Kinematics Analysis of a Novel 5-DOF Parallel Manipulator with Two Planar Limbs

Canguo Zhang[1], Yi Lu[2], Liang jianming[3], Peng Wang[4]

[1,2,4]College of Mechanical Engineering, Yanshan University, Qinhuangdao Hebei P.R.China
[1,3]College of Mechanical Engineering, Hebei University of architecture, Zhangjiakou Hebei P.R.China

Abstract— *It is significant to develop a limited-DOF parallel manipulator (PM) with high rigidity. However, the existing limited-DOF PMs include so many spherical joint which has less capability of pulling force bearing, less rotation range and lower precision under alternately heavy loads. A novel 5-DOF PM with two planar limbs is proposed and its kinematics is analyzed systematically. A 3-dimension simulation mechanism of the proposed manipulator is constructed and its structure characteristics are analyzed. The kinematics formulae for solving the displacement, velocity, acceleration of the platform, the active legs are established. An analytic example is given for solving the kinematics of the proposed manipulator and the analytic solved results are verified by the simulation mechanism. It provides the theoretical and technical foundations for its manufacturing, control and application.*

***Keywords—* kinematics, limited-DOF, parallel manipulator, planar limbs, singularity**

I. INTRODUCTION

Currently, various limited-DOF PMs are attracting much attention due to their fewer active legs, large workspace, simpler structure, easy control and simple kinematic solutions [1-2].Various limited-DOF parallel manipulators (PMs) have been applied in fields of rescue missions, industry pipe inspection, manufacturing and fixture of parallel machine tool, CT-guided surgery, health recover and training of human neck or waist and micro–Nano operation of bio-medicine [3–4]. In the aspects, Xie et al. [3] synthesized a class of limited-DOF PMs with several spherical joints(S). He and Gao [4] synthesized a class of 4-DOF PMs with 4 limbs, several S. S has the following disadvantages due to its structure: (1) the drag load capability is lower; (2) the rotation range is limited; (3) precision is lowed under alternately heavy loads. For this reason, The PMs with planar limbs have attracted many attentions because the planar limb only include revolute joints R and prismatic joint P. Wu and Gosselin [5] designed a PM with 3 planar limbs which are formed by a four-bar linkage. Lu et al. [6] proposed a novel 6-DOF PM with three planar limbs.

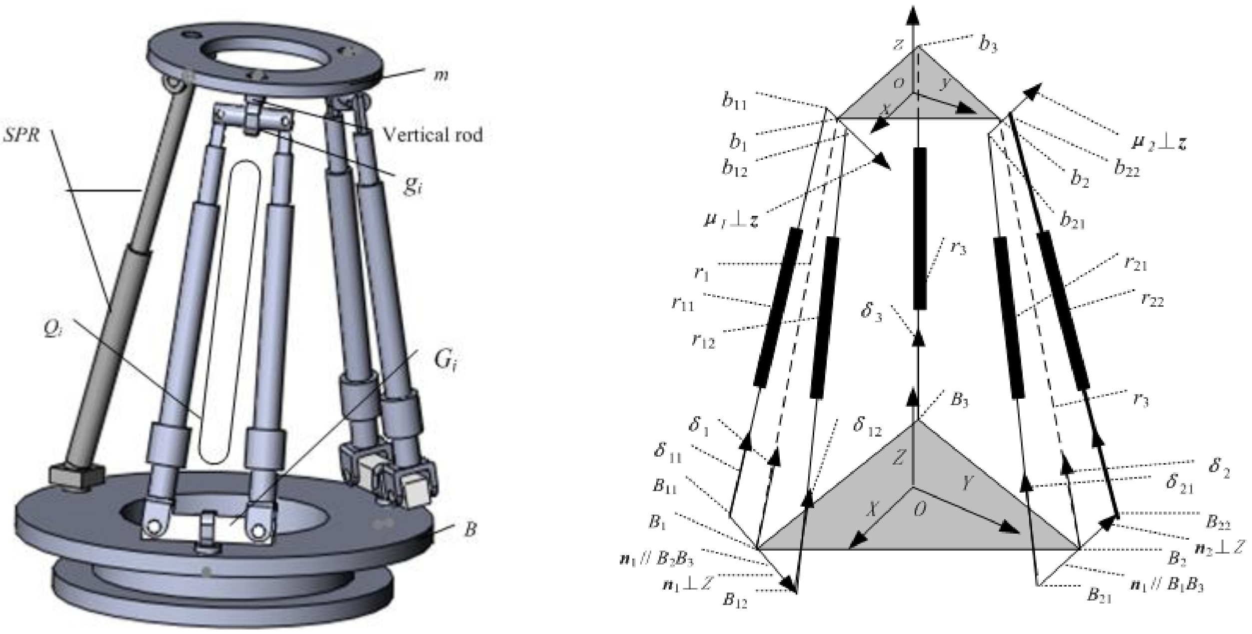
In the aspects of kinematics of PMs, Huang et al. [1] proposed the influence coefficient matrices. By screw theory, Gallardo-Alvarado [2] analyzed the kinematics of a hybrid PM. Kim and Merlet [7] studied the Jacobian matrix of various PMs by different approaches. Canfield et al. [8] analyzed the velocity of PMs by truss transformations. Zhou et al. [9] studied the kinematics of some limited-DOF PMs. Lu and Hu [10] derived unified and simple velocity and acceleration of some limited-DOF PMs with linear active legs.

