Precision geoid determination by Fast Fourier Transform solutions of the Kernel functions of the gridded gravity anomalies and distances in the Oman Gulf

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Abstract— *The geoid is the fundamental surface that defines the figure of the Earth. It is approximated by mean sea level and undulates due to spatial variations in the Earth's gravity field.*

The solution, that considers a global geopotential model (GM), gravity anomalies (Δg), and topographic effects, is used to determine the gravimetric geoid undulation. The EGM96 global geopotential model to degree 360 was used in order to determine the long wavelength effect of the geoid surface. By applying the remove-restore technique the geoid undulations were determined by combining a geopotential model, mean free-air gravity anomalies and height in a Digital Elevation Model (DEM). The long wavelength effects from a geopotential model and short wavelength effects from the topography are mathematically removed from the observed gravity anomalies in this technique. The Stoke's formulation of the residual parts of the gravity anomalies yields the medium wavelength of the geoid height. The geoidal height of a point is determined by restoring the long and short wavelength components. If the area for determining local geoid is chosen small and is considered as planar, it can be divided into M by N grids while distances Δx and Δy are the grid intervals. The geoid undulations can be calculated from Fast Fourier Transform (FFT) solutions of the Kernel functions of the gridded gravity anomalies and distances.

This paper deals with the precision geoid determination by a gravimetric solution in around the Oman Gulf. The size of the study area is about 600 km by 660 km. A number of data files were compiled for this work, containing now more than 88000 point gravity data on ocean areas. The EGM96 global geopotential model to degree 360 was used in order to determine the long wavelength effect of the geoid surface. By applying the remove-restore technique the geoid undulations were determined by combining a geopotential model, mean free-air gravity anomalies.

Keywords— Gravity, Geoid, Geopotential Model, FFT, Remove-restore technique Stokes-Kernel

I. INTRODUCTION

The geoid can be described as the equipotential surface of the Earth's gravity field which corresponds most closely with mean sea-level in the open oceans and ignores the effects of semi-dynamic sea surface topography. It defines the figure of the Earth and is used as the vertical datum surface in most countries. The determination of the geoid has attracted much attention within the discipline of geodesy, where both theoretical and practical problems are studied in order to improve the definition and accuracy of the geoid. This has been possible since the publication of the famous (to geodesists) integral formula by George Gabriel Stokes in 1849.At every point, the geoid surface is perpendicular to the local plumb line. It is then a natural datum or reference surface for orthometric heights measured along the curved plumb lines and, at the same time, the geoid is the best graphical representation of one equipotential surface of the Earth gravity field

Geoid determination is the process of calculation of the length of the ellipsoidal normal (geoid undulation) between the geoid surface and the reference ellipsoid. Various methods are used in determination of the geoid undulations.In Iran, Ardalan evaluated some geopotential models to determine the optimal reference field for the geoid solution. A comparison of the solutions with surface gravity data, GPS data and each models in Iran showed that the EGM96 and PGM2000A models to degree 360 gives the best fit (Ardalan et al 2006).

The NASA Goddard Space Flight Center and the Ohio State University (Lemoine et al., 1996) announced the EGM96, an improved degree 360 spherical harmonic model. The EGM96 was improved the data holdings over many of the world's land areas, including Alaska, Canada, parts of South America and Africa, Southeast Asia, Eastern Europe and the former Soviet Union. In addition to the above surface gravity data acquisitions, there have been major efforts to improve the National Imagery and Mapping Agency (NIMA)'s existing 30' mean anomaly data base through mean anomaly contributions over various countries in Asia (Lemoine et al., 1996). The distribution and extent of the surface gravity data is a major improvement on the data available for the OSU91A. The EGM96 model represents the latest development in high degree geopotential models which combine satellite data and the available surface and marine gravity data from NIMA.This study represents an updated version of the Oman gulf geoid using all available data, including the most recent EGM96 model and surface gravity data. By applying the remove-restore technique, high precision gravimetric geoids undulations were determined by combining the EGM96 model and residual gravity anomalies.

