International Journal of Engineering Research & Science (IJOER)
ISSN: [2395-6992]
[Vol-12, Issue-3, March- 2026]
Website: www.ijoer.com
Journal DOI: 10.25125/engineering-journal

Interdependent Networks with Higher-Order Structures

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Received: 01 March 2026/ Revised: 08 March 2026/ Accepted: 13 March 2026/ Published: 31-03-2026

Abstract— *In the construction of interdependent networks, the functionality of a group in one layer typically relies on the support of a group in another layer. To investigate the stability of such networks, we propose a framework comprising a bilayer interdependent hypergraph system, where the two layers exhibit mutual dependencies. Our core hypothesis is that the removal of nodes in one layer not only leads to node failures but, more critically, triggers the failure of hyperedges, resulting in iterative cascading failures across layers. Using a bilayer system characterized by a Poisson hyperdegree distribution as an example, we have proven through rigorous analysis how parameter changes affect the robustness of the target network. Overall, our study highlights the critical role of hyperedge interdependence mechanisms and network topological structures in mitigating cascading failures in systems with higher-order interactions, providing valuable insights for the design and optimization of network systems.*

Keywords— Interdependent Networks, Hypergraph, Hyperedge Dependency, Cascading Failure.

I. INTRODUCTION

With the exploration of real-world systems, we find that a single network can no longer meet our real needs[1,2], Networks in the real world usually interact with other networks to varying degrees[3-5] and jointly function to serve the real world. These networks often take the form of multi-layer interdependent networks[6].

In such a network, the cascading failure process is more worthy of study. It usually manifests as the removal of a few nodes triggering iterations through intra-layer failures or inter-layer failures, ultimately bringing the network to a stable state. However, merely the interdependence relationship is far from sufficient. Networks in the real world usually exhibit complex relationships and dependencies, not just pairwise connections. To address this issue, we introduce hypergraphs to reflect high-order interactions and dependencies[7], providing a more comprehensive representation. For example, in social networks[8,9], we can consider the interaction patterns among multiple users, explore the high-order associations among multiple users. In biological networks[10], we can effectively identify the strong and weak relationships between nodes.

Further study the robustness of the network. Percolation theory is a commonly used method to study network robustness[11], played a key role in the study of the collective behavior of systems. There are usually two types of percolation, one of which is bond percolation[12], one is point percolation[13]. So usually, the seepage situation is studied to observe the system behavior of the network. Zhou et al[14] analyzing the characteristics of interdependent networks, a related k-core percolation model is proposed. Given any degree distribution of edge-coupled interdependent networks, the relative sizes of the k-core and the corona cluster as well as the percolation threshold can be obtained through the derivation of self-consistent equations, and then the phase transition of k-core percolation can be analyzed. The proposed k-core percolation model and protection strategy not only help to understand the hierarchical structure of the network, but also provide some guidance for enhancing the resilience of the network against attacks. Secondly, the higher-order interactions in interdependent networks can usually be represented by hyperedges or simplicial complexes. For simplicial complexes, Peng et al[6,15] constructed a two-layer partial dependence network theoretical model with a simplicial complex, in which failures between nodes occur through the synergistic effects of pairwise interactions and high-order interactions. In this model, removing a node will cause all other nodes in the same simplex to be removed, and due to the dependence between the two networks, node failures will spread through the dependence links between the two networks. This process will occur recursively, ultimately leading to a cascading process. Four types of artificial MIHN models were further constructed, where high-order interactions are still described by simplicial complexes, and inter-layer dependencies are established through one-to-one matching dependence links. The robustness of MIHN was studied by investigating the largest connected component and the percolation threshold. We found that the density of the simplicial complex and the number of network layers affect its percolation behavior.

For hyperedges, Liu et al[16] proposed a percolation model that takes into account the dependence of hyperedges on their internal nodes, and the research reveals the different impacts of the hyperdegree distribution on the system robustness in single-layer and double-layer hypergraphs. In the real world, not only do nodes have interdependent relationships, but edges do as well. Qian et al[17] proposed an interdependent hypergraph model considering the inter-layer node dependency. The cascading failures in hypergraphs with different inter-layer dependencies were studied. The maximum attack intensity that the network can withstand was determined through theoretical analysis, as well as how its robustness changes under different attack intensities. The multi-layer hypergraph can represent the relationships between nodes more clearly. However, there is less research on identifying important nodes within this framework. Wang et al[18] proposed a method named HCT to fill this gap. The global centrality of nodes in the entire network can be calculated. Compared with other methods, the important nodes identified by HCT exhibit stronger propagation capabilities, and removing these nodes will seriously damage the connectivity and robustness of the network. To further reveal the profound impact of group support on the resilience of the system against cascading failures, Chen et al [19] designed a framework consisting of a two-layer interdependent hypergraph system, where the nodes in one layer can obtain support through the hyperedges in the other layer. The article derived the critical threshold of the initial node survival probability that marks the second-order phase transition point.