Up to now, no effort towards the kinematics analysis of the limited-DOF PMs with planar limbs is found. For this reason, the paper focuses on the kinematics analysis of a novel 5-DOF PM with 2 planar limbs. Its structure characteristics, kinematics and singularity are studied systematically.

II. CHARACTERISTICS OF THE PM WITH 2Q_i AND ITS DOF

A 5-DOF PM with 2 planar limbs includes a moving platform m, a fixed base B, 2 vertical rods, 2 identical planar limbs Q_i (i = 1, 2) and a SPR (spherical joint S-active prismatic joint P-revolute joint R) type active leg, see Fig1(a). Here, m is a regular triangle with 3 vertices b_i (i = 1, 2, 3), 3 sides $l_i = l$, and a central point o; B is a regular triangle, 3 sides $L_i = L$, and a central point O, see Fig 1(b).Each of planar limbs Q_i includes 1 upper beam g_i, 1 lower beam G_i and 2 linear active legs r_{ij}. Each of r_{ij} is composed 1 cylinder q_{ij},1 piston rod p_{ij} and 1 linear actuator. In each of planar limbs, the middle of lower beam G_i connects with B by a horizontal revolute joint R^{i1}at B_i; one end of vertical rod connects with m by a revolute joint R^{i4} at b_i, the other end of the vertical rod connects with the upper beam g_i by a revolute joint R^{i5}; the two ends of r_{ij} connect with the two ends of g_i and G_i by revolute joints R^{i2}. g_i, G_i, and 2 r_{ij} form a closed planar mechanism Q_i. The PM is named as the 5-DOF PM with 2Q_i for distinguishing other kinds of PM with different planar limbs.

Let |, ||, $\perp$ be collinear, parallel and perpendicular constraints respectively. Let {B} be a coordinate frame O-XYZ which is fixed on B, {m} be a coordinate frame o-xyz fixed on m, see Fig 1(b). The 5-DOFPM includes the following geometric conditions: $z \perp m$, $y \mid ob_2$, $x \| b_1b_3$, $Z \perp B$,$Y \| OB_2$,$\boldsymbol{R}^{i1} \| B$,$\boldsymbol{R}^{i2} \perp \boldsymbol{\delta}_i$, $\boldsymbol{R}^{i2} \perp \boldsymbol{\delta}_{ij}$,$\boldsymbol{R}^{i4} \perp \boldsymbol{R}^{i5}$, $\boldsymbol{R}^{i4} \| z$, $g_i \| m$, $G_i \| B$,(g_i, G_i, r_i, r_{ij}) being in Q_i, $b_{i1}b_{i2} = g_i$, $B_{i1}B_{i2} = G_i$, $ob_i = e$, $OB_i = E$.



(A) 3D model of the PM with 2Q_i **(B) kinematics model of the PM with *2Qi***

FIGURE 1. A 3D MODEL OF THE PM WITH 2Q_i AND ITS KINETOSTATICS MODEL

By inspecting the PM with 2planar limbs, it is known thatthe number of links is$n$ = 18 corresponding to 1 m, 5 cylinders, 5 piston rods, 2 lower beamsG_i , 2 upper beams g_i ,2 vertical rods and 1 B. The number of kinematic pairsis g = 21 corresponding to 5 prismatic joints,15 revolute joints and 1 spherical join. The number of redundant is μ=3×2= 6 corresponding to 2Q_i. The number of located DOF of joints is Σf_i =5 ×1 + 15×1+3 = 23 corresponding to 5 prismatic joints , 15 revolute joints and 1 spherical joint. Based on a revision Grübler–Kutzbach equation [5],DOF of the PM with 2Q_i is calculated formula as:

$$M = 6\times(n-g-1)+\sum_{i=1}^{g} f_i + \mu = 6\times(18-21-1)+23+3\times 2 = 5 \tag{1}$$

Comparing with the existing limited-DOF PMs, the proposed 5-DoF PM with 2Q_i possess the merits as follows:

1) Each of planar limbs Q_i only includes revolute joints R and prismatic joint P, therefore, it is simple in structure and is easy manufacturing.

2) Since all R in each of 2Q_i are parallel mutually, each of r_{ij} in Q_i is only subjected a linear force along its axis. Thus, the hydraulic translational actuator can be used for increasing a capability of large load bearing. In addition, a bending moment and a rotational torque between the piston rod and the cylinder can be avoided.

3) In each of planar limbs Q_i, The backlash of revolute joints$R$ can be eliminated easily, sorevolute joints$R$ has higher precision than S under cyclic loading. The workspace of the 5-DoF PM can be increased due to R having larger rotation range than S.

