Theory of Spin Waves in a Thin Ferromagnetic Film with a Periodic System of Circular Antidots. Solutions that Correspond to the Crystal Band Theory

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Abstract— *The paper continues study of dipole-exchange spin waves in a two-dimensional magnonic crystal (a thin ferromagnetic film with a periodic system of circular antidots) started by the author in the previous paper. The proposed model considers the magnetic dipole-dipole interaction, the exchange interaction and the anisotropy effects. An improved method of obtaining the dispersion relation as well as the values' spectra of the frequencies and wavenumbers for the investigated spin waves – the method based on using the Bloch-type solutions of the Landau-Lifshitz equation for the spin waves together with the Born–von Karman boundary conditions – is proposed. Exploitation of the above-mentioned method essentially extends the area of application of the obtained results compared to the previous paper. Newly obtained spectral characteristics are shown to exhibit crystal-type band structure with band gaps. The wave vector component that correspond to the wave propagating orthogonally to the film plane is shown to have narrow allowed bands, so its values' spectrum is near discrete.*

Keywords*— *Magnetic dynamics, Spin wave, Dipole-exchange theory, Ferromagnetic antidot, Magnonic crystal.

I. INTRODUCTION

Spin waves in nanosystems become an actual and promising topic of research because of their numerous applications - both current and prospective - in different fields of technology. These applications include mostly new devices for data storage, transfer and processing [1-4]. Such applications require precise theoretical models of excitation and propagation of spin waves in nanosystems of different configurations, thus causing these models to be extensively developed recently.

Prospective materials for applications in spin-wave technologies include, in particular, magnonic crystals [5,6] - composite materials whose magnetic properties change periodically along one, two or three directions. They are known to exhibit unique magnetic properties [5] making them prospective for creating novel magnonic devices [5,6]. As a result, spin waves in magnonic crystals of different configurations are studied extensively, both theoretically and experimentally [7-9].

Because of the parameters' periodicity, magnonic crystals often exhibit properties similar to those observed in crystals, such as appearance of crystal-like band structure in the spin waves' spectrum (see, e.g., [8]). Therefore, elements of crystals theory can be used in a theory of spin waves in such nanosystems in order to refine corresponding models and, therefore, obtain more precise results.

The paper extends theoretical study of dipole-exchange spin waves in a two-dimensional magnonic crystal (a thin ferromagnetic film with a two-dimensional periodic system of circular antidots) started by the author in the previous paper [10].The magnetic dipole-dipole interaction, the exchange interaction and the anisotropy effects are considered. Unlike in the previous paper, periodicity of the system is taking into account by applying the Bloch theorem and using the Bloch-type solutions of the Landau-Lifshitz equations for a spin wave together with Born–von Karman boundary condition. As a result, a refined dispersion relation and the wave vector components' spectrum of such waves is obtained. Analysis shows that a crystal-type band structure with band gaps appears in the resulting spectral characteristics.

II. PROBLEM STATEMENT: MODEL DESCRIPTION

Let us consider a ferromagnetic film with the thickness l composed of a uniaxial ferromagnet of the "easy axis" type containing a periodic two-dimensional system of identical circular antidots with the distances between the centers of neighboring antidots a and the radii R (see Fig. 1). Let us denote the ferromagnet parameters as follows: the exchange constant α, the uniaxial anisotropy parameter β (is considered constant), the gyromagnetic ratio γ (is considered constant). The easy magnetization axis of the ferromagnet (and hence the ground state magnetization $\vec{M}_0$, which is considered constant in the entire volume of the film) is directed orthogonally to the film and the Oz axis is chosen in this direction. The external magnetic field $\vec{H}_0^{(e)}$ is assumed to be homogeneous and directed along the Oz axis.

If the film is thin enough or the outer field is strong enough ($l \sim l_{ex}$ where l_{ex} is the exchange length of the ferromagnet, $l<<R$ or $H_0^{(e)} >> 4\pi M_0$), the ground state magnetization vector has an approximately uniform distribution, and the internal magnetic field of the film is also directed along the Oz axis and is approximately equal to the field inside a continuous film (without antidots) subjected to the same external magnetic field: $\vec{H}_0^{(i)} \approx \vec{H}_0^{(e)} - 4\pi \vec{M}_0$. (To be exact, the condition of the film thinness compared to the characteristic size of the antidot system should include not the antidot radius R, but the minimal distance d=2(a –R) between the antidots: $l<<d=2(a-R)$).

