Numerical Analysis of Crack Effect on Natural Frequency of Cantilever Composite Beam

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Abstract— *The presence of cracks on a beam causes changes in the physical properties which introduces flexibility, and reduce the natural frequency of the beam. The crack in a structural member always remains open during vibration. However, this assumption may not be valid when dynamic loadings are dominant.*

This study is an investigation of the effects of cracks on a cantilever composite beam, made of Aluminium- reinforced GFRP and Aluminium reinforced Nylon. . The mechanical properties of aluminum and fibers (Nylon and Glass fiber reinforcement polymer) are measured with universal testing machine. The beams are made of Aluminium and synthetic fibers of dimensions 500x30x6 mm. The Cracks are provided on the cantilever beam which is varying from 10 to 90% of beam length, and we investigate the natural frequency of all five mode shapes with zero cracks to ninth cracks on the beam. The finite element model agrees well with the analytical values.

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Keywords— GFRP, Nylon, Aluminium, Natural Frequency, Mode Shapes.

I. INTRODUCTION

Vision et al. (1975) have examined that the fibers composite orthotropic material properties exhibit to exemplify its suitability in high speed purposes of civil structures and mechanical engineering components [1]. Nikpur et al. (1988) have evaluated compliance matrix of composite cracked bodies [2]. Raciti and Kapania (1989) collected a report of developments in the vibration analysis of laminated composite beams. Classical laminate plate theory and first order shear deformation theory are used for analysis. The assumption of displacements as linear functions of the coordinate in the thickness direction has proved to be inadequate for predicting the response of thick laminates [3]. Yuan and Miller (1990) derived a new finite element model for laminated composite beams. The model includes sufficient degrees of freedom to allow the cross-sections of each lamina to deform into a shape which includes up through cubic terms in thickness co-ordinate. The element consequently admits shear deformation up through quadratic terms for each lamina but not interfacial slip or delamination [4]. Maiti & Sinha (1994) used higher order shear deformation theory for the analysis of composite beams. Nine noded iso parametric elements are used in the analysis. Natural frequencies of composite beam are compared for different stacking sequences, different (l/h) ratios and different boundary conditions. They had shown that natural frequency decreases with an increase in ply angle and a decrease in (l/h) ratio [5]. Teboub and Hajela (1995) approved the symbolic computation technique to analyze the free vibration of generally layered composite beam on the basis of a first-order shear deformation theory. The model used considering the effect of poisson effect, coupled extensional, bending and torsional deformations as well as rotary inertia [6]. Banerjee (1999) has investigated the free vibration of axially laminated composite Timoshenko beams using dynamic stiffness matrix method. This is accomplished by developing an exact dynamic stiffness matrix of a composite beam with the effects of axial force, shear deformation and rotatory inertia taken into account. The effects of axial force, shear deformation and rotator inertia on the natural frequencies are demonstrated. The theory developed has applications to composite wings and helicopter blades [7]. Bassiouni (1999) proposed a finite element model to investigate the natural frequencies and mode shapes of the laminated composite beams. The FE model needed all lamina had the same lateral displacement at a typical cross-section, but allowed each lamina to rotate to a different amount from the other. The transverse shear deformations were included [8]. Kisa (2003) the effects of the location and depth of the cracks, and the volume fraction and orientation of the fibers on the natural frequencies and mode shapes of the beam with transverse non-propagating open cracks, were explored. The results of the study leaded to conclusions that, presented method was adequate for the vibration analysis of cracked cantilever composite beams, and by using the drop in the natural frequencies and the change in the mode shapes, the presence and nature of cracks in a structure can be detected [9]. Sreekanth et al (2004) have studied on composite structure taking into account various forms of transverse cracks like, fibers fracture, matrix cracking and surface-breaking cracks with by spectral finite element methods. They established FEM-2D model is more suitable for complex cracked beam to detect crack location and crack depth [10]. Jafari and Ahmadian (2007) had done free vibration analysis of a cross-ply laminated composite beam on Pasternak Foundation. The model is designed in such a way that it can be used for single-stepped cross-section. For the first time to-date, the same analysis was conducted for a single-stepped LCB on Pasternak foundation. Stiffness and mass matrices of a cross-ply LCB on Pasternak foundation using the energy method are computed [11]. Ramanamurthy (2008) the cracks can be present in structures due to their limited fatigue strengths or due to the manufacturing processes. These cracks open for a part of the cycle and close when the vibration reverses its direction. These cracks will grow over time, as the load reversals continue, and may reach a point where they pose a threat to the integrity of the structure. As a result, all such structures must be carefully maintained and more generally, SHM denotes a reliable system with the ability to detect and interpret adverse “change” in a structure due to damage or normal operation. [12]. Lu and Law (2009) the finite beam element was formulated using the composite element method with a one-member–one-element configuration with cracks where the interaction effect between cracks in the same element was automatically included. The accuracy and convergence speed of the proposed model in computation were compared with existing models and experimental results. [13]. Gaith (2011) the effects of crack depth and location, fiber orientation, and fiber volume fraction on the flexibility and consequently on natural frequency and mode shapes for cracked fiber-reinforced composite beams are investigated [14]. Erdelyi et al. (2012) have described mostly composites have different characteristics, such as high strength to weight ratio, good buckling resistance, and high stiffness [15]. Mehdi et al (2014) the natural frequency found higher in the fifth mode shape for all composite and pure materials [16].