III. Inverse displacement analysis of the 5-DOF PM with 2Q_I

The derivation of displacement formulae of the proposed PM is a prerequisite for solving velocity, acceleration and statics of the PM. The coordinates of b_i of m in {m} and B_i of B in {B} are expressed as follows:

$$\boldsymbol{b}_i^m = \frac{e}{2}\begin{bmatrix} \pm q \\ -1 \\ 0 \end{bmatrix}, \boldsymbol{b}_2^m = \begin{bmatrix} 0 \\ e \\ 0 \end{bmatrix}, \quad q=\sqrt{3}, e=\frac{\sqrt{3}}{3}l, E=\frac{\sqrt{3}}{3}L \tag{2}$$

Here e is the distance from b_i to o, E is the distance from B_i to O. i = 1 or 3. As i = 1, ± is +; as i = 3, ± is −. This condition is also available for (i.e., (4)), (i.e., (5)) and (i.e., (7)).

Let X_o, Y_o, Z_o be the position components of m at o in $\{B\}$. Let φ be one of 3 Euler angles (α, β, γ). Set $s_\varphi = \sin\varphi$, $c_\varphi=\cos\varphi$, b_i of m in $\{B\}$ can be derived as follows:

$$\boldsymbol{b}_i = \boldsymbol{R}_m^B \boldsymbol{b}_i^m + \boldsymbol{o} \tag{3}$$

$$\boldsymbol{o} = \begin{bmatrix} X_o \\ Y_o \\ Z_o \end{bmatrix},\ \boldsymbol{R}_m^B = \begin{bmatrix} x_l & y_l & z_l \\ x_m & y_m & z_m \\ x_n & y_n & z_n \end{bmatrix} = \begin{bmatrix} c_\alpha c_\beta c_\gamma - s_\alpha s_\gamma & -c_\alpha c_\beta s_\gamma - s_\alpha c_\gamma & c_\alpha s_\beta \\ s_\alpha c_\beta c_\gamma + c_\alpha s_\gamma & s_\alpha c_\beta s_\gamma + c_\alpha c_\gamma & s_\alpha s_\beta \\ -s_\alpha c_\gamma & s_\alpha s_\gamma & c_\beta \end{bmatrix}$$

Here $\boldsymbol{R}_m^B$ is a rotation matrix from $\{m\}$ to $\{B\}$ in order ZYZ (about Z_1 by α, Y by β, Z_2 by γ); x_l, x_m, x_n, y_l, y_m, y_n, z_l, z_m, z_n are nine orientation parameters of $\{m\}$.

Based on (i.e., (2)) and (i.e., (3)),the coordinates of b_i in $\{B\}$ are derived as follows:

$$\boldsymbol{b}_i = \frac{1}{2}\begin{bmatrix} \pm qex_l - ey_l + 2X_o \\ \pm qex_m - ey_m + 2Y_o \\ \pm qex_n - ey_n + 2Z_o \end{bmatrix},\ \boldsymbol{b}_2 = \begin{bmatrix} ey_l + X_o \\ ey_m + Y_o \\ ey_n + Z_o \end{bmatrix} \tag{4}$$

Let $\boldsymbol{r}_i$ (i = 1, 2, 3) be the vector from B_i to b_i, $\boldsymbol{e}_i$ (i = 1, 2, 3) be the vector from o to b_i. They are derived from (i.e., (2)) and (i.e., (4)) as follows:

$$\boldsymbol{r}_i = \frac{1}{2}\begin{bmatrix} \pm(qex_l - \mathrm{q}E) - ey_l + 2X_o \\ \pm qex_m - ey_m + 2Y_o + E \\ \pm qex_n - ey_n + 2Z_o \end{bmatrix}, \boldsymbol{r}_2 = \begin{bmatrix} ey_l + X_o \\ ey_m + Y_o - E \\ ey_n + Z_o \end{bmatrix},\ \boldsymbol{e}_i = \frac{e}{2}\begin{bmatrix} \pm qx_l - y_l \\ \pm qx_m - y_m \\ \pm qx_n - y_n \end{bmatrix}, \boldsymbol{e}_2 = e\begin{bmatrix} y_l \\ y_m \\ y_n \end{bmatrix} \tag{5}$$

Let $\boldsymbol{n}_{0i}$ and $\boldsymbol{n}_i$ be the vector and unit vector of lower beam G_i. Based on the geometric condition, there are $\boldsymbol{n}_{01} \| B_2B_3$, $\boldsymbol{n}_{02} \| B_1B_3$, $\boldsymbol{n}_{0i}$, $\boldsymbol{n}_i$ can be represented by (i.e., (2)) as follows:

$$\boldsymbol{n}_{01} = \boldsymbol{B}_2 - \boldsymbol{B}_3 = \frac{E}{2}\begin{bmatrix} q \\ 3 \\ 0 \end{bmatrix}, \boldsymbol{n}_{02} = \boldsymbol{B}_1 - \boldsymbol{B}_3 = \begin{bmatrix} qE \\ 0 \\ 0 \end{bmatrix},\ \boldsymbol{n}_i = \frac{\boldsymbol{n}_{0i}}{|\boldsymbol{n}_{0i}|} \quad (i{=}1,2) \tag{6}$$