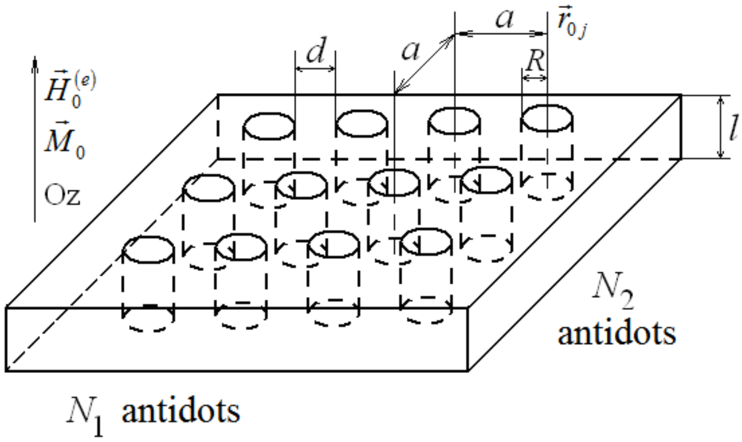

FIGURE 1: The considered antidot system.

Note that such system can be considered as an example of a two-dimensional magnonic crystal as its magnetic properties change periodically along 2 dimensions.

Let us consider a spin wave propagating in the above-described film and take into account both the magnetic dipole-dipole and exchange interaction as well as the anisotropy in the Landau-Lifshitz equation. The wave is considered linear so that the magnetization $\vec{m}$ and the magnetic field $\vec{h}$ of the wave are small perturbations of the overall magnetization $\vec{M}$ and the magnetic field inside the ferromagnet $\vec{H}^{(i)}$, correspondingly. Thus, the relations $|\vec{m}| << |\vec{M}_0|$, $|\vec{h}| << |\vec{H}_0^{(i)}|$ fulfill, where $\vec{H}_0^{(i)}$ is the ground state internal magnetic field (so that $\vec{M} = \vec{M}_0 + \vec{m}$, $\vec{H}^{(i)} = \vec{H}_0^{(i)} + \vec{h}$).

Let us note that the spin wave pattern in the system depends significantly on the minimum distance between adjacent antidots d. From the properties of the exchange length l_{ex} implies the fact that when $d< l_{ex}$, the studied system actually splits into a system of separate magnetic quantum dots and the spin wave propagates in directions orthogonal to vectors that connect neighboring antidots and in narrow adjacent sectors. Let us investigate spin waves in the system for the case $d>> l_{ex}$, so that these spin waves can be described similarly to spin waves in a continuous film. The task of the paper is to find the dispersion relation, the wavenumber values' spectrum and the frequency values' spectrum for the above-described spin waves.

A linearized Landau-Lifshitz equation (see, e.g., [11]) for such film together with the Maxwell equation $div\vec{h} = -4\pi \cdot div\vec{m}$ in a magnetostatic approximation (see, e.g., [11]) – where $\vec{h}$ is an internal magnetic field perturbation and $\vec{m}$ is a magnetization perturbation – forms a system of equations in which the spin wave magnetization vector can be eliminated. Therefore, the following equation for the amplitude Φ_0 of the magnetic potential Φ (so that $\Phi = \Phi_0 \exp(i\omega t)$) can be obtained:

$$\left(\frac{\omega^2}{\gamma^2 M_0^2}-\left(\frac{H_0^{(i)}}{M_0}+\beta-\alpha\Delta\right)\left(\left(\frac{H_0^{(i)}}{M_0}+\beta\right)+4\pi-\alpha\Delta\right)\right)\Delta\Phi_0+4\pi\left(\frac{H_0^{(i)}}{M_0}+\beta-\alpha\Delta\right)\frac{\partial^2\Phi_0}{\partial z^2}=0 \quad (1)$$

here ω is the spin wave frequency (see, e.g., [10]).