II. GLOBAL GEOPOTENTIAL MODEL

The contribution of the GM coefficients to the geoid undulation (NGM) of a point is computed by spherical harmonic expansion. A global gravity model (GM) describes the long wavelength characteristics of the earth's gravity field.Various geopotential models have been developed up to now. Currently, developed geopotential models have the maximum degree and order of 360 (Rapp and Cruz, 1986; Pavlis, 1996). The mostly used geopotential models are EGM96 and OSU91A. The contribution of geopotential model coefficient at a point on the earth is calculated with spherical harmonic expansions and given by spherical approach on geoid:

$$N_{GM} = R\sum_{n=2}^{n_{\max}}\sum_{m=0}^{n}(\bar{C}_{nm}\cos m\lambda + \bar{S}_{nm}\sin m\lambda)\bar{P}_{nm}(\sin\varphi) \quad (1)$$

Where R: is the mean radius of the Earth, $\bar{C}_{nm}, \bar{S}_{nm}$: are the fully normalized spherical harmonic coefficients of the disturbing potential, $\bar{P}_{nm}$: are the fully normalized associated Legendre functions (HEISKANEN and MORITZ, 1967), $n, n_{\max}$: denotes the maximum degree and order of expansion of the geopotential solution φ ,λ: geocentric latitude and longitude of a point.

Similarly, in spherical approach, gravity anomaly on a geoid can be computed from a geopotential model with following equation (Heiskanen and Moritz, 1967).

$$\Delta g_{GM} = G\sum_{n=2}^{n_{\max}}(n-1)\sum_{m=0}^{n}(\bar{C}_{nm}\cos m\lambda + \bar{S}_{nm}\sin m\lambda)\bar{P}_{nm}(\sin\varphi) \quad (2)$$

Where G is the mean gravity of the Earth.

III. METHOD DETERMINATION OF REDUCED GRAVITY ANOMALY

The computation of the gravimetric geoid model was based on the classical remove-predict-restore technique.The gravity measurements made at physical surface of the earth has to be reduced to geoid surface with a specific gravity reduction. Usually in sea, free-air anomalies at sea level are used in the Stokes' equation. Point gravity anomalies were computed from the EGM96 model using equation [2], and then the residual gravity anomalies (Δg) for all points in the network is found by subtracting gravity anomalies computed from the EGM96 global gravity model(Δg_{GM}) ,from the free air anomalies (Δg_{FA}) i.e.:

$$\Delta g = \Delta g_{FA} - \Delta g_{GM}$$

Δg_{GM} values were computed in a grid. From this grid, gravity anomalies values were linearly interpolated and removed from the observed free-air anomaly gravity points.

IV. PRACTICAL COMPUTATION

The well-known Stokes formula for the geoid undulation relative to the reference ellipsoid is:

$$N_{\Delta g} = \frac{R}{4\pi\gamma}\iint_{\sigma}\Delta g(\varphi,\lambda)\,\mathrm{S}(\psi)\,\mathrm{d}\sigma \quad (4)$$

Where S (ψ) is the Stokes' function, R is the mean radius of the Earth, and σ denotes the Earth's surface. By using Eq. (4), the geoid undulation, i.e. the physical figure of the earth, can be determined from the gravity observation. As we can easily understand from the equation, we need to know the gravity values on the entire surface of the earth for the geoid determination. It is not practical to get gravity data densely throughout the globe. When a

global geopotential model is available, Stokes'integral can be modified to integrate gravity anomalies over small cap σ.

In practice gravity data are only effective in limited point areas, equation (4) can be rewritten as equation (5) for gravity anomaly data on sphere (Li and Sideris, 1994).

$$N_{\Delta g} = \frac{R}{4\pi\gamma} \sum_{\varphi=\varphi_1}^{\varphi_B} \sum_{\lambda=\lambda_1}^{\lambda_L} \Delta g(\varphi,\lambda) S(\psi) \cos\Delta\varphi\Delta\lambda \quad (5)$$

Where, Δφ, Δλ : Grid intervals at latitude and longitude, L, B: The number of meridians and parallels and studying area are in a block.