Let $\boldsymbol{u}_{0i}$ and $\boldsymbol{u}_i$ be the vector and unit vector of upper beam g_i. It is known that both $\boldsymbol{u}_{0i}$ and $\boldsymbol{r}_i$ locate in the same plane Q_i and let $\boldsymbol{F}$ be the vector which is perpendicular to Q_i. Based on the geometric condition, $\boldsymbol{u}_{0i}$, $\boldsymbol{u}_i$ can be derived as follows:

$$\boldsymbol{F} = \boldsymbol{n}_{0i} \times \boldsymbol{r}_i,\ \boldsymbol{\mu}_{0i} = \pm \boldsymbol{n}_z \times \boldsymbol{F}, \quad \boldsymbol{n}_z = \begin{bmatrix} z_l & z_m & z_n \end{bmatrix}^T,\ \boldsymbol{\mu}_i = \frac{\boldsymbol{\mu}_{0i}}{|\boldsymbol{\mu}_{0i}|} \quad (i{=}1,2) \tag{7}$$

Let $b_ib_{i1} = b_ib_{i2} = d$, $B_{i1}B_i = B_iB_{i2} = D$, $\boldsymbol{r}_{ij}$ be the vector from B_{ij} to b_{ij}. $\boldsymbol{r}_{ij}$ are derived as follows:

$$\boldsymbol{B}_{i1}\boldsymbol{B}_i = \boldsymbol{B}_i\boldsymbol{B}_{i2} = D\boldsymbol{n}_i\ \boldsymbol{e}_{i1} = \boldsymbol{b}_i\boldsymbol{b}_{i1} = d\boldsymbol{\mu}_i\ \boldsymbol{e}_{i2} = \boldsymbol{b}_i\boldsymbol{b}_{i2} = -d\boldsymbol{\mu}_i$$

$$\begin{cases} \boldsymbol{r}_{i1} = \boldsymbol{B}_{i1}\boldsymbol{B}_i + \boldsymbol{B}_i\boldsymbol{b}_i + \boldsymbol{b}_i\boldsymbol{b}_{i1} \\ \boldsymbol{r}_{i2} = \boldsymbol{B}_i\boldsymbol{b}_i - \boldsymbol{B}_i\boldsymbol{B}_{i2} + \boldsymbol{b}_i\boldsymbol{b}_{i2} \end{cases} \Rightarrow \begin{cases} \boldsymbol{r}_{i1} = \boldsymbol{r}_i + d\boldsymbol{\mu}_i - D\boldsymbol{n}_i \\ \boldsymbol{r}_{i2} = \boldsymbol{r}_i - d\boldsymbol{\mu}_i + D\boldsymbol{n}_i \end{cases} \quad (i{=}1,2) \tag{8}$$

Let $\boldsymbol{\delta}_i$ be the unit vector of r_i, let $\boldsymbol{\delta}_{ij}$ be the unit vector of r_{ij}. Based on (i.e., (5))–(i.e., (8)), the formulae for solving $\boldsymbol{\delta}_i$, $\boldsymbol{\delta}_{ij}$ r_i, and r_{ij}, are represented as follows:

$$\boldsymbol{\delta}_i = \frac{\boldsymbol{r}_i}{r_i}, \boldsymbol{\delta}_{ij} = \frac{\boldsymbol{r}_{ij}}{r_{ij}}\ r_i^2 = r_{ix}^2 + r_{iy}^2 + r_{iz}^2,\ r_{ij}^2 = r_{ijx}^2 + r_{ijy}^2 + r_{ijz}^2 \tag{9}$$

Thus, $\boldsymbol{r}_3$ is the vector of *SPR* active leg. $\boldsymbol{r}_{ij}$(i=1,2,j=1,2) are the vectors of active leg in planer limbers.

IV. KINEMATICS ANALYSIS OF THE 5-DOF PM WITH 2Q_I.

4.1 Basic concepts and relative equations

The derivation of velocity formulae of a proposed PM is a key issue to establish the acceleration model and statics model of the proposed PM. Suppose there are a vector $\boldsymbol{\zeta}$ and a skew-symmetric matrix $\hat{\xi}$ or $s(\zeta)$, Theymust satisfy[10,11]:

$$\zeta\times = \hat{\zeta} = s(\zeta),\ \hat{\zeta}^T = -\hat{\zeta},\ -\hat{\zeta}^2 = E - \zeta\zeta \tag{10}$$

A kinematics model of the 5-DOF PM are shown in Fig 1（b）. Let $\boldsymbol{V}$ and $\boldsymbol{A}$ be the general output velocity and the general output acceleration of m,$\boldsymbol{v}$, and $\boldsymbol{a}$ be the linear velocity and linear acceleration of m at o,$\boldsymbol{\omega}$ and $\boldsymbol{\varepsilon}$ be the angular velocity and angular accelerations of m, respectively. They can be expressed as follows:

$$\boldsymbol{V} = \begin{bmatrix} \boldsymbol{v} \\ \boldsymbol{\omega} \end{bmatrix},\ \boldsymbol{A} = \begin{bmatrix} \boldsymbol{a} \\ \boldsymbol{\varepsilon} \end{bmatrix}, \boldsymbol{v} = \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix}, \boldsymbol{\omega} = \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}, \boldsymbol{a} = \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix}, \boldsymbol{\varepsilon} = \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \end{bmatrix} \tag{11}$$