III. SPECTRAL CHARACTERISTICS OF THE SPIN WAVES

First, similarly to [10] let us seek a solution of (1) that corresponds to the system symmetry. After choosing the orthogonal (in-plane) part of the solution in the form of a combination of the Bessel and Neumann functions and considering the system symmetry relative to the rotation transformation one can obtain:

$$\Phi_{sym}=\exp(-i\omega t)\left(\Gamma_1\cos(k_{||}z)+\Gamma_2\sin(k_{||}z)\right)\sum_{j,n}\left(A_n J_{4n}\left(k_\perp\left|\vec{r}_\perp-\vec{r}_{0j}\right|\right)+B_n N_{4n}\left(k_\perp\left|\vec{r}_\perp-\vec{r}_{0j}\right|\right)\right)\exp\left(4in\theta_j\right) \quad (2)$$

here the radius vector $\vec{r}_\perp$ lies in the plane normal to $\vec{M}_0$ (xOy plane), j is the antidot number, θ_j is the polar angle measured from the center of the antidot number j, $k_\perp$ and $k_{||}$ are the wavenumbers that correspond to propagation of the spin wave in the plane xOy and in the ortogonal direction, correspondingly, J_n and N_n are the Bessel and Neumann functions of the order n, correspondingly, while Γ_1, Γ_2, A_n and B_{jn} are constants. For every wave that enters the superposition (2) (and, therefore, for the entire superposition (2)) the Laplace equation $\Delta\Phi_{sym}=-\left(k_\perp^2+k_{||}^2\right)\Phi$ fulfills. After substituting this solution into the equation (1), one can obtain a dispersion relation (presented in [10]).

Now, let us take advantage of the fact that the considered system possesses space periodicity, so conditions of the Bloch theorem formally fulfill for the equation (1). Therefore, its solution can be chosen as a combination of the Bloch-type functions for the magnetic potential. Such solution can be written in the form $\Phi(\vec{r},t)=\exp(-i\omega t)\Phi_0(\vec{r})$ with the following expression for Φ_0:

$$\Phi_0=\left(\Gamma_1\cos(k_{||}z)+\Gamma_2\sin(k_{||}z)\right)\Phi_{0\perp}(\kappa,\vec{r}_\perp)=\left(\Gamma_1\cos(k_{||}z)+\Gamma_2\sin(k_{||}z)\right)\exp\left(i\vec{k}_\perp{}'\vec{r}_\perp\right)F_{per}\left(\vec{k}_\perp,\vec{r}_\perp\right) \quad (3)$$

where $\vec{k}_\perp$ and $\vec{k}_\perp{}'$ are in-plane wave vectors ($\vec{k}_\perp$ corresponds to the symmetrical solution), κ is a total in-plane wavenumber and the relation $\Delta_\perp\Phi_{0\perp}=-\kappa^2\Phi_{0\perp}$ (where the operator $\Delta_\perp=\partial^2/\partial x^2+\partial^2/\partial y^2$) fulfills. The function F_{per} has in-plane periodicity - but nevertheless, substitution of $\Phi_{0\perp}$ into the relation $\Delta_\perp\Phi_{0\perp}=-\kappa^2\Phi_{0\perp}$ shows that the function F_{per} doesn't coincide with the corresponding function in (2). After substituting (3) into (2) one can obtain the following dispersion relation:

$$\omega=\gamma M_0\sqrt{\alpha^2k^4+2\alpha\left(2\pi+\tilde{\beta}\right)k^2+\tilde{\beta}\left(4\pi+\tilde{\beta}\right)-4\pi k_{||}^2\left(\alpha+\frac{\tilde{\beta}}{k^2}\right)} \quad (4)$$

where $\tilde{\beta}=H_0^{(i)}/M_0+\beta$ (so for the considered thin film $\tilde{\beta}\approx\beta+\left(H_0^{(e)}-4\pi M_0\right)/M_0=\beta-4\pi+H_0^{(e)}/M_0$) and the total wavenumber $k^2=\kappa^2+k_{||}^2$. The dispersion relation is expressed by a formula similar to the one obtained in [10], however, an expression for the total wavenumber k that enters these dispersion relations differ and moreover, the parameter κ that enters k depends on both $\vec{k}_\perp$ and $\vec{k}_\perp{}'$ for Bloch-type solutions, thus making these dispersion relations essentially different.

As the wave vector of the investigated spin waves has both longitudinal and orthogonal components, for more complete specification of the spin wave pattern the dispersion relation (4) must be supplemented by either a spectrum of values of at least one of these components or a relation between them. The spectrum of values of the longitudinal wave number can be found from the condition of limited film thickness, of the orthogonal wave number - either from the periodicity feature of the system or from boundary conditions.