There is also an understanding of network resilience. Network resilience measures the degree of performance degradation of a network after being perturbed and its recovery ability, and is closely related to the ability to resist cascading failures. Li et al[20] proposed three resilience enhancement strategies based on the node capacity redundancy at different structural scales, and developed a network resilience evaluation method that considers both the structure and node load. The performance of the enhancement strategies is closely related to the node capacity redundancy. Specifically, when enhancing nodes with larger capacity redundancy, the enhancement efficiency is higher. In addition, the heterogeneity of node load has a profound impact on the enhancement efficiency. Lv et al [21] proposed a resilience assessment model to predict the performance of interdependent networks against cascading failures. This model can accurately monitor the activities of each node during the cascading process.

In the real world, systems often exist in the form of groups, so that the network not only needs to consider the intra-layer dependencies but also the inter-layer dependencies. Furthermore, it is necessary to consider whether this dependence is node dependence or hyperedge dependence. Therefore, we will combine these two aspects and use hypergraphs to study the robustness of two-layer partially interdependent networks.

II. MODEL AND THEORY

2.1 Model:

We construct a bilayer hypergraph system with hyperedge interdependencies, denoted as layers A and B. Layer A consists of N_A nodes and M_A hyperedges, while layer B comprises N_B nodes and M_B hyperedges. The hyperdegree of each node, denoted by k, represents the number of hyperedges to which the node belongs. The hyperdegree distributions of layers A and B follow $P_A(k)$ and $P_B(k)$, respectively. Similarly, the cardinality of a hyperedge, denoted by m, indicates the number of nodes contained in each hyperedge, with distributions following $Q_A(m)$ and $Q_B(m)$, respectively. Regarding interlayer dependencies, we define that hyperedges in layers A and B are interdependent with probability λ.

Due to the interdependence between network systems being maintained through hyperedges, the cascading failure process involves both node failures and hyperedge failures. First, we consider node failures. In layer A, nodes are initially present with probability p (equivalently, removed with probability $1-p$), where p represents the initial node survival probability. For a node to remain functional, it must satisfy the following conditions: (1) the node must belong to the Giant Connected Component (GCC) of its layer to ensure intralayer connectivity; (2) all hyperedges to which the node belongs must remain functional.

Second, we address hyperedge failures. Each hyperedge initially exists in a functional state. To remain functional after node failures, a hyperedge must satisfy the following conditions: (1) following node failures, the functional state of the hyperedge is reassessed; we define a tolerance coefficient $\alpha \in [0,1]$ (typically taken as $\alpha = 0.5$ in this study), and a hyperedge fails if the proportion of its constituent nodes that are functional falls below α; (2) if a hyperedge's interdependent partner hyperedge in the opposite layer fails, the hyperedge itself fails.

This cascading failure process, triggered by initial node removal, iterates between the two layers until no additional node or hyperedge failures occur. The final sizes of the Giant Connected Components in layers A and B are denoted as G_A and G_B, respectively.

The cascading failure process in the model is illustrated in Figure 1. (a) Initially, node 4 in layer A is removed. (b) Nodes (1, 2, 3) fail because they become disconnected from the GCC, and hyperedges e_1^A and e_2^A fail as their functional node proportions fall below $\alpha = 0.5$. (c) Due to the interdependence between layers A and B, hyperedges e_1^B and e_2^B in layer B fail, and hyperedge e_3^B fails because its functional node proportion equals exactly 0.5. (d) Layer B further affects layer A, causing hyperedge e_3^A in layer A to fail, at which point the system reaches a stable state.