Mehdi et al (2014) the maximum strength is found in composite GFRP instead of Aluminium and composite Nylon. Composite material has shown an improvement of mechanical properties when compared with individual materials [17]

II. GOVERNING EQUATION

The differential equation of the bending of a beam with a mid-plane symmetry (B_{ij} = 0) so that there is no bending-stretching coupling and no transverse shear deformation (ε_{xz}=0) is given by

$$IS_{11}\,(d^4\omega/d\omega^4) = q\,(x) \qquad (1)$$

It can easily be shown that under these conditions if the beam involves only a one layer, isotropic material, then $S_{11}=EI=Ebh^3/12$ and for a beam of rectangular cross-section Poisson’s ratio effects are ignored in beam theory, which is in the line [18]

In Equation 1, it is seen that the imposed static load is written as a force per unit length. For dynamic loading, if Alembert’s Principle are used then one can add a term to Equation.1 equal to the product mass and acceleration per unit length. In that case Equation.1 becomes

$$IS_{11}[d^4\omega(x,t)/d\omega^4 = q(x,t) - \rho F[\partial^2\omega(x,t)/\partial x^2 \qquad (2)$$

where ω and q both become functions of time as well as space, and derivatives therefore become partial derivatives, ρ is the mass density of the beam material, and here F is the beam cross- sectional area. In the above, $q(x, t)$ is now the spatially varying time-dependent forcing function causing the dynamic response, and could be anything from a harmonic oscillation to an intense one-time impact.

For a composite beam in which different lamina have differing mass densities, then in the above equations use, for a beam of rectangular cross-section,

$$\rho F = \rho bh = \Sigma\rho b\,(h_k - h_{k-1}) \qquad (3)$$

However, natural frequencies for the beam occur as functions of the material properties and the geometry and hence are not affected by the forcing functions; therefore, for this study let q(x,t) be zero.Thus, the natural vibration equation of a mid-plane symmetrical composite beam is given by

$$IS_{11}[d4\omega(x,t)/d\omega^4]+\rho F[\partial^2\omega(x,t)/\partial x^2]=0 \qquad (4)$$

It is handy to know the natural frequencies of beams for various practical boundary conditions in order to insure that no recurring forcing functions are close to any of the natural frequencies, because that would result almost certainly in a structural failure. In each case below, the natural frequency in radians /unit time is given as

$$\omega_n= \alpha^2\,(IS_{11} / \rho FL^4)^{1/2} \qquad (5)$$

Where α^2 is the co-efficient, which value is catalogued by Warburton, Young and Felgar and once ω_n is known then the natural frequency in cycles per second (Hertz) is given by $f_n= \omega_n /2\pi$, which is in the [21]

In general, governing equation for free vibration of the beam can be expressed as

$$[K]-\omega^2[M]\{q\}=0 \qquad (6)$$

Where, K = Stiffness matrix
M = Mass matrix , and
q = degrees of freedom.