The cylindrical functions that enter (2) are not periodical, but they asymptotically tend to periodical functions when the distance to the chosen central antidot increases (see [10]). As the considered system possess the translational symmetry, the resulting harmonical functions' phase change on the translation period of the system should be a multiple of 2π. The same considerations can be applied to the symmetrical part of the Bloch-type solutions (3) regardless of the exact form of the function F_{per}. Therefore, the "symmetrical" orthogonal wavenumber $k_\perp$ should have the same form as the single orthogonal wavenumber found in [10]:

$$k_\perp(s) = \frac{2\pi s}{a} \tag{5}$$

with $s \in \aleph \cup \{0\}$ being a number of the orthogonal mode. As the film is considered large (in Ox and Oy directions) and the total antidot number is considered big, spectral characteristics of the spin wave should not be sensitive to the exact form of boundary conditions, so Born–von Karman boundary conditions from the crystal electronic band theory can be used. Let us choose the ferromagnetic film to be a rectangle whose sides are parallel to the Ox and Oy axes and contain N_1 antidots along the Ox axis, N_2 antidots along the Oy axis. Then, using two-dimensional Born–von Karman boundary conditions at the boundaries of the rectangle, one can obtain $\vec{k}_\perp' = \frac{2\pi}{aN_1} q_1 \vec{e}_x + \frac{2\pi}{aN_2} q_2 \vec{e}_y$, where q_1 and q_2 are arbitrary integers. To obtain the total in-plane wavenumber κ, let us expand the function F_{per} into the Fourier series: $F_{sym}(\vec{r}_\perp) = \sum_{\vec{K}_\perp} A_{\vec{K}_\perp} \exp(i\vec{K}_\perp \vec{r}_\perp)$, where $\vec{K}_\perp = \frac{2\pi}{a} p_1 \vec{e}_x + \frac{2\pi}{a} p_2 \vec{e}_y$ are the reciprocal lattice vectors (p_1, p_2 are arbitrary integers). After substituting F_{per} in this form into the $\Delta_\perp \Phi_{0\perp} = -\kappa^2 \Phi_{0\perp}$ one can obtain the expression known from the theory of crystal solids $\kappa^2 = (\vec{K}_\perp + \vec{k}_\perp')^2$. Therefore, the sought spectrum of values of can be written as follows:

$$\kappa^2 = \left(\frac{2\pi}{a}\right)^2 \left(\left(p_1 + \frac{q_1}{N_1}\right)^2 + \left(p_2 + \frac{q_2}{N_2}\right)^2 \right) \tag{6}$$

After applying the standard magnetic boundary conditions together with the magnetic potential continuity condition (that should be applied twice - on the film boundary and on the antidots' boundaries) and substituting the solutions of the Laplace equations for the magnetic potential outside the film and inside the antidots, after some transformations one can obtain $\kappa = k_{||} \mathrm{tg}(k_{||} l/2)$, $\Gamma_1 \Gamma_2 = 0$. Therefore, the spin wave frequencies' spectrum is determined implicitly by the following system of conditions:

$$\begin{cases} \omega(p_1, p_2, q_1, q_2, k_{||}) = \gamma M_0 \sqrt{ \begin{aligned} & \alpha^2 \left(k_{||}^2 + \left(\frac{2\pi}{a}\right)^2 \left(\left(p_1 + \frac{q_1}{N_1}\right)^2 + \left(p_2 + \frac{q_2}{N_2}\right)^2 \right) \right)^2 + \\ & + 2\alpha(2\pi + \tilde{\beta}) \left(k_{||}^2 + \left(\frac{2\pi}{a}\right)^2 \left(\left(p_1 + \frac{q_1}{N_1}\right)^2 + \left(p_2 + \frac{q_2}{N_2}\right)^2 \right) \right) + \tilde{\beta}(4\pi + \tilde{\beta}) - \\ & - 4\pi k_{||}^2 \left(\alpha + \frac{\tilde{\beta}}{k_{||}^2 + (2\pi/a)^2 \left((p_1 + q_1/N_1)^2 + (p_2 + q_2/N_2)^2 \right)} \right) \end{aligned} } \\ k_{||} tg\left(\frac{k_{||} l}{2}\right) = \left(\frac{2\pi}{a}\right) \sqrt{ \left(p_1 + \frac{q_1}{N_1}\right)^2 + \left(p_1 + \frac{q_1}{N_1}\right)^2 } \end{cases} \tag{7}$$