Let $\boldsymbol{v}_{bi}$ be a velocity vector of m at b_i, $\boldsymbol{\omega}_{bi}$ be the angular velocity of upper beam g_i,$\boldsymbol{v}_{bij}$ be a velocity vector of the upper beam g_i at b_{ij}. Let v_{ri} be a scalar velocity along r_i,$\boldsymbol{\omega}_{ri}$ be the angular velocity of r_i. Let v_{rij} be the input scalar velocity along r_{ij}, $\boldsymbol{\omega}_{rij}$ be the angular velocity of r_{ij}. Let ω_{i1} and $\boldsymbol{R}_{i1}$ be a scalar angular velocity of the lower beam G_i about B at B_i and its unit vector. Let ω_{i2} and $\boldsymbol{R}_{i2}$ be a scalar angular velocity of r_i about G_i and its unit vector. Let ω_{i3} and $\boldsymbol{R}_{i3}$ be the scalar angular velocity of r_i about g_i and its unit vector. Let ω_{i4} and $\boldsymbol{R}_{i4}$ be the scalar angular velocity of vertical rod about m at b_i and its unit vector. Let ω_{i5} and $\boldsymbol{R}_{i5}$ be the scalar angular velocity of g_i about vertical rod at b_i and its unit vector. Based on the geometric condition, there are $\boldsymbol{R}_{i3} \| \boldsymbol{R}_{i2}$,$\boldsymbol{R}_{i3} \perp \boldsymbol{R}_{i4}$, $\boldsymbol{R}_{i3} \perp \boldsymbol{R}_{i5}$ .They can be expressed as follows:

$$\boldsymbol{R}_{i1} = \boldsymbol{n}_i, \boldsymbol{R}_{i2} = \frac{\boldsymbol{R}_{i1}\times\boldsymbol{\delta}_i}{|\boldsymbol{R}_{i1}\times\boldsymbol{\delta}_i|}, \boldsymbol{R}_{i3} = \boldsymbol{R}_{i2},\ \boldsymbol{R}_{i4} = \boldsymbol{n}_z, \boldsymbol{R}_{i5} = \boldsymbol{\mu}_i, \boldsymbol{v}_{bi} = \boldsymbol{v} + \boldsymbol{\omega}\times\boldsymbol{e}_i \tag{12}$$

$$\boldsymbol{v}_{bij} = \boldsymbol{v}_{bi} + \boldsymbol{\omega}_{bi}\times\boldsymbol{e}_{ij} = \boldsymbol{v}_{rij} + \boldsymbol{\omega}_{rij}\times\boldsymbol{r}_{ij},\ \ \boldsymbol{\omega}_{bi} = \boldsymbol{\omega} + \omega_{i4}\boldsymbol{R}_{i4} + \varphi_{i5}\boldsymbol{R}_{i5} = \boldsymbol{\omega}_{ri} + \omega_{i3}\boldsymbol{R}_{i3}$$

$$\boldsymbol{\omega}_{ri} = \omega_{i1}\boldsymbol{R}_{i1} + \omega_{i2}\boldsymbol{R}_{i2},\ \ v_{ri} = \boldsymbol{v}_{bi}\cdot\boldsymbol{\delta}_i, v_{rij} = \boldsymbol{v}_{bij}\cdot\boldsymbol{\delta}_{ij}(i = 1,2; j = 1,2)$$

4.2 General input velocity V_{rij} and angular velocity ω_{rij}.

In the active legs of the planer limbs. Let v_{rij} (i=1, 2, j=1, 2) and $\boldsymbol{V}_{rij}$ be the input velocity along r_{jj} and the general velocity input of the planer limbs. Let $\boldsymbol{\omega}_{rij}$ be the angular velocity of r_{ij}. The formulae for solving $\boldsymbol{\omega}_{rij}$ and v_{rjj} can be derived as follows:

$$\boldsymbol{\omega}_{rij} = \boldsymbol{J}_{\omega ij}\boldsymbol{V} \qquad (i=1,2; j=1,2) \tag{13}$$

$$v_{rij} = \boldsymbol{v}_{bij}\cdot\boldsymbol{\delta}_{ij} = \left(\boldsymbol{v}_{bi} + \boldsymbol{\omega}_{bi}\times\boldsymbol{e}_{ij}\right)\cdot\boldsymbol{\delta}_{ij} = (\boldsymbol{v} + \boldsymbol{\omega}\times\boldsymbol{e}_i)\cdot\boldsymbol{\delta}_{ij} + (\boldsymbol{J}_{\omega bi}\boldsymbol{V}\times\boldsymbol{e}_{ij})\cdot\boldsymbol{\delta}_{ij} = \boldsymbol{J}_{vij}\boldsymbol{V} \tag{14}$$

$$\boldsymbol{V}_{rij} = \boldsymbol{J}_{rij}\boldsymbol{V},\ \boldsymbol{V}_{rij} = \begin{bmatrix} v_{r11} & v_{r12} & v_{r21} & v_{r22} \end{bmatrix}^T \boldsymbol{J}_{rij} = \begin{bmatrix} \boldsymbol{J}_{v11} & \boldsymbol{J}_{v12} & \boldsymbol{J}_{v21} & \boldsymbol{J}_{v22} \end{bmatrix}^T \tag{15}$$

Here, $\boldsymbol{J}_{\omega ij}$ is a 3×6 matrix; $\boldsymbol{J}_{vij}$ is a 1×6 matrix; $\boldsymbol{J}_{rij}$ is a 4×6 matrix.