III. RESULT AND DISCUSSION

The natural frequencies in both the cases either with or without the cracks are compared for the different materials and their composites and it had been found that the natural frequency of aluminium is minimum and that of GFRP is highest while the natural frequencies of nylon, GFRP composite and Nylon composite lie in between in all mode shapes.

TABLE 1:- EFFECT ON NATURAL FREQUENCY WITH DIFFERENT CRACK FOR ALUMINIUM BEAM

Aluminium					
Cracks	Natural Frequency				
	First Mode	Second Mode	Third Mode	Fourth Mode	Fifth Mode
Beam with 0 Crack	0.28664	1.4253	1.795	5.0215	8.3529
Beam with 1 Crack	0.28542	1.4227	1.7942	5.028	8.3531
Beam with 2 Cracks	0.28447	1.4197	1.7945	5.0237	8.335
Beam with 3 Cracks	0.28148	1.414	1.7877	4.9919	8.2979
Beam with 4 Cracks	0.28278	1.4156	1.7855	4.9955	8.3117
Beam with 5 Cracks	0.281	1.413	1.7745	4.9826	8.2783
Beam with 6 Cracks	0.28264	1.4155	1.7735	4.9899	8.3015
Beam with 7 Cracks	0.28051	1.4122	1.7596	4.9383	8.2693
Beam with 8 Cracks	0.2805	1.4126	1.7569	4.9178	8.2689
Beam with 9 Cracks	0.2806	1.4132	1.7571	4.9152	8.2711

TABLE 2:- EFFECT ON NATURAL FREQUENCY WITH DIFFERENT CRACK FOR GFRP BEAM

GFRP					
Cracks	Natural Frequency				
	First Mode	Second Mode	Third Mode	Fourth Mode	Fifth Mode
Beam with 0 Crack	0.37805	1.8816	2.3675	6.6229	11.255
Beam with 1 Crack	0.37439	1.8737	2.3592	6.6188	11.228
Beam with 2 Cracks	0.37202	1.8684	2.3592	6.6078	11.203
Beam with 3 Cracks	0.37112	1.8665	2.3578	6.583	11.18
Beam with 4 Cracks	0.3702	1.8645	2.3497	6.5684	11.162
Beam with 5 Cracks	0.37047	1.8651	2.3399	6.5706	11.154
Beam with 6 Cracks	0.36943	1.863	2.3269	6.5503	11.143
Beam with 7 Cracks	0.36981	1.8641	2.3197	6.5104	11.142
Beam with 8 Cracks	0.36981	1.8645	2.3163	6.4834	11.141
Beam with 9 Cracks	0.36993	1.8654	2.3165	6.4798	11.144

TABLE 3:- EFFECT ON NATURAL FREQUENCY WITH DIFFERENT CRACK FOR COMPOSITE GFRP BEAM

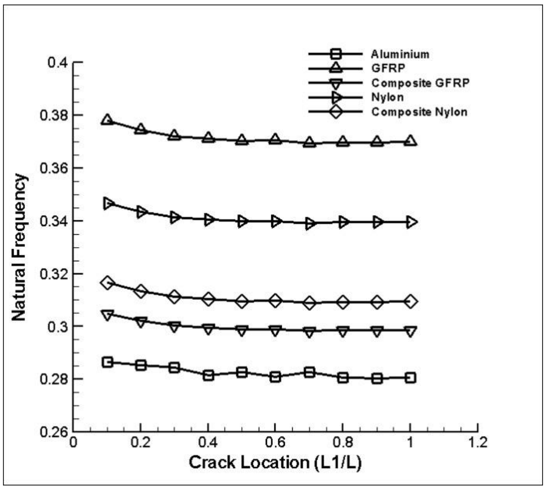
Composite GFRP					
	Natural Frequency				
Cracks	First	Second	Third	Fourth	Fifth
Beam with 0 Crack	0.30496	1.5524	1.9097	5.3429	8.9061
Beam with 1 Crack	0.30213	1.5464	1.9033	5.3396	8.8858
Beam with 2 Cracks	0.3003	1.5424	1.9033	5.3314	8.8659
Beam with 3 Cracks	0.29956	1.5408	1.9023	5.3124	8.8488
Beam with 4 Cracks	0.29885	1.5393	1.896	5.301	8.8345
Beam with 5 Cracks	0.29906	1.5398	1.8886	5.3027	8.8287
Beam with 6 Cracks	0.29827	1.5382	1.8785	5.2872	8.8208
Beam with 7 Cracks	0.29857	1.5391	1.873	5.2567	8.8201
Beam with 8 Cracks	0.29858	1.5396	1.8702	5.2353	8.8202
Beam with 9 Cracks	0.29871	1.5404	1.8705	5.2326	8.8231