Let us note that band gaps (associated with the diffraction of the wave on the lattice of antidots) may appear in the frequencies' spectrum of the investigated spin waves. Such gaps may appear near the edges of the Brillouin zones ($\vec{k}_\perp' = \vec{K}_\perp$), so in these areas of the spectrum the dispersion relation (4) and the spin wave frequencies' spectrum (7) should be refined. For this, let us note that the magnetic potential both inside the film and inside an antidot satisfy the equation $\Delta_\perp \Phi_{0\perp} = -(\kappa^2 - (\kappa^2 + k_{||}^2) f_R(\vec{r}_\perp)) \Phi_{0\perp}$ (the function $f_R(\vec{r}_\perp) = \sum_j \chi(R - |\vec{r}_\perp - \vec{r}_{0j}|)$ takes into account the periodic structure of the antidots, here χ is the Heaviside function) that is mathematically similar to the two - dimensional Schrödinger equation $\Delta_\perp \psi = -\frac{2m}{\hbar^2}(E - U(\vec{r}_\perp))\psi$ for the electron (with the energy E) wave function ψ in a two - dimensional periodic crystal lattice potential $U(\vec{r}_\perp)$ after the following replacements: $E \to \frac{\hbar^2\kappa^2}{2m}$, $U(\vec{r}_\perp) \to \frac{\hbar^2(\kappa^2+k_{||}^2)}{2m} f_R(\vec{r}_\perp)$. In the first zone of $k_{||}$ values - where $k_{||}$ is less than or of the same order with κ - a simple model of the theory of crystal solids can be used to take into account this diffraction. Namely, let us make the following replacement (taken from the above-mentioned theory) in the dispersion relation (4) near the boundary of the Brillouin zone:

$$\kappa^2 \to \frac{1}{2}\left(\left(\vec{\kappa}(\vec{k}_\perp')\right)^2 + \left(\vec{\kappa}(\vec{k}_\perp' - \vec{K}_\perp)\right)^2\right) \pm \sqrt{\frac{1}{4}\left(\left(\vec{\kappa}(\vec{k}_\perp')\right)^2 - \left(\vec{\kappa}(\vec{k}_\perp' - \vec{K}_\perp)\right)^2\right)^2 + \left|U_{\vec{K}_\perp}\right|^2} \tag{8}$$

here $U_{\vec{K}_\perp} = \int\limits_{[-a,a)^2} d\vec{r}_\perp \frac{\kappa^2+k_{||}^2}{a^2} f_R(\vec{r}_\perp) e^{-i\vec{K}_\perp \vec{r}_\perp} = (\kappa^2+k_{||}^2) B(\vec{K}_\perp)$, $B(\vec{K}_\perp) = \frac{1}{a^2}\int\limits_{|\vec{r}_\perp|\le R} d\vec{r}_\perp e^{-i\vec{K}_\perp \vec{r}_\perp}$ with the substitution of the constant value $\kappa^2 = k_{||}^2 tg^2(k_{||}l/2)$. After some transformations, one can obtain the following expressions in the vicinity of the edge of the first Brillouin zone:

$$\omega^2 = \gamma^2 M_0^2 \Bigg(\left(\alpha^2(\kappa_1^2+k_{||}^2)^2 + 2\alpha(2\pi+\tilde{\beta})(\kappa_1^2+k_{||}^2) + \tilde{\beta}(4\pi+\tilde{\beta}) - 4\pi k_{||}^2\left(\alpha + \frac{\tilde{\beta}}{\kappa_1^2+k_{||}^2}\right)\right)\cos^2\varphi(\vec{k}_\perp') +$$
$$+\left(\alpha^2(\kappa_2^2+k_{||}^2)^2 + 2\alpha(2\pi+\tilde{\beta})(\kappa_2^2+k_{||}^2) + \tilde{\beta}(4\pi+\tilde{\beta}) - 4\pi k_{||}^2\left(\alpha + \frac{\tilde{\beta}}{\kappa_2^2+k_{||}^2}\right)\right)\sin^2\varphi(\vec{k}_\perp')\Bigg) \tag{9}$$