In the *SPR* type active leg, let v_{r3} be the input velocity along r_3, Let $\boldsymbol{\omega}_{r3}$ be the angular velocity of r_3.The formulae for solving v_{r3} and $\boldsymbol{\omega}_{r3}$ have been derived in [10] as follows:

$$v_{r3} = (\boldsymbol{v} + \boldsymbol{\omega} \times \boldsymbol{e}_3) \cdot \boldsymbol{\delta}_3 = \boldsymbol{J}_{r3}\boldsymbol{V} \quad \boldsymbol{J}_{r3} = [\boldsymbol{\delta}_3^T \quad \boldsymbol{e}_3 \times \boldsymbol{\delta}_3]_{1\times6} \tag{16}$$

$$\boldsymbol{\omega}_{r3} = \frac{1}{r_3}\left(\hat{\boldsymbol{\delta}}_3\boldsymbol{v} - \hat{\boldsymbol{\delta}}_3\hat{\boldsymbol{e}}_3\boldsymbol{\omega} + r_3\boldsymbol{\delta}_3\boldsymbol{\delta}_3^T\boldsymbol{\omega}\right) = \boldsymbol{J}_{\omega3}\boldsymbol{V} \quad \boldsymbol{J}_{\omega3} = \left[\hat{\boldsymbol{\delta}}_3 \quad -\hat{\boldsymbol{\delta}}_3\hat{\boldsymbol{e}}_3 + r_3\boldsymbol{\delta}_3\boldsymbol{\delta}_3^T\right]_{3\times6} \tag{17}$$

Here, $\boldsymbol{J}_{r3}$is a 1×6 matrix; $\boldsymbol{J}_{\omega3}$is a 3×6 matrix.

In the 5–DOF PM there are constrained wrench ($\boldsymbol{F}_y$, $\boldsymbol{T}_c$) in the *SPR* type active leg limited the movement of the PM. The constrained wrench do not do any power during the movement of PM. Let $\boldsymbol{f}_3$be the unit vector of $\boldsymbol{F}_y$, $\boldsymbol{d}_3$ is the vector of the arm from o to $\boldsymbol{F}_y$, thus the constrained wrench have been derived in [10].An auxiliary velocity equation is derived as:

$$0=\boldsymbol{J}_{vy}\boldsymbol{V} \quad \boldsymbol{J}_{vy} = \left[\boldsymbol{f}_3^T \quad (\boldsymbol{d}_3 \times \boldsymbol{f}_3)^T\right]_{1\times6} \tag{18}$$

Here, $\boldsymbol{J}_{vy}$ is a 1×6 matrix. By combining (i.e., (15)), (i.e., (16))with (i.e., (18)), a general inverse velocity $\boldsymbol{v}_r$ can be derived as:

$$\boldsymbol{v}_r = \boldsymbol{J}\boldsymbol{V} \ ,\boldsymbol{v}_r = \left[v_{r11} \quad v_{r12} \quad v_{r21} \quad v_{r22} \quad v_{r3} \quad 0\right]^T, \quad \boldsymbol{J} = \left[\boldsymbol{J}_{v11} \quad \boldsymbol{J}_{v12} \quad \boldsymbol{J}_{v21} \quad \boldsymbol{J}_{v22} \quad \boldsymbol{J}_{r3} \quad \boldsymbol{J}_{vy}\right]^T \tag{19}$$

Here, $\boldsymbol{J}$ is a 6×6 Jacobian matrix of the 5–DOF PM with 2 planer limbers.

4.3 Acceleration of the PM

The establishment of acceleration model of the proposed PM is a prerequisite to establish dynamics model of the proposed PM. Let $\boldsymbol{a}_{rij}$ be the input scalar acceleration along r_{ij}. By differentiating (i.e., (15)) with respect to time, the acceleration matrix of the active legs in planer limbers equation is derived as:

$$\boldsymbol{a}_{rij} = \boldsymbol{J}_{rij}\boldsymbol{A} + \dot{\boldsymbol{J}}_{rij}\boldsymbol{V} = \boldsymbol{J}_{rij}\boldsymbol{A} + \boldsymbol{V}^{\mathrm{T}}\boldsymbol{H}_{vij}\boldsymbol{V}, \boldsymbol{a}_{rij} = \begin{bmatrix} a_{r11} \\ a_{r12} \\ a_{r21} \\ a_{r22} \end{bmatrix}, \ \boldsymbol{H}_{vij} = \begin{bmatrix} \boldsymbol{H}_{11} \\ \boldsymbol{H}_{12} \\ \boldsymbol{H}_{21} \\ \boldsymbol{H}_{22} \end{bmatrix}_{4\times6\times6} \tag{20}$$

Here, $\boldsymbol{a}_{rij}$ is a 4×1 matrix, $\boldsymbol{H}_{ij}$ (i=1,2;j=1,2)are 6×6 sub-Hessian matrixes, $\boldsymbol{H}_{vij}$ is a 4×6×6sub-Hessian matrix.