TABLE 4:- EFFECT ON NATURAL FREQUENCY WITH DIFFERENT CRACK FOR NYLON BEAM

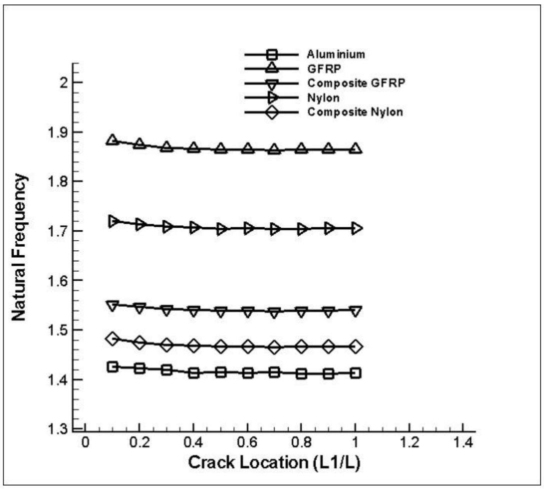
Nylon					
	Natural Frequency				
Cracks	First	Second	Third	Fourth	Fifth
Beam with 0 Crack	0.34655	1.72	2.1701	6.0716	9.7726
Beam with 1 Crack	0.34342	1.7132	2.1627	6.0672	9.7495
Beam with 2 Cracks	0.3414	1.7087	2.1627	6.0584	9.7273
Beam with 3 Cracks	0.34058	1.7069	2.1615	6.0373	9.7081
Beam with 4 Cracks	0.33977	1.7051	2.1546	6.0243	9.6918
Beam with 5 Cracks	0.34003	1.7058	2.1465	6.0265	9.6851
Beam with 6 Cracks	0.33914	1.7039	2.1353	6.0092	9.6759
Beam with 7 Cracks	0.33948	1.7049	2.1297	5.9765	9.6747
Beam with 8 Cracks	0.33947	1.7053	2.1262	5.952	9.6742
Beam with 9 Cracks	0.3396	1.7061	2.1265	5.949	9.6769

TABLE 5:- EFFECT ON NATURAL FREQUENCY WITH DIFFERENT CRACK FOR COMPOSITE NYLON BEAM

Composite Nylon					
	Natural Frequency				
Cracks	First Mode	Second Mode	Third Mode	Fourth Mode	Fifth Mode
Beam with 0 Crack	0.31648	1.4824	1.9814	5.5406	9.0497
Beam with 1 Crack	0.31324	1.475	1.974	5.537	9.0293
Beam with 2 Cracks	0.31118	1.4702	1.974	5.5279	9.0077
Beam with 3 Cracks	0.31043	1.4685	1.9729	5.5073	8.9889
Beam with 4 Cracks	0.30964	1.4666	1.9661	5.4949	8.9733
Beam with 5 Cracks	0.30992	1.4673	1.958	5.497	8.9674
Beam with 6 Cracks	0.30898	1.4653	1.9466	5.4793	8.9587
Beam with 7 Cracks	0.30934	1.4663	1.9407	5.4458	8.9583
Beam with 8 Cracks	0.30936	1.4668	1.9375	5.4222	8.9583
Beam with 9 Cracks	0.3095	1.4676	1.9378	5.4192	8.9615



(a)