Stokes-Kernel function is given by:

$$S(\psi) = \frac{1}{\sin\frac{\psi}{2}} - 4 - 6\sin\frac{\psi}{2} + 10\sin^2(\frac{\psi}{2}) - \left[3 - 6\sin^2\left(\frac{\psi}{2}\right)\right] - \ln\left[\sin\frac{\psi}{2} + \sin^2\left(\frac{\psi}{2}\right)\right] \quad (6)$$

Where,

$$\sin^2\left(\frac{\psi}{2}\right) = \sin^2\left(\frac{\varphi_p - \varphi}{2}\right) + \sin^2\left(\frac{\lambda_p - \lambda}{2}\right)\cos\phi_p \cos\phi \quad (7)$$

The use of Fast Fourier Transform. (FFT) requires that the random residual gravity anomalies must be interpolated on a grid and then they are converted into residual height anomalies by spherical FFT. FFT is an algorithm for calculation of Fourier transform of discretely gridded data. Due to the suitability to manage data the spectral domain, by the use of the some of the properties of the Fourier transform, its application is mostly used for numerical solutions in physical geodesy, usually in planar approximation. The 2D continuous Fourier transform (CFT) is given as follows (Schwarz et al., 1990).

$$G(\mathrm{u},\mathrm{v}) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(\mathrm{x},\mathrm{y})\,\mathrm{e}^{-i2\pi(\mathrm{ux}+\mathrm{vy})}\,dxdy = F\left[g(\mathrm{x},\mathrm{y})\right] \quad (8)$$

Here G, is called spectrum of the function g(x,y); u and v are are spatial frequencies in the directions of x and y respectively; i is the imaginary number.

The function g(x,y) can be expressed in the space domain by an inverse operation to its Fourier transform by :

$$g(\mathrm{x},\mathrm{y}) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} G(\mathrm{u},\mathrm{v})\,\mathrm{e}^{i2\pi(\mathrm{ux}+\mathrm{vy})}\,dudv = F^{-1}\left[G(\mathrm{u},\mathrm{v})\right] \quad (9)$$

Where, F^{-1} is 2 Dimensional Fourier inverse operator.

In practice our data are only given at discrete points and are of limited extent. Therefore we need a formulation for discrete case. If we estimate of the spectrum for a function on a finite interval, Formulation can be expressed with data given the interval (-X/2≤x≤X/2, - Y/2≤y≤Y/2) as follows (Schwarz et al., 1990).

$$G_F(\mathrm{u},\mathrm{v}) = \int_{\frac{-X}{2}}^{\frac{X}{2}}\int_{\frac{-Y}{2}}^{\frac{Y}{2}} g_F(\mathrm{x},\mathrm{y})\,\mathrm{e}^{-i2\pi(\mathrm{ux}+\mathrm{vy})}\,dxdy \quad (10)$$

If we now consider the data to represent a periodic process, the spectrum becomes discrete and the corresponding discrete Fourier transform for discrete gridded data, in the directions x and y, can be approximated by transforming the integrals in equations (8) and (9) into the respective summations as follows:

$$G(\mathrm{m}\,\Delta\,\mathrm{u},\mathrm{n}\,\Delta\,\mathrm{v}) = \Delta\,\mathrm{x}\,\Delta\,\mathrm{y}\sum_{k=0}^{M-1}\sum_{l=0}^{N-1} g(\mathrm{k}\,\Delta\,\mathrm{x},\mathrm{l}\,\Delta\,\mathrm{y})\,\mathrm{e}^{-i2\pi\left[\frac{mk}{M}+\frac{nl}{N}\right]} = DFT\left[g\right] \qquad (11)$$
$$m = 0,1,\ldots M-1, n = 0,1,\ldots.N-1$$

$$g(\mathrm{k}\,\Delta\,\mathrm{x},\mathrm{l}\,\Delta\,\mathrm{y}) = \Delta\,\mathrm{u}\,\Delta v\sum_{m=0}^{M-1}\sum_{n=0}^{N-1} G(\mathrm{m}\,\Delta\,\mathrm{u},\mathrm{n}\,\Delta\,\mathrm{v})\,\mathrm{e}^{-i2\pi\left[\frac{mk}{M}+\frac{nl}{N}\right]} = DFT^{-1}\left(G\right) \qquad (12)$$
$$k = 0,1,\ldots,M-1, l = 0,1,\ldots,\mathrm{N}-1$$

Where, $\Delta u = \dfrac{1}{M\Delta x}, \Delta v = \dfrac{1}{N\Delta y}$, *M* and *N* are number of data points in the direction of *x* and *y* respectively.