Let a_{r3} be the input scalar acceleration along the *SPR* type active leg. By differentiating (i.e., (16)) with respect to time, a standard formula for solving the general input acceleration $a_{r3}$of the *SPR* type active leg is derived as follows:

$$a_{r3} = \left[\boldsymbol{\delta}_3^T \quad \left(\boldsymbol{e}_3 \times \boldsymbol{\delta}_3\right)^T\right]\boldsymbol{A} + \frac{1}{r_3}\boldsymbol{V}^T\boldsymbol{H}_{\alpha3}\boldsymbol{V} \quad \boldsymbol{H}_{\alpha3} = \frac{1}{r_i}\begin{bmatrix} -\boldsymbol{\delta}_3^2 & \hat{\boldsymbol{\delta}}_3^2\hat{\boldsymbol{e}}_3 \\ -\hat{\boldsymbol{e}}_3\hat{\boldsymbol{\delta}}_3^2 & r_3\hat{\boldsymbol{e}}_3\hat{\boldsymbol{\delta}}_3 + \hat{\boldsymbol{e}}_3\hat{\boldsymbol{\delta}}_3^2\hat{\boldsymbol{e}}_3 \end{bmatrix}_{6\times6} \tag{21}$$

Here, $a_{r3}$is a scalar acceleration along r_3, $\boldsymbol{H}_{\alpha3}$ is 6×6 sub-Hessian matrixes.

By differentiating (i.e., (18)) with respect to time, an auxiliary acceleration matrix equation is derived as:

$$0 = \left[\boldsymbol{R}_3^T \quad \left(\boldsymbol{e}_3 \times \boldsymbol{R}_3\right)^T\right]\boldsymbol{A} + \left[\dot{\boldsymbol{R}}_3^T \quad \dot{\boldsymbol{e}}_3 \times \boldsymbol{R}_3 + \boldsymbol{e}_3 \times \dot{\boldsymbol{R}}_3\right]\boldsymbol{V} = \boldsymbol{J}_{vy}\boldsymbol{A} + \boldsymbol{V}^T\boldsymbol{H}_v\boldsymbol{V} \tag{22}$$

Here $\boldsymbol{H}_v$ is 6×6 sub-Hessian matrixes.

By combining (i.e., (20)),(i.e., (21))with(i.e., (22)), a general inverse acceleration $\boldsymbol{a}_r$ is derived as:

$$\boldsymbol{a}_r = \boldsymbol{J}\,\boldsymbol{A} + \boldsymbol{V}^{\mathrm{T}}\boldsymbol{H}\boldsymbol{V}, \quad \boldsymbol{a}_r = \left[a_{11} \quad a_{12} \quad a_{21} \quad a_{22} \quad a_{r3} \quad 0\right]^{\mathrm{T}}_{6\times1}, \boldsymbol{H} = \begin{bmatrix} \boldsymbol{H}_{vij} \\ \boldsymbol{H}_{\alpha3} \\ \boldsymbol{H}_v \end{bmatrix}_{6\times6\times6} \tag{23}$$

Here, $\boldsymbol{H}$ is a 6 × 6 × 6 Hessian matrix of the 5-DOF PM with $2Q_i$.

V. ANALYTIC SOLVED EXAMPLE

For the 5-DOF PM with two planer limbers, set L=240mm, l=120mm, 2D=80mm, 2d=50mm. Based on relevant analytic equations above. A Matlab program is compiled for solving the inverse/forward velocity and acceleration of the 5-DOF PM with 2 planar limbs.

When the pose variables (X_o, Y_o,Z_o) (see Fig. 2a) and (α, β, γ) (see Fig. 2b) is given. The linear and angular velocities (v_x, v_y, v_z ,ω_x, ω_y, ω_z) (see Fig. 2c,d)of the moving platform m is given. The linear acceleration $\boldsymbol{a}$ and the angular acceleration $\boldsymbol{\varepsilon}$ of m are solved (see Fig. 2e,f). The extension r_{ij} (i = 1, 2, j=1, 2), (see Fig. 2g) the velocity v_{rij} and the acceleration a_{rij} of the active legs are solved (see Fig.2 h,i).

The angular velocity $\boldsymbol{\omega}_{rij}$ and the angular acceleration $\boldsymbol{\varepsilon}_{rij}$ of the active legs are solved (see Fig. 2j,k).The solved results are verified by its simulation mechanism in the Simulink/Mechanics.

