Model Formulation for the Rupture Mechanism of Syringes Produced in A Typical Syringe Plant

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Abstract— *In a previous research work, attempt was made to improve upon the productivity of the syringe plant of First Medical & Sterile Product, Port Harcourt, Nigeria, by using time and motion study. The innate requirement for quality assurance and integrity of the manufactured products medically when put to use sensitize the consciousness of the researchers to delve into the design and rupture mechanism of the syringe barrel. Two design models were proposed. These are cylindrical barrel with flat ends and cylindrical barrel with hemispherical ends. Stress and force analysis projected the cylindrical barrel with flat ends as more suitable design on the basis of crack propagation, bursting or mode of failure by rupture. For the cylindrical barrel, force equilibrium to prevent bursting in transverse direction was confirmed to be 5.4711N while force equilibrium to stop bursting in the longitudinal direction was 89.01N. The force equilibrium was established on the basis of equality of the stress force and pressure force of the enclosed fluid.*

The results of cylindrical barrel with hemispherical ends was catastrophic in the sense that equilibrium of stresses and forces to avoid crack propagation or busting could only hold in the longitudinal direction. So to say that, equilibrium of stresses and forces failed in transverse directions, resulting in longitudinal rupture or bursting of the cylinder. The numerical values of the stress force and fluid pressure force being 99.95N and 94.48N respectively.

To this end the design and rupture mechanism of the syringe barrel of First Medical & Sterile Product had been established with cylindrical barrel having flat ends being the approved design.

Keywords— Quality assurance and integrity; Design and rupture mechanism; Cylindrical barrel with flat ends; Cylindrical barrel with hemispherical ends; Equilibrium of stresses and forces.

I. INTRODUCTION

In the work titled, "improvement of productivity and work measurement in a typical syringe plant, the researchers had been able to produce practicable master piece of work to impart upon the productive capacity of the case study company. The quest for quality assurance, quality specification and safety of the product when in use led to the development of formulation for the design and rupture mechanism of the syringe barrel of First Medical & Sterile Product. The company manufactures three different grades of syringe barrels. The three variations and unit price are as indicated:

0.5 ml = ₦ 17.50

2 ml = ₦ 17.50

5 ml = ₦ 18.50

The design and rupture mechanism was performed on the 2ml syringe barrel [1]. Two models of barrels were envisaged; cylindrical barrel with flat ends and cylindrical barrel with hemispherical ends. The barrel of the syringe is considered as thin walled cylinder. The cylinder encloses a fluid at constant internal pressure, P. In practice, a cylinder is classed thin walled cylinder if the ratio of its walls thickness, t and internal diameter, D, is within the limits $\frac{1}{25} \leq \frac{t}{D} \leq \frac{1}{20}$. Subject to the condition of constant internal pressure, three types of stresses are prevailing. The stresses are circumferential or hoop stress, longitudinal stress and radial stress. Usually the radial stress is negligible and neglected in the design analysis of thin walled cylinder. The radial stress is equal to the internal fluid pressure at the inner wall of the cylinder and it zero at the outer wall of the cylinder. The circumferential or hoop stress and longitudinal stress are constant over the thickness of the cylinder. If however the cylinder or tube is long and braced by stays or carried on brackets, the longitudinal stress may be negligible and hence neglected.

The design and rupture mechanism of the syringe barrel is hinged on two different barrels models, cylindrical barrel with flat ends and cylindrical barrel with hemispherical ends. Applying the physical geometric data of the case study company and operational parameters the barrel rupture analysis would be carried out and the best syringe design that would be reliable and safe in service recommended.

II. RESEARCH SIGNIFICANCE

Medical equipment or components must be highly reliable even with extreme medical certification. They are to applied on patients, more so their health and life should not be endangered. Hence different design models of syringe barrels had been proposed to enable the selection of the best design option prone to be safer under service conditions.

III. MATERIALS AND METHODS

3.1 Mathematical Models Deduction for the Syringe Barrel:

3.1.1 Deduction for Cylindrical Barrel with Flat Ends:

The barrel-plunger arrangement of this design option is depicted in Figure 1.