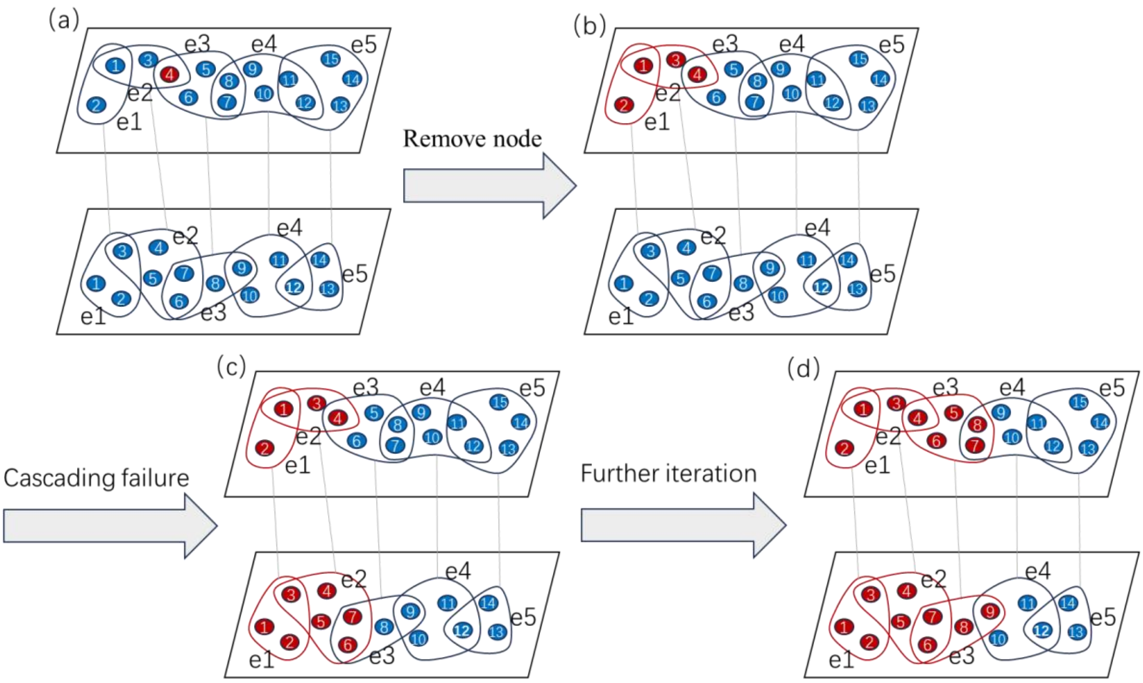

FIGURE 1: Schematic diagram of the cascading failure process in bilayer hypergraphs A and B. The connections between the two layers represent the interdependence established between a hyperedge in one layer and its corresponding dependent hyperedge in the other layer. Blue nodes indicate functional nodes, black hyperedges represent functional hyperedges, red nodes denote failed nodes, and red hyperedges indicate hyperedges that have failed due to degraded functionality.

2.2 Theory

To characterize the structure of the two-layer network and the interdependence between them, we introduce generating functions to simplify the key information regarding the hyperdegree distribution and cardinality distribution in each layer[19].

Since this study considers a bilayer network, the generating functions for the hyperdegree distributions of networks A and B are similar and are both functions of k:

$$\begin{cases} G_{k0}^{A}(x)=\sum_{k=0}^{\infty}P^{A}(k)x^{k} \\ G_{k0}^{B}(x)=\sum_{k=0}^{\infty}P^{B}(k)x^{k} \end{cases} \tag{1}$$

The generating function for the distribution of excess hyperdegree is similarly expressed as:

$$\begin{cases} G_{k1}^{A}(x)=\sum_{k=1}^{\infty}\frac{kP^{A}(k)}{\langle k\rangle}x^{k-1}=G_{k0}^{A'}(x)/G_{k0}^{A'}(1) \\ G_{k1}^{B}(x)=\sum_{k=1}^{\infty}\frac{kP^{B}(k)}{\langle k\rangle}x^{k-1}=G_{k0}^{B'}(x)/G_{k0}^{B'}(1) \end{cases} \tag{2}$$

The generating function for the cardinality distribution is defined as:

$$\begin{cases} G_{m0}^{A}(x)=\sum_{m=0}^{\infty}Q^{A}(m)x^{m} \\ G_{m0}^{B}(x)=\sum_{m=0}^{\infty}Q^{B}(m)x^{m} \end{cases} \tag{3}$$

The generating function for the distribution of excess bases is defined as:

$$\begin{cases} G_{m1}^{A}(x)=\sum_{m=1}^{\infty}\frac{mQ^{A}(m)}{\langle m\rangle}x^{m-1}=G_{m0}^{A'}(x)/G_{m0}^{A'}(1) \\ G_{m1}^{B}(x)=\sum_{m=1}^{\infty}\frac{mQ^{B}(m)}{\langle m\rangle}x^{m-1}=G_{m0}^{B'}(x)/G_{m0}^{B'}(1) \end{cases} \tag{4}$$

As shown in Figure 1, the cascading failure process unfolds in a discrete iterative manner, progressing from stage $n=1$ to $n=\infty$ in both layers of the network.