Equations (11) and (12) are base for FFT algorithm in order to evaluate DFT (Schwarz et al., 1990). If the area for determining local geoid is chosen small and is considered as planar, Stokes formulation can be expressed approximately as follows:

$$N(\mathrm{x}_{p,}\mathrm{y}_p) = \frac{1}{2\pi\gamma}\iint_E \Delta g(\mathrm{x},\mathrm{y})\,\mathrm{l}_N(\mathrm{x}_p,\mathrm{y}_p,\mathrm{x},\mathrm{y})\,\mathrm{dxdy} \qquad (13)$$

Where,

$$l_{N(\mathrm{x}_p,\mathrm{y}_p,\mathrm{x},\mathrm{y})} = \frac{1}{\sqrt{(\mathrm{x}_p-\mathrm{x})^2-(\mathrm{y}_p-\mathrm{y})^2}} \qquad (14)$$

l_N Is given as above equation and it is called approximated planar kernel. If the eare is divided into M×N elements with grid interval Δx, Δy, using the gridded gravity anomalies the geoid undulation at point (k, l) can be computed by:

$$N(\mathrm{k},\mathrm{l}) = \frac{1}{2\pi\gamma}\sum_{i=0}^{M-1}\sum_{j=0}^{N-1}\Delta g(\mathrm{x}_{i,}\mathrm{y}_j)\,l_N(\mathrm{x}_k-\mathrm{x}_i,\mathrm{y}_l-\mathrm{y}_j)\Delta\,\mathrm{x}\,\Delta\,\mathrm{y} \qquad (15)$$

Where, lN is:

$$l_N(\mathrm{k}-\mathrm{i},\mathrm{j}-1) = \begin{cases}(2\pi)^{-1}\left[(\mathrm{x}_k-\mathrm{x}_i)^2+(\mathrm{y}_l-\mathrm{y}_j)^2\right],(x_k\neq x_i, veya, y\neq y_i)\\ 0,\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(\mathrm{x}_{k=x_i}, veya, \mathrm{y}_{l=}\mathrm{y}_i)\end{cases} \qquad (16)$$

In frequency domain equation (15) can be expressed by:

$$N(\mathrm{x}_k,\mathrm{y}_l) = \frac{1}{2\pi\gamma}F^{-1}\left\{\Delta G(\mathrm{u}_m,\mathrm{v}_n)\,\mathrm{L}_N(\mathrm{u}_m,\mathrm{v}_n)\right\} \qquad (17)$$

In equation (17) $\Delta G(\mathrm{u}_m,\mathrm{v}_n)$ and $\mathrm{L}_N(\mathrm{u}_m,\mathrm{v}_n)$, are Fast Fourier Transform of the Δg and L_N as shown below equations (Sideris, 1994).

$$\Delta G(u_m, v_n) = F\{\Delta g(x_k, y_l)\} = \sum_{k=0}^{M-1}\sum_{l=0}^{N-1} \Delta g(x_k, y_l)\, e^{-i2\pi(mk/M+nl/N)}\, \Delta x \Delta y \quad (18)$$

$$L_N(u_m, v_n) = F\{l_N(x_k, y_l)\, e^{-i2\pi(mk/M+nl/N)}\, \Delta x \Delta y\} \quad (19)$$

V. RESULTS AND ANALYSIS

The residual gravity anomalies were calculated by simply subtracting the contribution of EGM96 model complete to degree and order 360 from the reduced gravity anomalies. Figure(1) shows the residual gravity anomalies. The gridded, reduced gravity anomalies have subsequently been converted to geoid undulations by using two dimensional FFT methods. The data are gridded by minimum curvature spline.

The FFT was carried out on a grid of 274 × 324 points, using 100% zero-padding to limit the periodicity effects. The 100% zero-padding consists of putting zeros around the values of the original field (input matrix).

Figure (2) represents Residual and EGM geoids height Map for planar approach and Figure (3) represents Residual and EGM geoids height Map for spherical approach. their statistics are tabulated in Table1.The major contributions to the final geoid is coming from the EGM96 geopotential model with values ranging from -27.92 m to -56.48 m and a standard deviation of 5.92 m.

The final geoid undulations were obtained by adding two effects. Figures 4 and 5 shows the final geoid surface referred to the GRS84 ellipsoid for planar and spherical approach respectively. It is noteworthy that the trend of SE-NW is recognized in the directions of convergence between the oceanic part of the Arabian Plate presently beneath the Gulf of Oman and the continental Eurasian Plate to the north, that generating an accretionary sediment wedge along the Makran continent margin of the plate Iran .

