

A Novel Technique for Designing Multirate Filter Banks Exploiting Quasi-Newton Optimization

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Abstract— This paper proposes a new technique for the design of multirate filter banks with linear phase in frequency domain. To match the ideal system response, low-pass analysis prototype filter response is optimized to minimize an objective function. The objective function is formulated as a weighted sum of pass-band error and stop-band residual energy of low-pass analysis filter, the square error of the overall transfer function at the quadrature frequency and amplitude distortion of the filter bank. Quasi-Newton optimization method is used to minimize the objective function by optimizing the filter tap weights of the prototype filter. Simulation results show that the proposed method is able to perform better than other existing methods.

Keywords— Quadrature mirror filter bank, Quasi-Newton optimization method, Peak reconstruction error, Linear phase.

I. INTRODUCTION

Quadrature mirror filter (QMF) banks are multirate filter bank and have been extensively used for sub-band coding, where the signal is split into two or more sub-bands in the frequency domain, so that each sub-band signal can be processed in an independent manner and sufficient compression may be achieved [1]. At the receiver end the sub-band signals are recombined such that the original signal is properly reconstructed [2]. QMF banks find applications in many areas, such as analog to digital conversion [3], design of wavelet bases [4,5], image compression [6,7], digital trans-multiplexers [8], discrete multi-tone modulation systems [9], 2-D short-time spectral analysis [10], antenna systems [11], digital audio industry [12], biomedical signal processing [13,14,15].

Alias free efficient design of two-channel QMF banks while keeping minimum dimensions is a tough task. Therefore, various constrained and unconstrained optimization based techniques [16–31] have been developed for the design of linear phase QMF banks. Iterative methods [22–27] and genetic algorithms [28–31] have been proposed for the design problem of QMF based on multi-objective or single objective nonlinear optimization.

Fig. 1(a) shows the analysis and synthesis section of a popular multirate filter bank known as two-channel QMF bank. The discrete input signal $x(n)$ is divided into two sub-band signals having equal band width, using the low-pass and high-pass analysis filters $H_0(z)$ and $H_1(z)$, respectively. Typical frequency responses of these filters are depicted in Fig. 1(b). The outputs of the synthesis filters are combined to obtain the reconstructed signal $\hat{x}(n)$. The reconstructed signal $\hat{x}(n)$ suffers from three types of errors: aliasing distortion (ALD), phase distortion (PHD), and amplitude distortion (AMD), due to the fact that the filters $H_0(z)$, $H_1(z)$, $F_0(z)$, and $F_1(z)$ are not ideal [32]. Therefore, the main stress of most of the researchers while designing the prototype filter for two-channel QMF bank has been on the elimination or minimization of these three distortions to obtain a perfect reconstruction (PR) or nearly perfect reconstruction (NPR) system [2, 16-21].

The overall transfer function of such an alias and phase distortion free system turns out to be a function of the filter tap coefficients of the low-pass analysis filter only [23]. Then, the AMD can only be minimized by optimizing the filter tap weights of the low-pass analysis filter using computer assistance techniques [2].