$$\kappa_{1,2}^2 = \left(\frac{\vec{K}_{1,2}^2}{2} + \vec{K}_{1,2}\vec{k}_\perp' + (\vec{k}_\perp')^2\right) \pm \sqrt{\left(\frac{\vec{K}_{1,2}^2}{2} - \vec{K}_{1,2}\vec{k}_\perp'\right)^2 + k_{||}^4\left(1+tg^2\left(\frac{k_{||}l}{2}\right)\right)^2 \left|B(\vec{K}_{1,2})\right|^2} \tag{10}$$

where $\varphi = arctg\left((k_\perp')_y/(k_\perp')_x\right)$, $\vec{K}_1 = \begin{pmatrix} 2\pi/a \\ 0 \end{pmatrix}$, $\vec{K}_2 = \begin{pmatrix} 0 \\ 2\pi/a \end{pmatrix}$. As a result, the frequency band gap can be written as follows:

$$\Delta\omega = \omega\left(\kappa^2 = \frac{\pi^2}{a^2} + k_{||}^2\left(1+tg^2\left(\frac{k_{||}l}{2}\right)\right)\left|B(\vec{K}_{1,2})\right|\right) - \omega\left(\kappa^2 = \frac{\pi^2}{a^2} - k_{||}^2\left(1+tg^2\left(\frac{k_{||}l}{2}\right)\right)\left|B(\vec{K}_{1,2})\right|\right) \tag{11}$$

For the higher Brillouin zones, the spin wave spectrum near the zones edge can be described by the same relations (9), (10) but with another values of $\vec{K}_{1,2}$ that correspond to the investigated zone.

IV. DISCUSSION

Let us make a graphical representation of the obtained results in the absence of an external magnetic field. Dependence of the spin wave frequency on the longitudinal wave number $k_{||}$ (that implies from the dispersion relation (4) for the Bloch solution taking into account the relation $\kappa = k_{||}tg(k_{||}l/2)$) for l=10 nm is given on the Fig. 2. Dependence of the spin wave frequency on the in-plane wave vector (that implies from the refined relations (9), (10)) for l=10 nm, a=50 nm, R=20 nm (so that the value $\left|B(\vec{K}_{1,2})\right| \sim 0.4$) for the first interval of values of $k_{||}$ (according to the Fig. 2) and the analogues of the first and the second Brillouin zones are shown on the Fig. 3. Both graphs are plotted for typical values of the ferromagnet parameters (presented in the captions). Numerical estimations for these nanosystem parameters show that the band gap for the boundaries of the first Brillouin zone is approximately $2\cdot10^{10}$ Hz. The width of the allowed bands is of the same order of magnitude ($3\cdot10^{10}$ Hz).

As it can be seen from the graphs, the values' spectrum of $k_{||}$ contains band gaps. They correspond to the values of $k_{||}$ for which the boundary conditions for the magnetization cannot be satisfied. They are not a classic analogue of the Brillouin band gaps. Their presence causes the values' spectrum of $k_{||}$ to be near discrete one-dimensional $k_{||}(p) = 2\pi p/l$ (with

$p \in \aleph \cup \{0\}$ being a number of the longitudinal mode). As it can be seen from the graph, this approximate discreteness becomes more pronounced with increasing number of the longitudinal mode (branches of the dependence $\omega(k_{\|})$). On the other hand, band gaps in the dependence $\omega(\vec{k}_{\perp}')$ are analogous to the Brillouin band gaps of crystal solids theory. The band gap becomes significant starting from the second longitudinal mode of the dependence $\omega(k_{\|})$, and the spin wave frequencies' spectrum becomes a set of narrow bands. For the first branch, however, one can use the spectrum (7).

