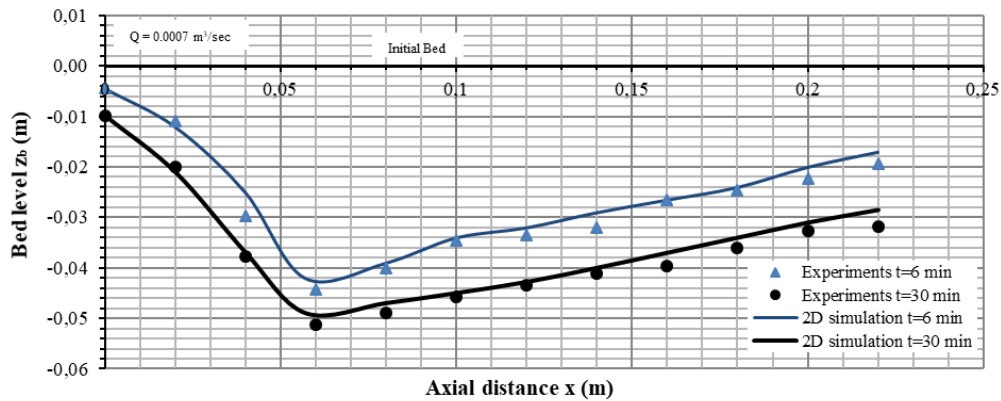
(a)



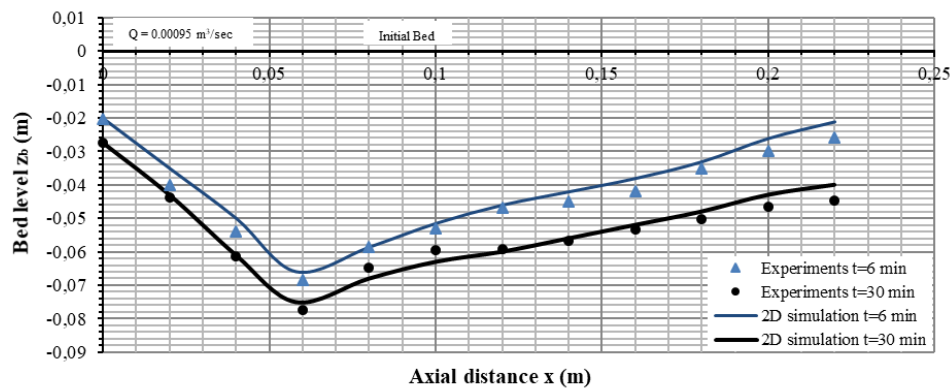
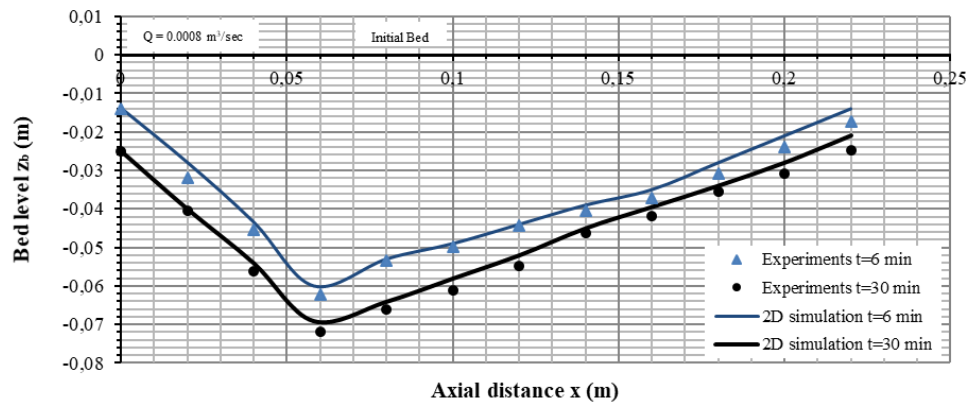
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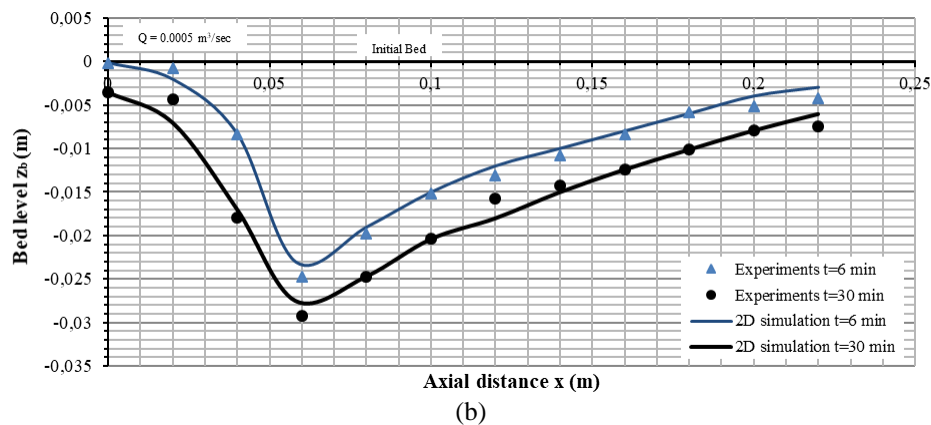
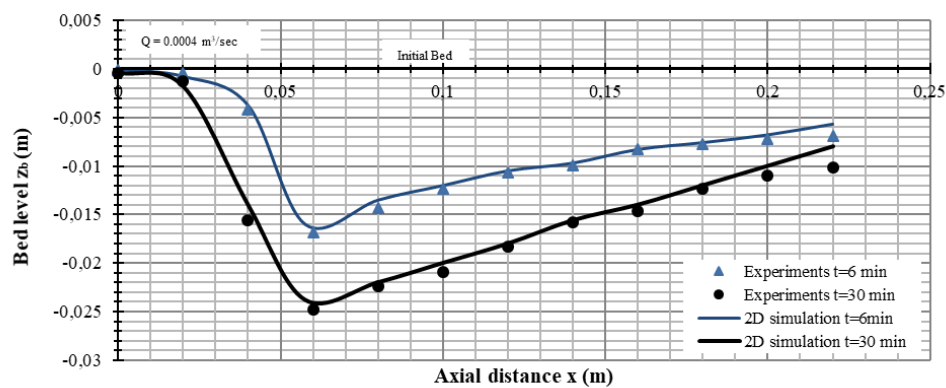
(c)