We denote the probabilities of hyperedges functioning normally in layers A and B at the n stage as T_n^A and T_n^B, respectively. Correspondingly, the probability that a dependent hyperedge in layer $A(B)$ lacks support from normally functioning hyperedges in layer $B(A)$ at the n stage is given by the following expression:

$$\begin{cases} u_n^A=q\left(1-T_{n-1}^{B}\right) \\ u_n^B=q\left(1-T_{n-1}^{A}\right) \end{cases} \tag{5}$$

The proportion of nodes that maintain their functions in layer $A(B)$ of the nth stage can be expressed as:

$$\begin{cases} p_n^A=r^A\left(1-qu_n^A\right) \\ p_n^B=r^B\left(1-qu_n^B\right) \end{cases} \tag{6}$$

T_n^A and T_n^B can be expressed as:

$$\begin{cases} T_n^A = \sum_{m=0}^{\infty} Q^A(m) \sum_{j=0}^{m} \binom{m}{j} \left\{1 - \left[G_{k1}^A\left(1 - f_n^A\right)\right]^{m-j}\right\} \left(p_n^A\right)^{m-j} \left(1 - p_n^A\right)^j I\left(\frac{m-j}{m} > \delta\right) \\ T_n^B = \sum_{m=0}^{\infty} Q^B(m) \sum_{j=0}^{m} \binom{m}{j} \left\{1 - \left[G_{k1}^B\left(1 - f_n^B\right)\right]^{m-j}\right\} \left(p_n^B\right)^{m-j} \left(1 - p_n^B\right)^j I\left(\frac{m-j}{m} > \delta\right) \end{cases} \tag{7}$$

Among them, f_n^A and f_n^B represents the probability that a randomly selected hyper-edge through a random node can be connected to the giant connected components (GCC) of each layer. First, select a random hyperedge with cardinality m according to the distribution $Q^A(m)$ or $Q^B(m)$.Then, $\binom{m}{j}$ represents the situation where there are j failed nodes among the m nodes of the selected hyperedge. $1 - \left[G_{k1}^A\left(1 - f_n^A\right)\right]^{m-1-j}$ or $1 - \left[G_{k1}^B\left(1 - f_n^B\right)\right]^{m-1-j}$ represents the probability that at least one of the $m-j$ functional nodes on the remaining hyperedges can be connected to the Giant Connected Component (GCC) in a random hyperedge with a base of m. $I(\cdot)$ is an indicator function. When the condition in the parentheses holds, it takes the value of 1; otherwise, it takes the value of 0. It is used to determine whether the hyperedge state exceeds the tolerance coefficient δ. Generally, the tolerance coefficient δ is first taken as 0.5.

f_n^A and f_n^B can be expressed as:

$$\begin{cases} f_n^A = \sum_{m=1}^{\infty} \frac{mQ^A(m)}{\langle m \rangle} \sum_{j=0}^{m-1} \binom{m-1}{j} \left\{1 - \left[G_{k1}^A\left(1 - f_n^A\right)\right]^{m-1-j}\right\} \left(p_n^A\right)^{m-1-j} \left(1 - p_n^A\right)^j I\left(\frac{m-1-j}{m-1} > \delta\right) \\ f_n^B = \sum_{m=1}^{\infty} \frac{mQ^B(m)}{\langle m \rangle} \sum_{j=0}^{m-1} \binom{m-1}{j} \left\{1 - \left[G_{k1}^B\left(1 - f_n^B\right)\right]^{m-1-j}\right\} \left(p_n^B\right)^{m-1-j} \left(1 - p_n^B\right)^j I\left(\frac{m-1-j}{m-1} > \delta\right) \end{cases} \tag{8}$$

Therefore, the proportion of nodes S_n^A and S_n^B in the Giant Connected Component (GCC) can be obtained from the following formula:

$$\begin{cases} S_n^A = p_n^A\left[1 - G_{k0}^A\left(1 - f_n^A\right)\right] \\ S_n^B = p_n^B\left[1 - G_{k0}^B\left(1 - f_n^B\right)\right] \end{cases} \tag{9}$$