Difference of about 28.63 meters from the SE to the NW on Oman Gulf is apparent.

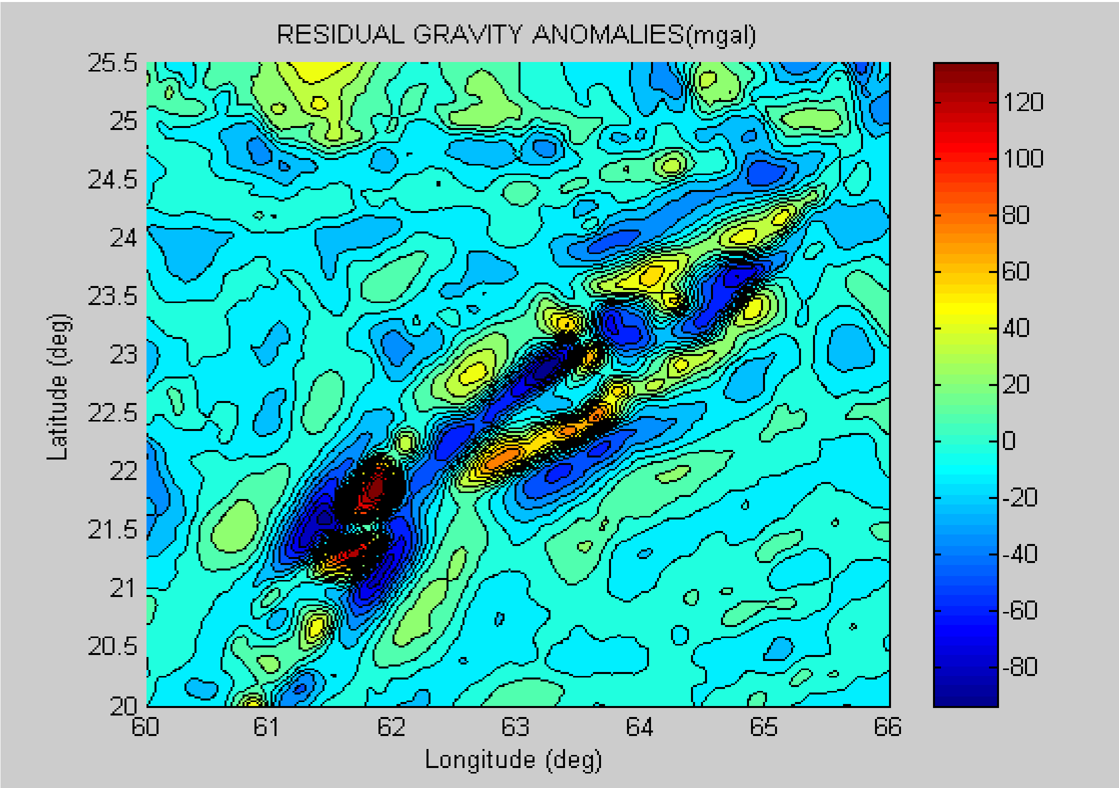

FIGURE 1: RESIDUAL GRAVITY ANOMALY MAP

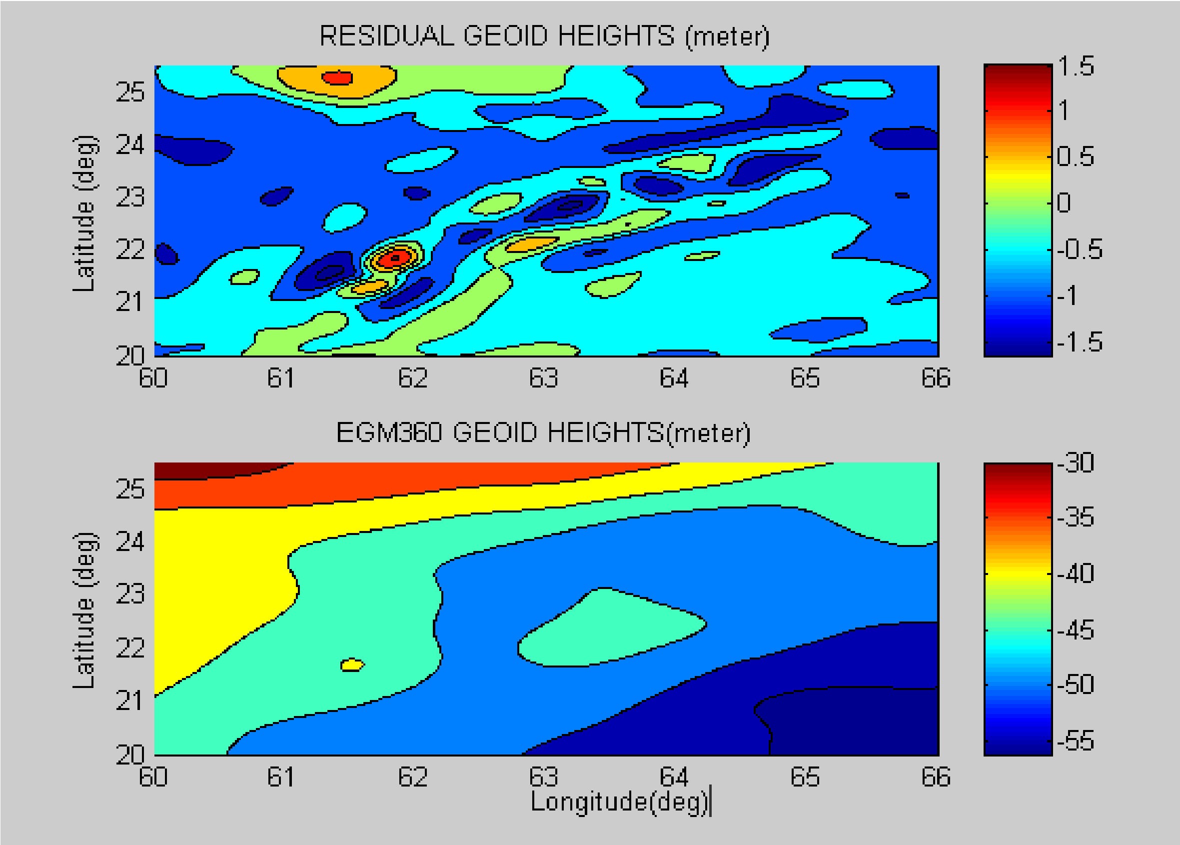

FIGURE 2: RESIDUAL AND EGM GEOID HEIGHT MAP FOR PLANAR APPROACH

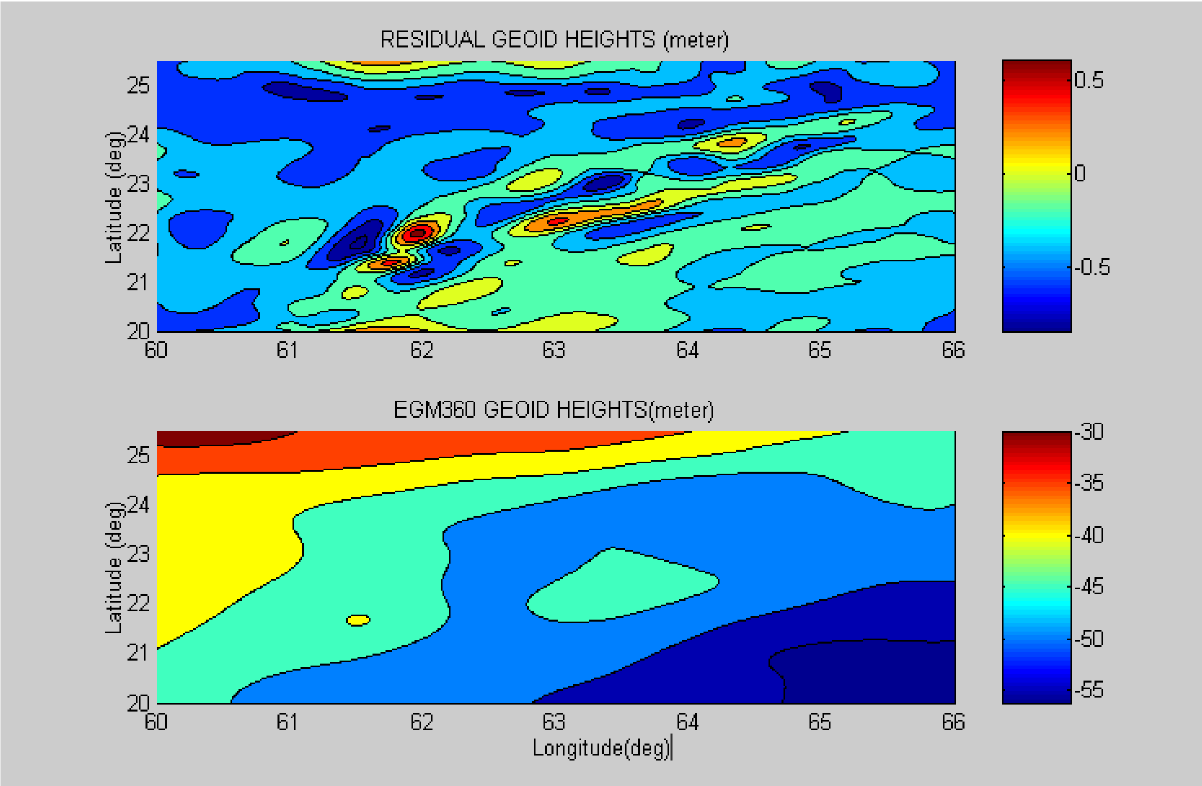

FIGURE 3 : RESIDUAL AND EGM GEOID HEIGHT MAP FOR SPHERICAL APPROACH

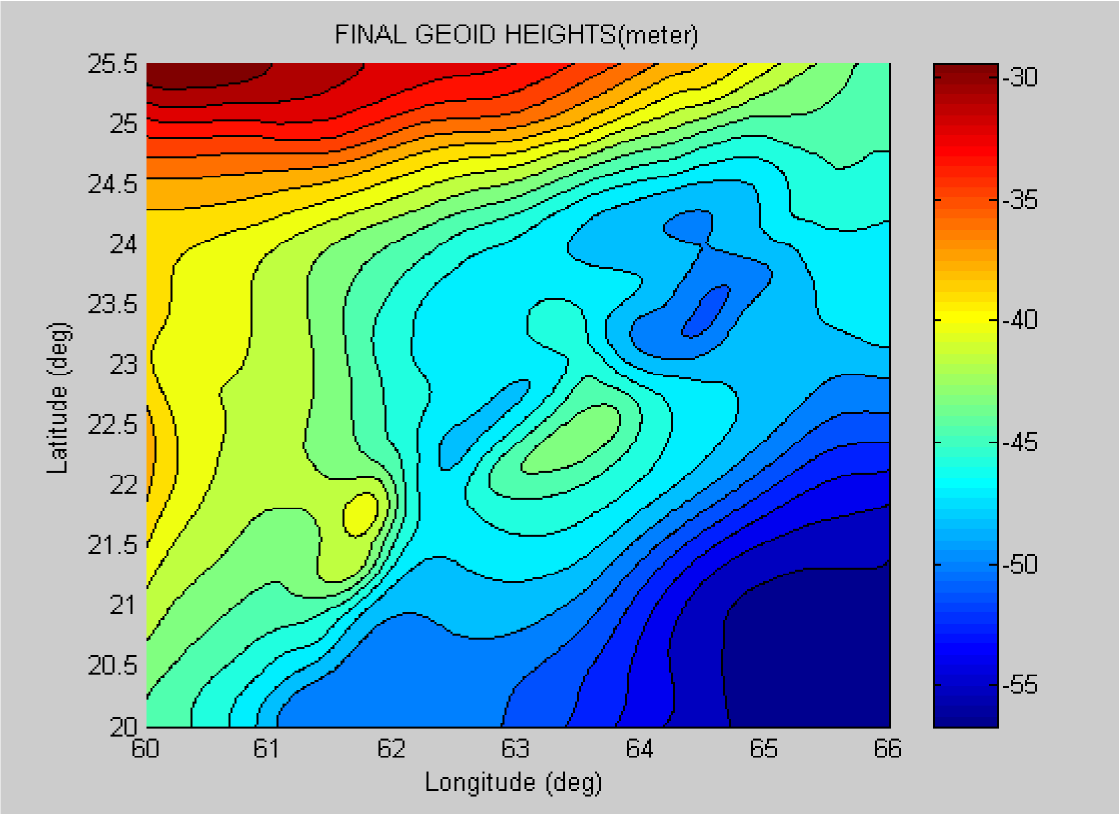

FIGURE 4: FINAL GEOID HEIGHT MAP FOR PLANAR APPROACH