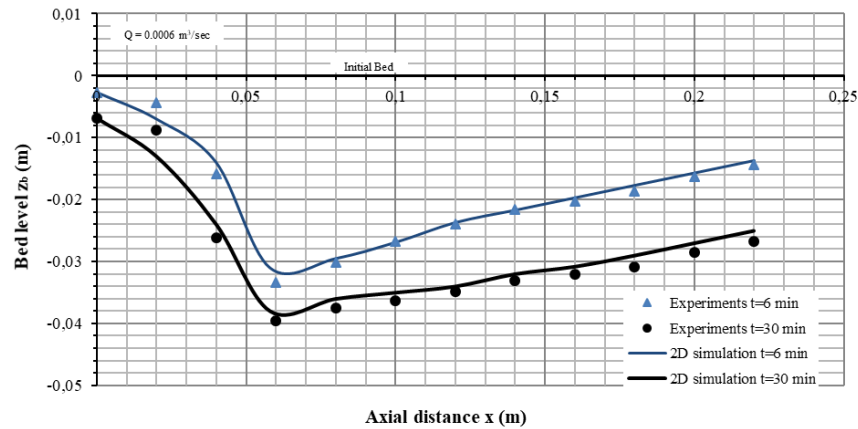


(d)

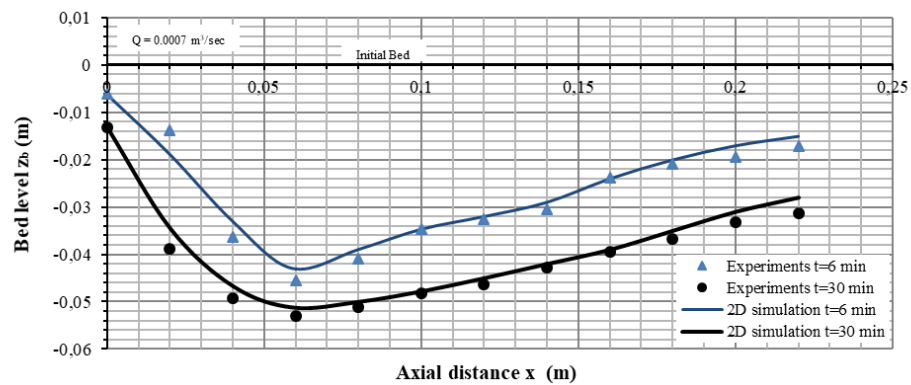


**FIG. 2. COMPARISON BETWEEN COMPUTED AND MEASURED BED LEVEL VARIATION ALONG THE STREAMWISE FACE OF THE ABUTMENT, FOR  $B=0.048$  M AND DIFFERENT INFLOW DISCHARGES**