When the cascading failure process terminates, they converge to their steady state values respectively: $p_\infty^A, p_\infty^B, T_\infty^A, T_\infty^B, f_\infty^A, f_\infty^B, S_\infty^A, S_\infty^B$. All the above formulas can form a self-consistent system of equations, written in the form of a generating function:

$$\begin{cases} T_\infty^A = I\left(\frac{m-j}{m} > \delta\right)\left(1 - G_{m0}^A\left(1 - p_\infty^A + p_\infty^A G_{k1}^A\left(1 - f_\infty^A\right)\right)\right) \\ f_\infty^A = I\left(\frac{m-j}{m} > \delta\right)\left(1 - G_{m1}^A\left(1 - p_\infty^A + p_\infty^A G_{k1}^A\left(1 - f_\infty^A\right)\right)\right) \end{cases} \tag{10}$$

$$\begin{cases} T_\infty^B = I\left(\dfrac{m-j}{m} > \delta\right)\left(1 - G_{m0}^B\left(1 - p_\infty^B + p_\infty^B G_{k1}^B\left(1 - f_\infty^B\right)\right)\right) \\ f_\infty^B = I\left(\dfrac{m-j}{m} > \delta\right)\left(1 - G_{m1}^B\left(1 - p_\infty^B + p_\infty^B G_{k1}^B\left(1 - f_\infty^B\right)\right)\right) \end{cases} \tag{11}$$

$$\begin{cases} S_\infty^A = p_\infty^A\left[1 - G_{k0}^A\left(1 - f_\infty^A\right)\right] \\ S_\infty^B = p_\infty^B\left[1 - G_{k0}^B\left(1 - f_\infty^B\right)\right] \end{cases} \tag{12}$$

First, assume that both layers follow a Poisson cardinality distribution: $e^{-<m>} < m >^m / m!$, This simplifies the solution of the equation. Therefore, we have:

$$\begin{cases} G_{m0}^A(x) = G_{m1}^A(x) \\ G_{m0}^B(x) = G_{m1}^B(x) \end{cases} \tag{13}$$

$$\begin{cases} f_\infty^A = T_\infty^A \\ f_\infty^B = T_\infty^B \end{cases} \tag{14}$$

$$\begin{cases} f_\infty^A = I\left(\dfrac{m-j}{m} > \delta\right)\left(1 - G_{m0}^A\left(1 - p_\infty^A + p_\infty^A G_{k1}^A\left(1 - f_\infty^A\right)\right)\right) \\ f_\infty^B = I\left(\dfrac{m-j}{m} > \delta\right)\left(1 - G_{m0}^B\left(1 - p_\infty^B + p_\infty^B G_{k1}^B\left(1 - f_\infty^B\right)\right)\right) \end{cases} \tag{15}$$

Furthermore, if the two hypergraph layers have identical cardinality and hyperdegree distributions, we can derive:

$$\begin{cases} G_{m0}^A(x) = G_{m0}^B(x) = G_{m0}(x) \\ G_{k0}^A(x) = G_{k0}^B(x) = G_{k0}(x) \end{cases} \tag{16}$$

If $q_A = q_B = q, r^A = r^B = r$, we can derive:

$$f_\infty^A = f_\infty^B = f \tag{17}$$

$$f = I(\frac{m-j}{m} > \delta)\left(1 - G_{m0}\left(1 - r\left[1 - q^2(1-f)\right]\left[1 - G_{k1}(1-f)\right]\right)\right) \tag{18}$$

$$S = r\left[1 - q^2(1-f)\right]\left[1 - G_{k0}(1-f)\right] \tag{19}$$

III. RESULTS AND DISCUSSION

In Figure 2, we can observe that, under the condition of fixing the average hyperdegree, the average hyperedge cardinality, and the tolerance coefficient, as the proportion of node removal increases, the change of the giant connected component (GCC). For the overall trend, all curves gradually decrease as $1-r$ increases and then tend to 0, which conforms to the general law when the network is under attack. The smaller the q is, the size of GCC can still maintain a relatively high proportion for a period of time; conversely, the larger the q is, the more sensitive the network is to node removal, and a small-scale node damage will lead to network collapse. As q decreases, the network characteristics tend to transform into a more continuous phase transition.