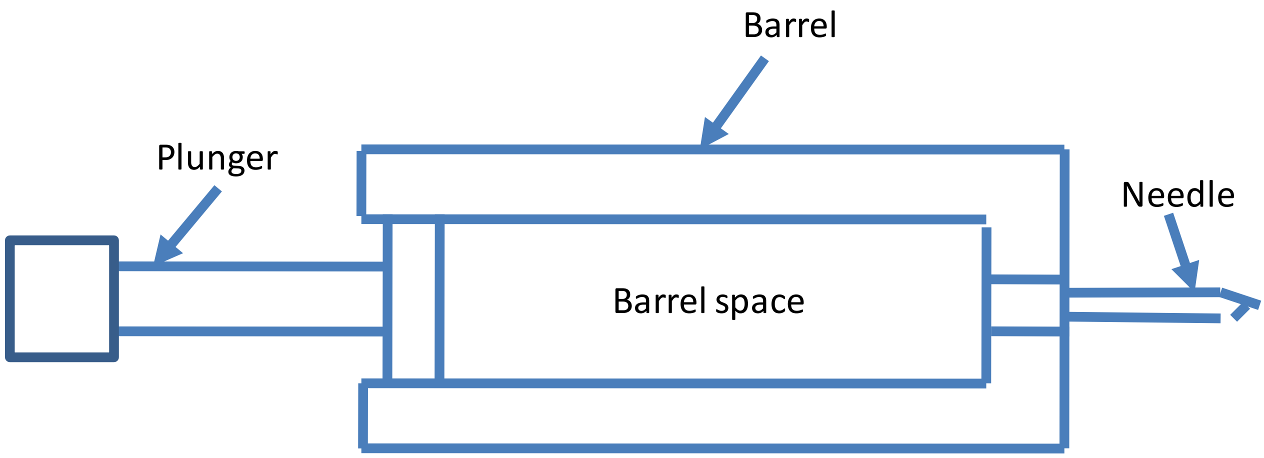

FIGURE 1: Schematic representation of cylindrical barrel-plunger syringe with flat ends

Thin cross section of thin walled cylindrical vessel subjected to internal pressure is as shown in Figure 2.

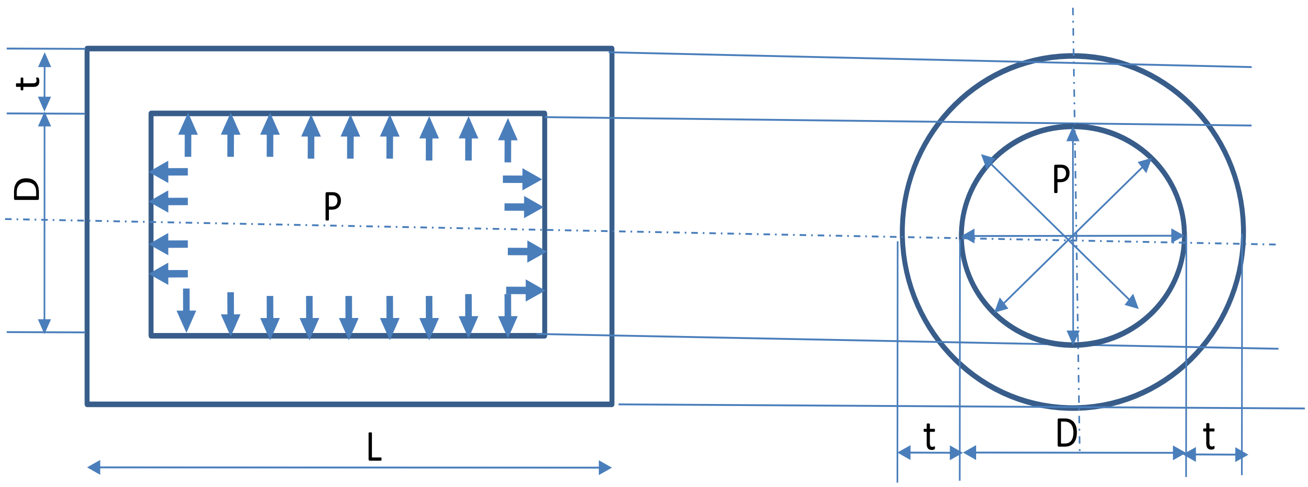

FIGURE. 2: Thin cylindrical vessel in which a fluid is stored at constant pressure.

The stresses and pressure acting on the thin walled cylinder sliced through the diametral plane and when cut with a plane perpendicular to the axis of the of the cylinder are as shown in Figures 3 and 4 respectively.