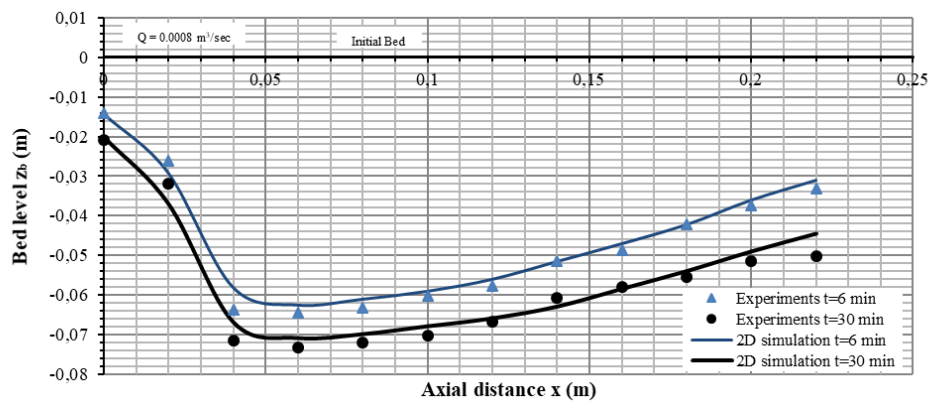




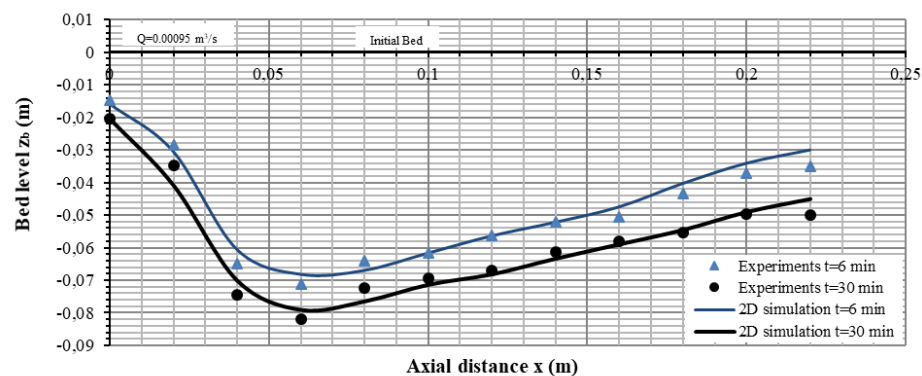
(c)



(d)



(e)



(f)

**FIG. 3. COMPARISON BETWEEN COMPUTED AND MEASURED BED LEVEL VARIATION ALONG THE STREAMWISE FACE OF THE ABUTMENT, FOR  $B=0.051\text{M}$  AND DIFFERENT INFLOW DISCHARGES**



#### IV. CONCLUSION

A two-dimensional, explicit, finite-volume numerical model has been applied to simulate bed level variation and maximum scour around bridge abutments in alluvial channels. The numerical predictions were backed by available experimental measurements and a sensitivity analysis was performed in order to test which is the most convenient empirical bed-load formula for the current hydraulic and sediment transport conditions. The applied numerical technique, directly coupling hydrodynamic and bed morphology equations, proved to be computational time consuming. It is stable, reliable, and accurate and can be applied to problems with complicated geometries. The numerical technique itself turned out to be flexible concerning its response to handle rapid changes of sediment transport at the boundaries and especially at regions of bridge constructions. Comparisons between computed results with measurements of scour depths at the region of vertical-wall abutments, in uniform sediments under clear water scour conditions, are graphically presented and can be used by other researchers to assist in the development of new and the refinement of existing codes for computing river bed morphology variations.

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