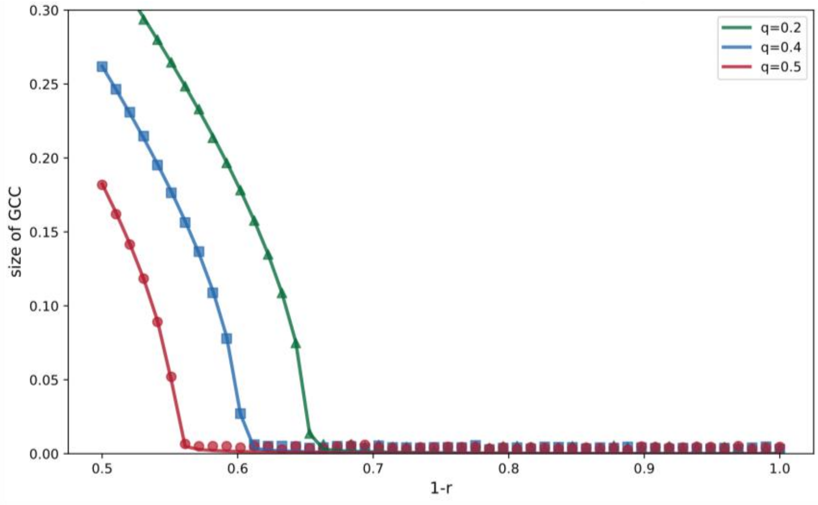

FIGURE 2: The trend graph of the GCC size changing with the node removal ratio. Different colors represent the change of the GCC size with the node removal ratio under different dependency ratios.

From Figure 3, we can see that as the dependency ratio increases, the network is more likely to collapse. Moreover, the larger the average hyperdegree, or the average hyperedge cardinality, the stronger the network's invulnerability, and the less likely it is to collapse as increases. From the figure, we can further learn that, excluding the situations where the dependency ratio, average hyperdegree, and average hyperedge cardinality are relatively small, as the dependency ratio increases and the average hyperdegree, and average hyperedge cardinality become larger, the network characteristics tend to transform into a more continuous phase transition.

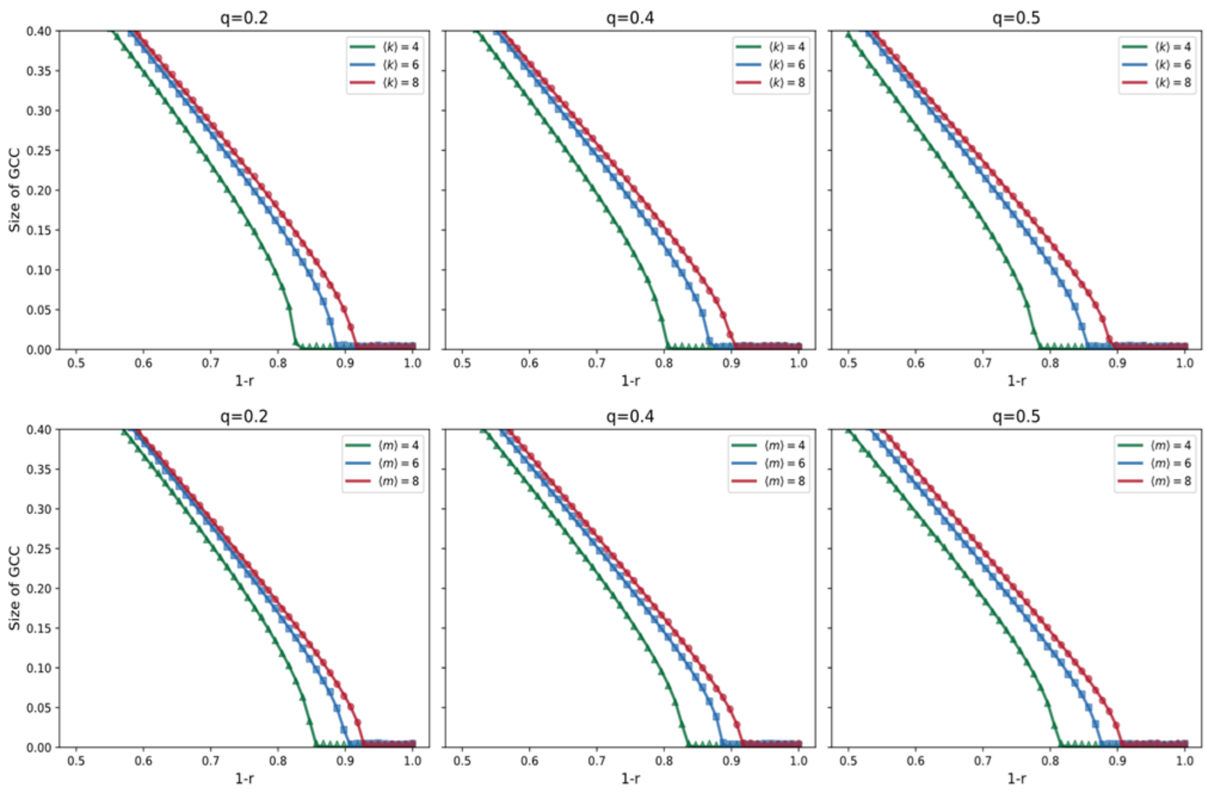

FIGURE 3: It shows the variation of the GCC size with the increase of the removal proportion under different parameter settings, that is, under different average hyperdegree and different average hyperedge cardinalities.