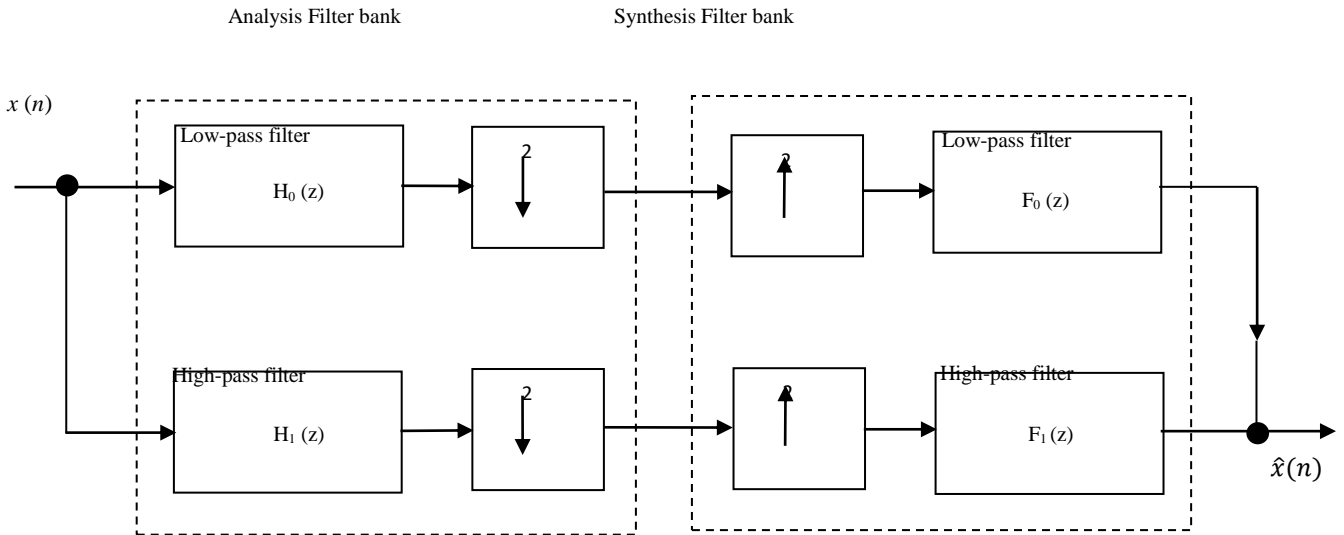


FIGURE 1 (a): Two-channel quadrature mirror filter bank

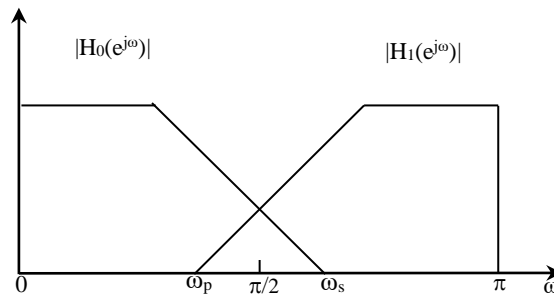


FIGURE 1 (b): Typical frequency responses of the analysis filters $H_0(z)$ and $H_1(z)$

It is well known [23] that the relation between output and input of an alias free two-channel QMF bank can be expressed. According to [23], the condition for perfect reconstruction can be written as:

$$|T(\omega)| = |H_r(\omega)|^2 + |H_r(\pi - \omega)|^2 = c, \text{ for all } \omega. \tag{1}$$

If the amplitude response of QMF bank in transition band is optimized, then the overall performance can be improved. Therefore, the aim is to optimize the coefficients of $H_0(z)$ by systematic computer-aided optimization technique, such that the filters satisfy the perfect reconstruction condition of Eq. (1) approximately.

In the above context, this paper presents a method to design a multirate filter bank. The objective function to be minimized [23] is formulated as a weighted sum of pass-band error and stop-band residual energy of low-pass prototype filter, the square error of the overall transfer function at the quadrature frequency $\omega = \pi/2$ and amplitude distortion of the QMF bank. Quasi-Newton optimization method is used to minimize the objective function by optimizing the filter tap weights of the prototype filter. The organization of rest of paper is as follows. Section 2 describes formulation of the design problem. Section 3 presents the proposed algorithm for design of prototype filter. Section 4 discusses the design results of the filter bank and comparison with already existing methods. Finally, conclusions are drawn in section 5.

II. DESIGN PROBLEM FORMULATION

To satisfy the perfect reconstruction condition of Eq. (1), an overall error function 'E' to be minimized is formulated as a weighted sum of four terms shown below:

$$E = \alpha_1 \cdot E_p + \alpha_2 \cdot E_s + \alpha_3 \cdot E_t \tag{2}$$

where α_1, α_2 , and α_3 are real constants, and E_p, E_s , and E_t are the measure of pass-band error, stop-band residual energy, square error of the overall transfer function at $\pi/2$, in transition band, and amplitude distortion, respectively. The objective function E is to be minimized iteratively using BFGS method by optimizing the coefficients of prototype filter.