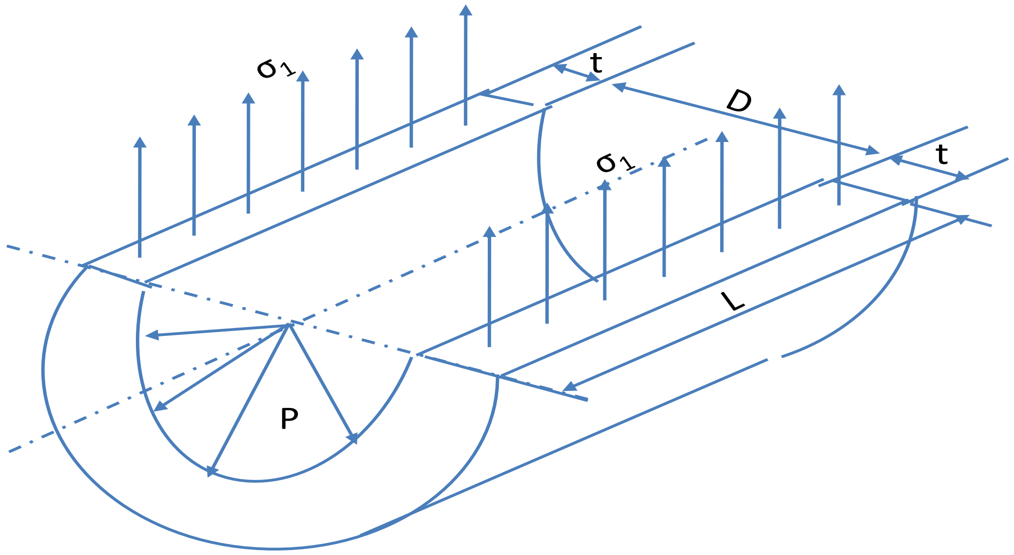

FIGURE 3: Stress and pressure acting on a thin walled cylinder when cut through the diametral plane

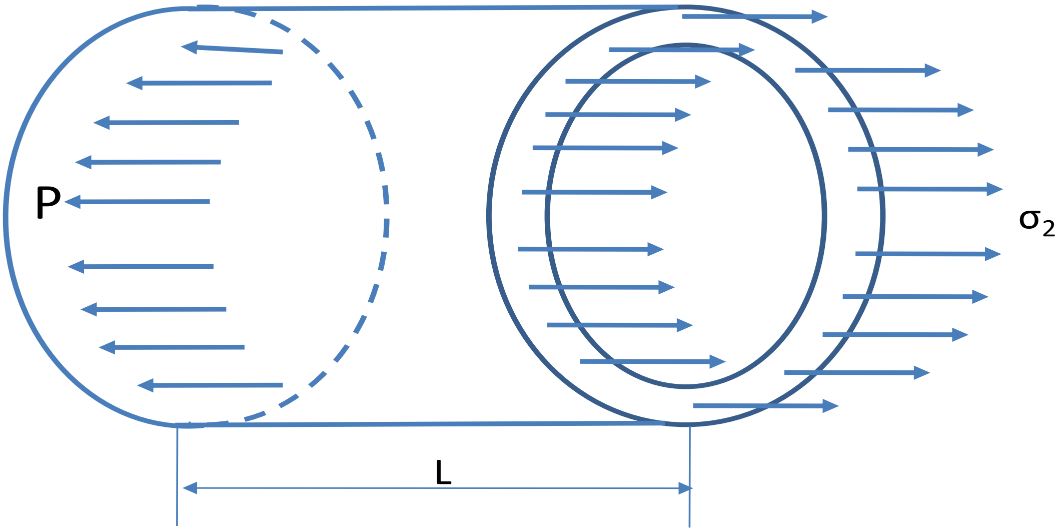

FIGURE 4: Stress and pressure acting on a thin walled cylinder when cut

By a plane perpendicular to the axis of the cylinder

The force acting on the two rectangular surfaces in Figure 3 due to the hoop or circumferential is expressed as:

$$F_1 = 2\sigma_1 Lt \tag{1}$$

Where,

σ_1—The hoop or circumferential stress N/m^2)

L—Length of cylinder (m)

t—cylinder wall thickness (m)

F_1—Force due to hoop stress (N)

The force due to the internal fluid pressure is given as:

$$F_2 = PDL \tag{2}$$

Where,

P—Internal pressure of the enclosed fluid (N/m^2)

D—Internal diameter of the cylinder (m)

L—Cylinder length (m)

F_2—Force due to fluid pressure (N/m^2)