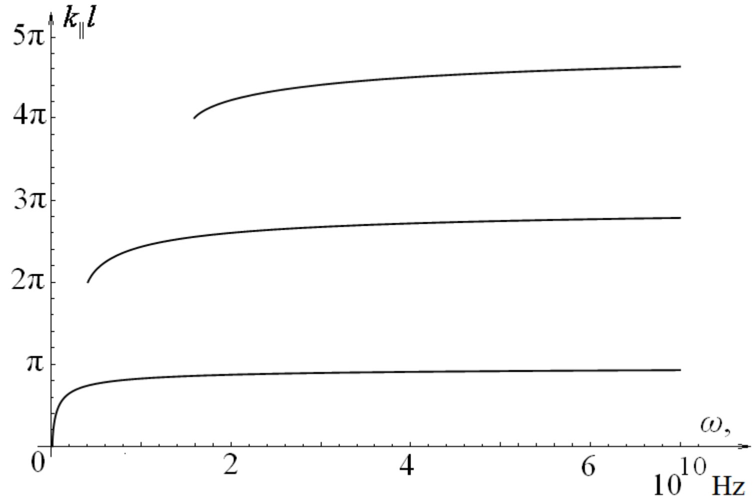

FIGURE 2: Dependence of the spin wave frequency on the longitudinal wavenumber $k_{\|}$ for the following nanosystem parameters: α=10^{-12} cm^{-2}, β=1, γ=10^5 Hz/Gs, M_0=10^3 Gs and l=10 nm.

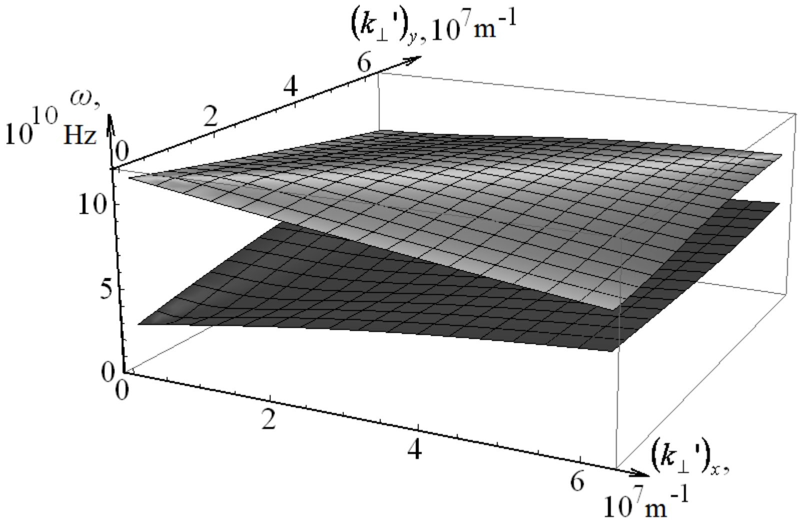

FIGURE 3. Dependence of the spin wave frequency on the in-plane wave vector $\vec{k}_{\perp}'$ for the first branch of the dependence shown on the Fig.2 and the following nanosystem parameters: α=10^{-12} cm^{-2}, β=1, γ=10^5 Hz/Gs, M_0=10^3 Gs, l=10 nm, a=50 nm and R=20 nm. The area represented on the graph corresponds to analogues of the first and the second Brillouin zones.

V. CONCLUSION

Therefore, the paper extends the study of the dipole-exchange linear spin waves in a thin ferromagnetic film with a two-dimensional periodic system of identical circular antidots started by the author in the previous paper [10]. The film is assumed to be composed of the uniaxial "easy axis"-type ferromagnet, with the axis of easy magnetization directed orthogonally to the film plane. For such waves, the differential equation for the magnetic potential in the magnetostatic approximation is written. The equation is solved for the case when either the external magnetic field is strong enough or the film is thin enough (l<<2(a–R)) to ignore the inhomogeneity of the equilibrium magnetization and magnetic field - and, additionally, the antidots are far enough from each other, so the minimum distance between them is much bigger than the exchange length.

Unlike the previous paper [10], the solution of the Landau-Lifshitz equations for the above-described spin wave in this paper is sought in the form of a two-dimensional function of Bloch type (for the in-plane wave propagation). For such solution, the dispersion relation and the relation between planar and longitudinal wavenumbers are obtained. Then, the crystal solid state formalism is used to obtain the values' spectra of the wave vector components and (after combining with the above-mentioned relations) of the spin waves' frequencies. The obtained results are refined near the edge of the Brillouin zones using the electronic band theory.

It is shown that the values' spectrum of the longitudinal wave numbers has band gaps and is near discrete. It is also shown that a band structure - which is analogous to the electronic band structure of a crystal solid - is inherent for the investigated spin waves.

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