For perfect reconstruction, the overall amplitude response of QMF bank, for all ω , must be equal to square of amplitude response of prototype filter at zero frequency. Consequently, the PR condition of Eq. (1) at quadrature frequency $\omega = \pi/2$, is reduced to:

$$H_r\left(\frac{\pi}{2}\right) = \frac{1}{\sqrt{2}} H_r(0) \quad (3)$$

where $H_r(0)$ and $H_r(\pi/2)$ are the amplitude response of prototype filter at zero frequency and quadrature frequency, respectively. The square error E_t is given by:

$$E_t = \left[H_r\left(\frac{\pi}{2}\right) - \frac{1}{\sqrt{2}}H_r(0)\right]^2 \quad (4)$$

Further, E_p , E_s and E_{am} are defined as follows:

$$E_p = \frac{1}{\pi} \int_0^{\omega_p} \left[|H_r(0)| - |H_r(\omega)|\right]^2 d\omega \quad (5)$$

$$E_s = \frac{1}{\pi} \int_{\omega_s}^{\pi} |H_0(e^{j\omega})|^2 d\omega \quad (6)$$

$$E_{am} = \max_{\omega} |T(e^{j\omega})| - \min_{\omega} |T(e^{j\omega})| \quad (7)$$

For even filter length $(N+1)$, the frequency response of low-pass prototype filter $H_0(e^{j\omega})$ is given by [2]

$$H_0(e^{j\omega}) = \left[\sum_{n=0}^{(N-1)/2} 2h_0(n) \cos \omega((N/2) - n) \right] e^{-j\omega N/2} \quad (8)$$

$$= H_r(\omega) e^{-j\omega N/2} \quad (9)$$

where $b(n) = 2h_0(n)$ and $H_r(\omega)$ is the amplitude function written as

$$H_r(\omega) = \mathbf{b}^T \mathbf{c}(\omega) \quad (10)$$

where vectors \mathbf{b} and $\mathbf{c}(\omega)$ are:

$$\mathbf{b} = [b_0 \ b_1 \ b_2 \ \dots \dots \ b_{(N-1)/2}]^T, \quad \mathbf{c}(\omega) = [\cos \omega(N/2) \ \cos \omega((N/2) - 1) \ \dots \dots \dots \ \cos(\omega/2)]^T \quad (11)$$

The amplitude function at zero frequency $\omega = 0$ is calculated as

$$H_r(0) = \mathbf{b}^T \mathbf{c}(0) = \mathbf{b}^T \mathbf{1}, \quad (12)$$

where $\mathbf{1}$ is the vector with all $(N+1)/2$ elements equal to unity.

Now, E_t , E_p and E_s can be realized as:

$$E_t = [\mathbf{b}^T \mathbf{c}(\pi/2) - \frac{1}{\sqrt{2}} \mathbf{b}^T \mathbf{c}(0)]^2 = [\mathbf{b}^T \mathbf{d} - H_{r1}]^2 \quad (13)$$

where vector \mathbf{d} and $\mathbf{c}(0)$ are equal to vector $\mathbf{c}(\omega)$, when it is evaluated at $\omega = \pi/2$ and $\omega = 0$, respectively, and $H_{r1} = 0.707 \mathbf{b}^T \mathbf{c}(0)$.

Similarly E_p can be realized

$$E_p = \mathbf{b}^T \mathbf{F} \mathbf{b} \quad (14)$$

where \mathbf{F} is a real, symmetric and positive definite matrix, given by

$$\mathbf{F} = \frac{1}{\pi} \int_0^{\omega_p} (\mathbf{c}(0) - \mathbf{c}(\omega))(\mathbf{c}(0) - \mathbf{c}(\omega))^T d\omega \quad (15)$$

Stop band error E_s is given as:

$$E_s = \mathbf{b}^T \mathbf{G} \mathbf{b} \tag{16}$$

where \mathbf{G} is a real, symmetric and positive definite matrix, calculated as

$$\mathbf{G} = \frac{1}{\pi} \int_{\omega_s}^{\pi} \mathbf{c}(\omega)\mathbf{c}(\omega)^T d\omega \tag{17}$$