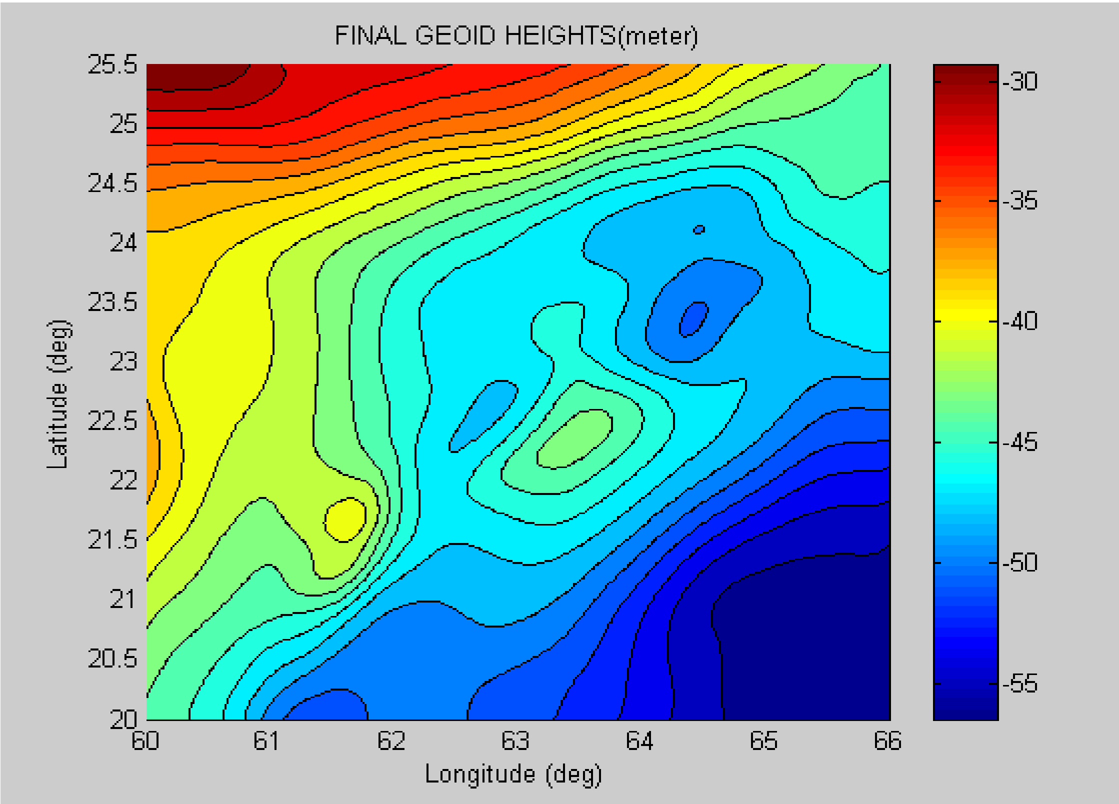

FIGURE 5: FINAL GEOID HEIGHT MAP FOR SPHERICAL APPROACH

TABLE 1
MAXIMUM AND MINIMUM VALUE OF GEOID UNDULATIONS AND THEIR STANDARD DEVIATIONS

Geoid undulation	Min. value(m)	Max. value(m)	Standard deviation(m)
$N_{\Delta g}(spherical)$	-0.83	0.67	.053
$N_{\Delta g}(plannar)$	-1.64	1.53	0.11
$N_{spherical}$	-56.48	-27.92	5.78
$N_{plannar}$	-56.68	-28.05	5.87
Difference $(N_{spherical} - N_{plannar})$	-1.32	1.37	0.31

VI. CONCLUSION

In this study a gravimetric geoid solution was determined in Oman Gulf using all available gravity. This involves EGM96 to degree and order 360 as a reference model, and the computations are conducted by two dimensional spherical and plannar FFT. The comparison two methods yields the standard deviation of 5.78 m and 5.87 m for two methods respectively. The resulting geoid shows that the difference from the south-East to the North-West is about 28.63 meters across the Oman Gulf.

As a result of this study, the minimum and maximum absolute values of the differences between geoid undulations obtained from two approaches at a point -1.32 m and 1.37 m respectively. Obtained geoid undulations from combined solution showed that variation between maximum and minimum values and standard deviation are smaller on spherical approach with respect to planar approach.

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