To avoid rupture or bursting of the cylinder in the longitudinal direction these two forces must be equal and opposite. Therefore,

$F_1 = F_2$

$2\sigma_1 Lt = PDL$

$$\sigma_1 = \frac{PD}{2t} \tag{3}$$

Considering Figure 4, the force in the longitudinal direction due to the longitudinal stress is expressed as:

$$F_3 = \sigma_2 \pi D t \tag{4}$$

The pressure force due to the internal pressure of the enclosed fluid is given as:

$$F_4 = \frac{\pi P D^2}{4} \tag{5}$$

To avoid bursting of the cylinder in the transverse direction, there should be equality of Equations 4 and 5. Therefore,

$F_4 = F_5$

$\sigma_2 \pi D t = \frac{\pi P D^2}{4}$

$$\sigma_2 = \frac{PD}{4t} \tag{6}$$

Where,

F3—Force due to the longitudinal stress (N)

F4—Force due to the internal fluid pressure (N/m^2)

σ_2 --Longitudinal stress (N/m^2)

3.1.2 Deduction for Cylindrical Barrel with Hemispherical Ends

For the second design option the barrel-plunger arrangement is as shown in Figure 5.

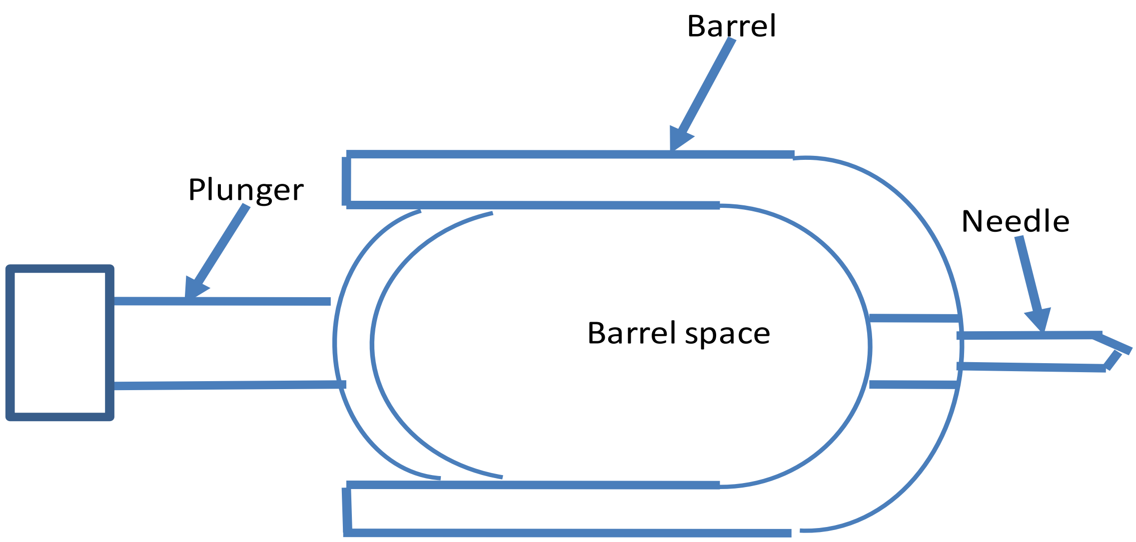

FIGURE 5: Barrel-plunger arrangement for cylinder with hemispherical ends

The stresses and the constant internal pressure acting on the hemispherical cylinder is represented in Figure 6.