III. PROPOSED METHOD FOR DESIGN OF LOW- PASS PROTOTYPE FILTER

By using Eqs. (13), (14) and (16) the objective E can be rewritten as:

$$E = \alpha_1 \mathbf{b}^T \mathbf{F} \mathbf{b} + \alpha_2 \mathbf{b}^T \mathbf{G} \mathbf{b} + \alpha_3 [\mathbf{b}^T \mathbf{d} - H_{r1}]^2 \tag{18}$$

$$\begin{aligned} &= \alpha_1 \mathbf{b}^T \mathbf{F} \mathbf{b} + \alpha_2 \mathbf{b}^T \mathbf{G} \mathbf{b} + \alpha_3 [\mathbf{b}^T \mathbf{D} \mathbf{b} - 2H_{r1} \mathbf{b}^T \mathbf{d} + H_{r1}^2] \\ &= \mathbf{b}^T \mathbf{R} \mathbf{b} + \alpha_3 [-2H_{r1} \mathbf{b}^T \mathbf{d} + H_{r1}^2] \end{aligned} \tag{19}$$

where matrix \mathbf{R} and \mathbf{D} are

$$\mathbf{R} = \alpha_1 \mathbf{F} + \alpha_2 \mathbf{G} + \alpha_3 \mathbf{D} \text{ and } \mathbf{D} = \mathbf{d} \mathbf{d}^T$$

The objective function given by Eq. (18) is a quadratic function and matrix \mathbf{R} is a Hermitian positive definite matrix, therefore, E can be minimized by Quasi Newton optimization method [33]. If \mathbf{b}_i is the approximation of the minimum point at the i th stage of iteration and λ_i is the optimal step length in the search direction, then the new or improved approximation in the $(i+1)$ th stage of iteration using BFGS method can be calculated as

$$\mathbf{b}_{i+1} = \mathbf{b}_i - \lambda_i [\mathbf{B}_i] \nabla E_i = \mathbf{b}_i + \lambda_i \mathbf{s}_i \tag{20}$$

where ∇E_i is the gradient of the objective function E and \mathbf{s}_i is the search direction, when evaluated at the design vector \mathbf{b}_i , both are given by

$$\nabla E_i = 2\mathbf{R} \mathbf{b}_i + \alpha_3 [-2H_{r1} \mathbf{d}] \text{ and } \mathbf{s}_i = -[\mathbf{B}_i] \nabla E_i \tag{21}$$

and matrix $[\mathbf{B}_i]$ is the estimate of inverse of Hessian matrix. Initially the matrix $[\mathbf{B}_i]$ is taken as the identity matrix $[\mathbf{I}]$ and updation of this matrix is done using BFGS formula [33].

$$[\mathbf{B}_{i+1}] = [\mathbf{B}_i] + \left[1 + \frac{\mathbf{g}_i^T [\mathbf{B}_i] \mathbf{g}_i}{\mathbf{d}_i^T \mathbf{g}_i} \right] \frac{\mathbf{d}_i \mathbf{d}_i^T}{\mathbf{d}_i^T \mathbf{g}_i} - \frac{\mathbf{d}_i \mathbf{g}_i^T [\mathbf{B}_i]}{\mathbf{d}_i^T \mathbf{g}_i} - \frac{[\mathbf{B}_i] \mathbf{g}_i \mathbf{d}_i^T}{\mathbf{d}_i^T \mathbf{g}_i} \tag{22}$$

where

$$\mathbf{d}_i = \mathbf{b}_{i+1} - \mathbf{b}_i = -\lambda_i [\mathbf{B}_i] \nabla E_i \text{ and } \mathbf{g}_i = \nabla E_{i+1} - \nabla E_i \tag{23}$$

The optimum step length λ_i in the direction of \mathbf{s}_i can be obtained by equating the derivate of objective function $E(\mathbf{b}_i + \lambda_i \mathbf{s}_i)$ with respect to λ_i to zero. The derivate $dE/d\lambda = 0$, gives following expression for optimum step length

$$\lambda_i = \frac{\{\mathbf{b}_i^T \mathbf{R} \mathbf{s}_i - \alpha_3 H_{r1} \mathbf{d}^T \mathbf{s}_i\}}{\{\mathbf{s}_i^T \mathbf{R} \mathbf{s}_i\}} \tag{24}$$

The unit energy constraint on the filter coefficients is also imposed within some prespecified limit as proposed in [17,27]. The design algorithm that minimizes the objective function in step by step manner proceeds through following steps:

- (1) Specify filter length $(N+1)$, stop band edge frequency (ω_s) and pass band edge frequency (ω_p) .
- (2) Assume initial values of $\alpha_1, \alpha_2, \alpha_3$ and α_4 .
- (3) Start with an initial design vector $\mathbf{h}_0 = [0 \ 0 \ 0 \ 0 \ 0 \ \dots \dots \dots \frac{1}{\sqrt{2}}]$, \mathbf{h}_0 is zero except $\mathbf{h}_0((N-1)/2) = \frac{1}{\sqrt{2}}$; satisfying the unit energy constraint within a pre-specified tolerance given by

$$g_1 = \left| 1 - 2 \sum_{k=0}^{(N-1)/2} h_0^2(k) \right| \leq \epsilon_1$$

- (4) Set the iteration number, $i = 1$, and $\mathbf{b}_i = 2\mathbf{h}_0$.

- (5) Compute the objective function E_i , by using Eq. (18), at the design vector \mathbf{b}_i .
- (6) Compute ∇E_i and the search direction \mathbf{s}_i , at the design vector \mathbf{b}_i .
- (7) Determine the optimum step length λ_i , by using Eq. (24).
- (8) Compute the new approximation

$$\mathbf{b}_{i+1} = \mathbf{b}_i - \lambda_i [\mathbf{B}_i] \nabla E_i = \mathbf{b}_i + \lambda_i \mathbf{s}_i$$
- (9) Obtain the constraint g_1 , at the point \mathbf{b}_{i+1} , if it is violated then choose the optimum point as \mathbf{b}_i , stop the algorithm and go to step (13).
- (10) Compute the amplitude distortion E_{am} using Eq. (13) at the design vector \mathbf{b}_{i+1} .
- (11) Compute the objective function E_{i+1} and ∇E_{i+1} at the design vector \mathbf{b}_{i+1} . Also compute matrix $[\mathbf{B}_{i+1}]$. If $E_{i+1} \geq E_i$, choose the optimum point as \mathbf{b}_i , stop the procedure and go to step (13). If $E_{i+1} < E_i$, set $E_i = E_{i+1}$, and $\mathbf{b}_i = \mathbf{b}_{i+1}$.
- (12) Set the new iteration number as $i = i + 1$, and go to step (5).
- (13) The optimum solution is $\mathbf{h}_0 = (1/2) \mathbf{b}_i$, and stop the procedure.

IV. RESULTS AND DISCUSSION

Design example is presented in this section to illustrate and examine the effectiveness of the proposed algorithm. The performance of the algorithm is evaluated in terms of following important parameters:

Mean square error in the pass band (E_p);

Stop band error (E_s);

stop-band first lobe attenuation (A_L);

stop-band edge attenuation (A_s) = $-20 \log_{10}(H_0(\omega_s))$;

maximum reconstruction error (ϵ) in dB = $\max_{\omega} |10 \log |T(e^{j\omega})||$

A MATLAB program has been written which implements the design technique described above.

4.1 Design example:

Example: For filter length $(N+1) = 48$, $\omega_s = 0.6\pi$, $\omega_p = 0.4\pi$, $\alpha_1 = 0.95$, $\alpha_2 = 0.04$, $\alpha_3 = 1$ and $\alpha_4 = 10^{-4}$, the following 24-filter coefficients for the FIR low-pass prototype filter are obtained:

$$\begin{aligned}
 h_0(0) &= 0.000208516673605 & h_0(1) &= -0.000277085005004 \\
 h_0(2) &= -0.000485925710620 & h_0(3) &= 0.000939830506683 \\
 h_0(4) &= 0.000828146074712 & h_0(5) &= -0.002254669530186 \\
 h_0(6) &= -0.001114175660144 & h_0(7) &= 0.004518779506026 \\
 h_0(8) &= 0.001102671159255 & h_0(9) &= -0.008074502198911 \\
 h_0(10) &= -0.000416487855403 & h_0(11) &= 0.013299020942940 \\
 h_0(12) &= -0.001522982128226 & h_0(13) &= -0.020679775981083 \\
 h_0(14) &= 0.005591974419610 & h_0(15) &= 0.030998856797507 \\
 h_0(16) &= -0.013310060758246 & h_0(17) &= -0.045983099224892 \\
 h_0(18) &= 0.027954420876814 & h_0(19) &= 0.070646028734991 \\
 h_0(20) &= -0.059744465465400 & h_0(21) &= -0.126933649477169 \\
 h_0(22) &= 0.172673420719003 & h_0(23) &= 0.593622292543939
 \end{aligned}$$