The first sub-figure in Figure 4 illustrates the variation of the size of the Giant Connected Component (GCC) under different tolerance coefficients as the proportion of removed nodes increases. Under a low tolerance coefficient, when the initial failure proportion is relatively small, the size of the GCC has already started to decrease significantly; as gradually increases, the GCC approaches 0. When the tolerance coefficient is small, the network is extremely sensitive to the initial failure, and the critical value of the phase transition is low. As the tolerance coefficient becomes larger and larger, a larger initial failure proportion is required for the size of the GCC to decrease, which indicates that the network can withstand a high-proportion initial failure, and the critical value of the phase transition is high, that is, only a strong disturbance can trigger the collapse of the network.

The second sub-figure shows the variation of the size of the GCC under the combined effect of the node retention proportion and the tolerance coefficient. It can be known from the color bar that the redder the color, the larger the size of the GCC. Fixing, it can be seen that a larger is better; fixing, it can be seen that a larger is better. Therefore, only when both and become larger and larger will the size of the GCC become larger and larger. The white lines in the figure represent contour lines, which means that for any combination of and on the line segment, the size of the GCC is constant.

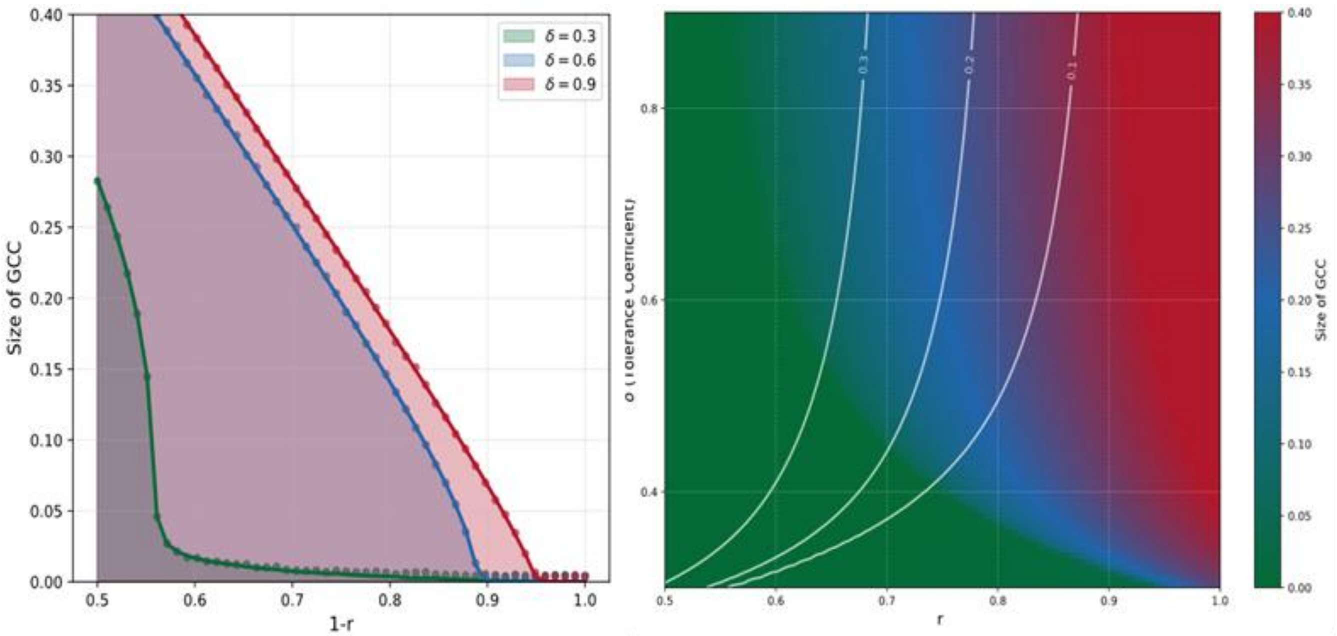

FIGURE 4: Respectively show the changes in the size of GCC under different tolerance coefficient conditions as the dependency proportion increases, and the changes in the size of GCC under the combined effect of the node retention proportion and the tolerance coefficient.