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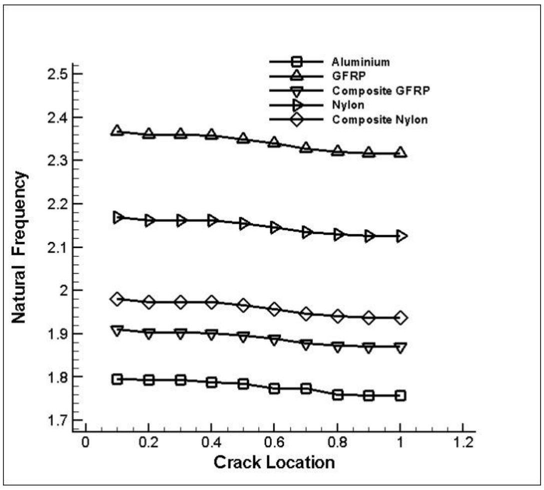

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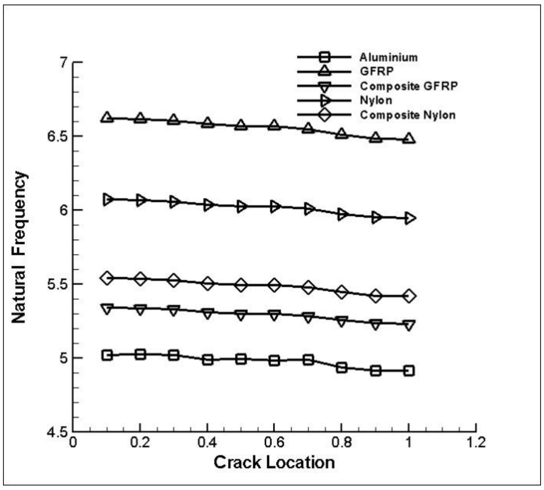

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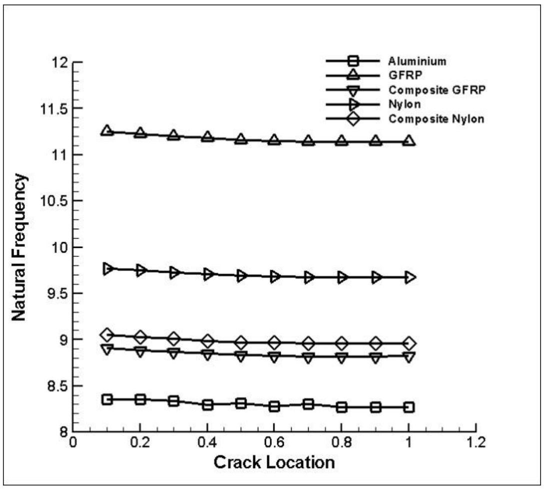

(e)

FIGURE 1:- VARIATION BETWEEN NATURAL FREQUENCY AND CRACK LOCATION (A) MODE-1, (B) MODE-2, (C) MODE-3, (D) MODE-4, (E) MODE-5

Figure-1 Shows the variation of natural frequency and crack location of composite cantilever beam made of Al-

GFRP-Al, Al-Nylon-Al, GFRP, Nylon, and Aluminum. In this figure it can be concluded that the natural frequency decreases when we increase the number of cracks. The main motive of this paper is to decrease the natural frequency of the beam. The natural frequency of Nylon and GFRP is high in all modes of vibration as shown in table 2 and 4, for decrease this natural frequency we made the composite beam with aluminum and found that the natural frequency with or without crack was decreases. The minimum natural frequency was found in Aluminum in mode-1 i.e 0.2806 Hz with 9 crack while maximum natural frequency was found in GFRP in mode-5 i.e. 11.255 Hz in No Crack

IV. CONCLUSION

- The Natural frequency of GFRP and Nylon is much higher, but when they are bonded with aluminium then their natural frequencies decrease in all modes of vibration.
- The natural frequencies of GFRP and Nylon in first mode of vibrations with 9 cracks are 0.36993Hz and 0.3396 Hz and when they bonded with aluminium then their natural frequency decreases i.e. 0.2987 Hz, and 0.3095 Hz
- It has been observed that when number of cracks increases the natural frequencies decreases.

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