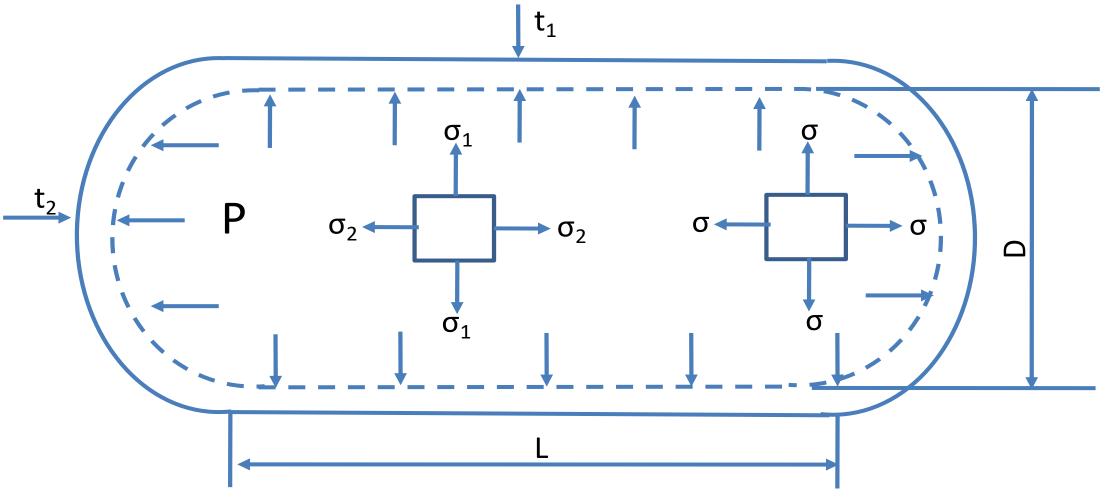

FIGURE 6: Internal fluid pressure and stresses acting on hemispherical cylinder

The thickness of the cylinder is t_1 while that of the hemisphere is t_2. The cylinder and the hemisphere might not necessarily be of the same thickness, i.e.$t_1 \pm t_2$. The cylindrical part of the barrel experiences both hoop stress, σ_1, and longitudinal stress, σ_2, just the same way cylindrical barrel does. The stress distribution of the hemispherical ends must be equal through the thickness of the shell and must be equal i.e., $\sigma_1=\sigma_2=\sigma$. The hoop stress and longitudinal stress at constant internal fluid pressure are expressed as:

$$\text{Hoop stress, } \sigma_1 = \frac{PD}{2t_1} \tag{7}$$

$$\text{Longitudinal stress, } \sigma_2 = \frac{PD}{4t_1} \tag{8}$$

The hoop strain is expressed as:

$$\epsilon_1 = \frac{1}{E}(\sigma_1 - \nu\sigma_2) = \frac{PD}{4Et_1}(2-\nu) \tag{9}$$

The hemispherical ends experiences only hoop stress expressed as:

$$\sigma = \frac{PD}{4t_2} \tag{10}$$

The hoop strain is given as:

$$\epsilon_2 = \frac{1}{E}(\sigma - \nu\sigma) = \frac{PD}{4Et_2}(1-\nu) \tag{11}$$

For the junctions not to distort under the prevailing stresses and loads, the hoop strain in the cylindrical portion must equate that in the hemispherical portion. Hence, $\epsilon_1 = \epsilon_2$, and,

$$\frac{t_1}{t_2} = \frac{2-\nu}{1-\nu} \tag{12}$$

If the hoop stress in the cylindrical part and hemispherical part should remain maximum and equal, the ratio of t_2 to t_1 will be 0.5 and poisson's ratio, $\nu = 0$.

For cylindrical part, force due to hoop stress is expressed as:

$$F_1 = \sigma_1(2Lt_1 + \pi D t_1) \tag{13}$$

Force due to fluid pressure is given as:

$$F_2 = P\left(LD + \frac{\pi D^2}{4}\right) = PD\left(L + \frac{\pi D}{4}\right) \tag{14}$$

Force due to longitudinal stress is stated as:

$$F_3 = \sigma_2 \pi D t_1 \tag{15}$$

Force due to fluid pressure goes thus:

$$F_4 = \frac{\pi P D^2}{4} \tag{16}$$

With reference to the hemispherical ends, the only acting stress is the hoop stress. It is expressed as:

$$\sigma = \frac{PD}{4t_2} \tag{17}$$

Force due hoop stress is expressed as:

$$F_5 = \sigma \pi D t_2 \tag{18}$$

Force due to fluid pressure is given as:

$$F_6 = \frac{\pi P D^2}{4} \tag{19}$$

3.2 Input Data to the Mathematical Models:

Size of syringe=2ml

Internal fluid pressure, P=86000N/m^2

Length of cylinder, L=115mm=0.115m

Cylinder diameter, D=9mm=0.009m

$$\frac{1}{25} \leq \frac{t}{D}\ ratio \leq \frac{1}{20}$$

$$\frac{t_2}{t_1} = \frac{1}{2}$$

3.3 Computational Results:

3.3.1 Considering the cylindrical barrel with flat ends:

Assuming a t/D ratio of 1/25:

Cylinder wall thickness is given as:

$$\frac{t}{D} = \frac{1}{25}$$

$$t = \frac{D}{25} = \frac{9}{26} = 0.36mm = 0.00036m$$

The hoop stress is given as,

$$\sigma_1 = \frac{PD}{2t} = \frac{86000 \times 9 \times 10^{-3}}{2 \times 0.3610^{-3}} = 1.075 \times 10^6 N/m^2$$

Force due to the hoop stress,

$$F_1 = 2\sigma_1 L t = 2 \times 86000 \times 115 \times 10^{-3} \times 0.36 \times 10^{-3} = 89.01N$$

Force due to the fluid pressure,

$F_2 = PDL = 86000 \times 9 \times 10^{-3} \times 115 \times 10^{-3}$=89.01N

To determine the longitudinal stress,

$$\sigma_2 = \frac{PD}{4t} = \frac{\sigma_1}{2} = \frac{1.075 \times 10^6}{2} = 537500 N/m^2$$

Force due to the longitudinal stress,

$F_3=\sigma_2\pi Dt = 537500 \times \pi\times 9\times 10^{-3} \times 0.36 \times 10^{-3} = 5.4711N$

Force due to the fluid pressure,

$F_4 = \frac{\pi PD^2}{4} = \frac{\pi\times 86000\times 81\times 10^{-6}}{4} = 5.4711N$

Assuming a t/D ratio of 1/20:

Cylinder wall thickness is given as:

$\frac{t}{D} = \frac{1}{20}$

$t = \frac{D}{20} = \frac{9}{20} = 0.45mm$

The hoop stress is given as,

$\sigma_1 = \frac{PD}{2t} = \frac{86000\times 9\times 10^{-3}}{2\times 0.4510^{-3}} = 860000N/m^2$

Force due to the hoop stress,

$F_1 = 2\sigma_1 Lt = 2 \times 860000 \times 115 \times 10^{-3} \times 0.36 \times 10^{-3} = 89.01N$

Force due to the fluid pressure,

$F_2 = PDL = 86000 \times 9 \times 10^{-3} \times 115 \times 10^{-3}$=89.01N

To determine the longitudinal stress,

$\sigma_2 = \frac{PD}{4t} = \frac{\sigma_1}{2} = \frac{860000}{2} = 430000N/m^2$

Force due to the longitudinal stress,

$F_3=\sigma_2\pi Dt = 430000 \times \pi\times 9\times 10^{-3} \times 0.36 \times 10^{-3} = 5.4711N$

Force due to the fluid pressure,

$F_4 = \frac{\pi PD^2}{4} = \frac{\pi\times 860000\times 81\times 10^{-6}}{4} = 5.4711N$

3.3.2 Considering the cylindrical barrel with Hemispherical ends:

Taking the t/D ratio=1/25

To determine the cylinder thickness, t_1:

$\frac{t_1}{D} = \frac{1}{25}$

$t_1 = \frac{D}{25} = \frac{9}{25} = 0.36mm$

To determine the thickness t_2 of the hemispherical ends::

$\frac{t_2}{t_1} = \frac{1}{2}$

$t_2 = \frac{t_1}{2} = \frac{0.36}{2} = 0.18mm$

The hoop stress in the cylindrical part is given as:

$\sigma_1 = \frac{PD}{2t_1} = \frac{86000\times 9\times 10^{-3}}{2\times 0.36\times 10^{-3}} = 1.075 \times 10^6 N/m^2$

Force due to the hoop stress,

$F_1 = \sigma_1(2Lt_1 + \pi Dt_1) = 1.075 \times 10^6(2 \times 115 \times 10^{-3} \times 0.36 \times 10^{-3}) + \pi \times 9 \times 10^{-3} \times 0.36 \times 10^{-3} = 99.95N$

Force due to fluid pressure,

$$F_2 = PD\left(L + \frac{\pi D}{4}\right) = 86000 \times 9 \times 10^{-3}\left(115 \times 10^{-3} + \frac{\pi \times 9 \times 10^{-3}}{4}\right) = 94.48N$$

The longitudinal stress is given as:

$$\sigma_2 = \frac{PD}{4t_1} = \frac{\sigma_1}{2} = \frac{1.075 \times 10^6}{2} = 537500N/m^2$$

Force sue to the longitudinal stress,

$$F_3 = \sigma_2 \pi D t_1 = 537500 \times \pi \times 9 \times 10^{-3} \times 0.36 \times 10^{-3} = 5.4711N$$