From Figure 5, the presented results demonstrate the variations in the size of GCC under different dependency ratios and various parameter combinations. Firstly, a larger dependency ratio renders the network more prone to collapse. Secondly, with other parameters kept constant, increasing the tolerance coefficient emerges as the optimal approach to enhance the network's invulnerability. When other parameters are kept constant and only the average hyperdegree or the average hyperedge cardinality is changed, the phase transition of the network will change. The larger and are, the slower the GCC decreases with and the network can withstand a higher proportion of node removal. In addition, in the case of a relatively small dependency ratio, increasing the average hyperedge cardinality can enhance the invulnerability of the network more effectively than increasing the average hyperdegree.

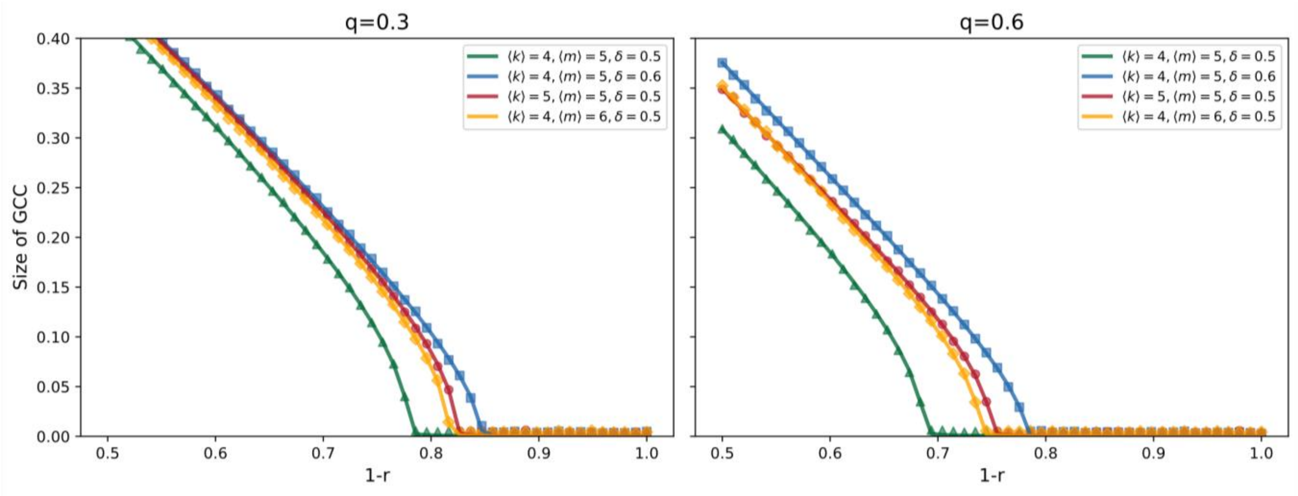

FIGURE 5: Results of parameter combinations under different dependency ratio conditions.

IV. CONCLUSION

This study examines cascading failures and phase transitions in a bilayer hypergraph system with hyperedge interdependencies. Key findings include: cascading failures arise from coupled node and hyperedge failures, driven by hyperedges dependence on both constituent node functionality and interdependent hyperedges across layers, iterating until a steady state. The giant connected component (GCC) exhibits phase transitions with increasing initial node removal. Lower dependency ratios or larger delay critical transitions, and makes the phase transition exhibit more continuous characteristics. Higher or smaller heighten sensitivity to initial failures, lowering collapse thresholds. Node retention proportion and synergistically affect GCC size: larger values of both preserve GCC integrity, with contour lines indicating (,) combinations yielding equivalent GCC sizes. Larger average hyperdegree and cardinality enhance invulnerability, mitigating collapse under higher. With other parameters remaining unchanged, increasing the tolerance coefficient is the optimal way to improve the network's invulnerability, In the case of a relatively small dependency ratio, increasing the average hyperedge cardinality yields better results than increasing the average hyperdegree. These results highlight hyperedge interdependencies and topology in bilayer hypergraph resilience, aiding network optimization against cascading failures.

ACKNOWLEDGEMENT

The authors have no acknowledgements to declare.

V. CONFLICT OF INTEREST

The authors declare that there is no conflict of interest regarding the publication of this paper.

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