The corresponding normalized magnitude plots of analysis filters $H_0(z)$ and $H_1(z)$ are displayed in Fig. 2a. Figures 2b and 2c, respectively, depict the amplitude of distortion function and attenuation characteristics of low-pass filter $H_0(z)$. The

reconstruction error (in dB) of QMF bank is plotted in Fig. 2d. The corresponding important parameters are $E_p = 3.144 \times 10^{-10}$, $E_s = 2.96 \times 10^{-8}$, $A_s = 55.06$ dB, $A_L = 64.09$ dB, and PRE (ϵ) in dB = 0.0092 dB.

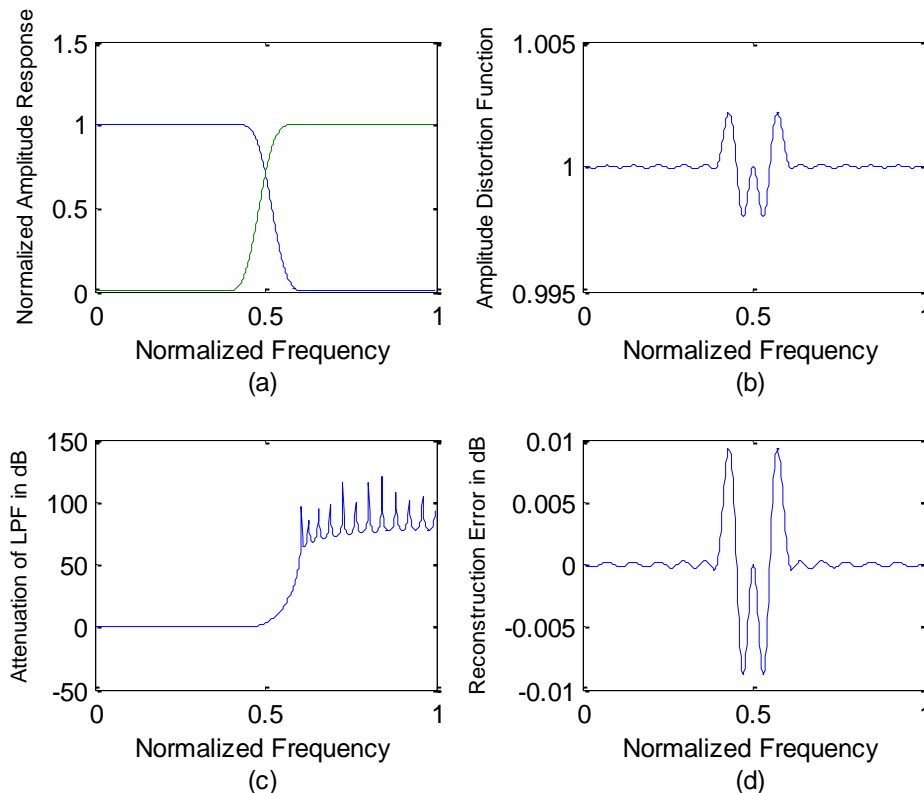


FIGURE 2 (a) Amplitude response of analysis filters for filter length = 48, (b) Amplitude of distortion function, (c) Attenuation characteristics of low-pass analysis filter, (d) Reconstruction error in dB.

From the above results, it is clearly indicate that the performance of the proposed method significantly improved when compared than to earlier known techniques in terms of PRE, E_p and A_s with similar design specifications.

V. CONCLUSION

A new iterative method for the design of multirate filter banks has been developed by formulating the perfect reconstruction condition in the frequency domain. The quadratic objective function is minimized without any matrix inversion which generally affects the effectiveness of some methods. Design example shows that the proposed technique is very effective in designing the quadrature mirror filters. The peak reconstruction error is minimum by the proposed method that makes it suitable for real time applications. Further, it is possible to extend this approach for the design of QMF banks with more than two bands.

CONFLICT OF INTEREST

The authors declare no conflict of interest.

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