For due to fluid pressure,

$$F_4 = \frac{\pi PD^2}{4} = \frac{\pi \times 86000 \times 81 \times 10^{-6}}{4} = 5.4711N$$

Hoop stress in the hemispherical ends,

$$\sigma = \frac{PD}{4t_2} = \frac{86000 \times 9 \times 10^{-3}}{4 \times 0.18 \times 10^{-4}} = 1.075 \times 10^6 N/m^2$$

Force due to the hoop stress,

$$F_5 = \sigma \pi D t_2 = 1.075 \times 10^6 \times \pi \times 9 \times 10^{-3} \times 0.18 \times 10^{-3} = 5.4711N$$

Force due to fluid pressure,

$$F_6 = \frac{\pi PD^2}{4} = \frac{\pi \times 86000 \times 81 \times 10^{-6}}{4} = 5.4711N$$

3.4 Output Results

Results for cylindrical barrel with flat ends is as in Table 1 when t/D ratio is 1/25.

TABLE 1
OUTPUT RESULTS FOR CYLINDRICAL BARREL WITH FLAT ENDS AT t/D RATIO OF 1/25

Hoop stress, σ_1 (N/m^2)	$1.075 \times 10^6 N/m^2$
Force due to the hoop stress, F_1 (N)	$89.01N$
Force due to pressure of the fluid, F_2(N)	$89.01N$
Longitudinal stress, σ_2 (N/m^2)	537500N
Force due to the longitudinal, stress, F_3 (N)	$5.4711N$
Force due to pressure of the fluid, F_4 (N)	$5.4711N$

TABLE 2
OUTPUT RESULTS FOR CYLINDRICAL BARREL WITH FLAT ENDS AT t/D RATIO OF 1/20

Hoop stress, σ_1 (N/m^2)	$860000N/m^2$
Force due to the hoop stress, F_1 (N)	$89.01N$
Force due to pressure of the fluid, F_2(N)	$89.01N$
Longitudinal stress, σ_2 (N/m^2)	$430000N/m^2$
Force due to the longitudinal, stress, F_3 (N)	$5.4711N$
Force due to pressure of the fluid, F_4 (N)	$5.4711N$

TABLE 3
OUTPUT RESULTS FOR CYLINDRICAL BARREL WITH HEMISPHERICAL ENDS AT t/D RATIO OF 1/25

Hoop stress, σ_1 (N/m^2)	$1.075 \times 10^6 N/m^2$
Force due to the hoop stress, F_1 (N)	$99.95N$
Force due to pressure of the fluid , F_2(N)	$94.48N$
Longitudinal stress, σ_2 (N/m^2)	$537500N/m^2$
Force due to the longitudinal, stress, F_3 (N)	$5.4711N$
Force due to pressure of the fluid, F_4 (N)	$5.4711N$
Hoop stress in hemispherical portion, σ(N/m^2)	$1.075 \times 10^6 N/m^2$
Force due to the longitudinal, stress, F_5 (N)	$5.4711N$
Force due to pressure of the fluid, F_6 (N)	$5.4711N$

IV. DISCUSSIONS

Within the limits of t/D ratio in the range $\frac{1}{25} \leq \frac{t}{D}\ ratio \leq \frac{1}{20}$, cylindrical barrel with flat ends confirms force equilibrium in the transverse and longitudinal directions. The numerical values for forces due to hoop stress and internal pressure of enclosed fluid being 89.01N. That for longitudinal stress force and fluid pressure force being 5.4711N.

Cylindrical barrel with hemispherical ends at t/D ratio of 1/25, disobeyed the condition of force equilibrium in the hoop stress force analysis. The hoop stress force and fluid pressure force being 99.95N and 94.48N respectively. Longitudinal stress and hoop stress forces analysis remain constant at 5.4711N. Hence bursting or rupture is likely to take place in the longitudinal direction.

V. RECOMMENDATION FOR FUTURE RESEARCH

Syringe barrel design for other geometrical configurations such as rectangular and elliptical cross sections should be looked into. This is to confirm their suitability and reliability and medical certification under service conditions relative to the two different worked upon in this research.

VI. CONCLUSION

The cylindrical syringe barrel design with flat ends is the preferred option to the cylindrical barrel with hemispherical ends. The cylindrical barrel design with hemispherical ends is prone to the case of developing crack or bursting in the longitudinal plane. The reason being that the hoop stress force and internal fluid pressure force do not agree. That is to say the hoop stress force is 99.95N while the fluid pressure force is 94.48N.

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