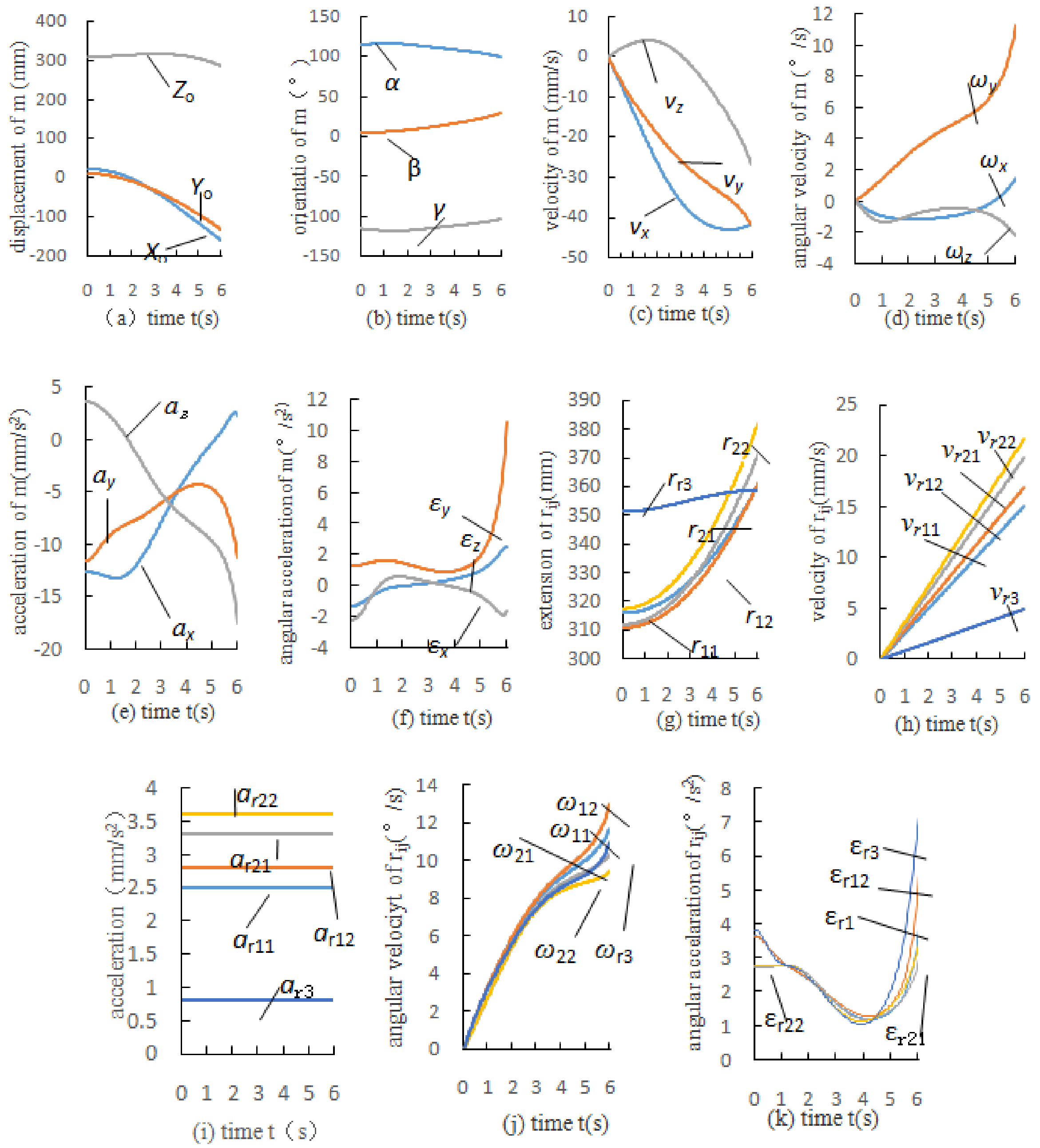


FIGURE 2. ANALYTIC KINEMATICS SOLUTIONS OF THE PM WITH 2 PLANAR LIMBS

VI. SINGULARITY ANALYSIS

Let |**J**| denote the determinant of the Jacobian matrix ***J*** of the 5-DOFPM. When |**J**| = 0, |**J**| → 0/0and |**J**| → ∞are satisfied, respectively, a boundary singularity, a structure singularity and a local singularity of the PM occurs[5].

In the active legs of the planer limbs, when$\boldsymbol{R}_{i3}$|| $\boldsymbol{R}_{i4}$are satisfied(see Fig. 3), it leads to:

$$\boldsymbol{R}_{i3}\cdot(\boldsymbol{R}_{i4}\times\boldsymbol{R}_{i5})=\boldsymbol{R}_{i5}\cdot(\boldsymbol{R}_{i3}\times\boldsymbol{R}_{i4})=0$$

$$D_0=\boldsymbol{r}_i\cdot\boldsymbol{D}_1,\boldsymbol{D}_1=\boldsymbol{R}_{i1}\times\boldsymbol{R}_{i2},\boldsymbol{D}_2=\boldsymbol{R}_{i1}\boldsymbol{R}_{i2}^{\mathrm{T}}-\boldsymbol{R}_{i2}\boldsymbol{R}_{i1}^{\mathrm{T}}\ \boldsymbol{D}_3=\frac{\boldsymbol{R}_{i3}(\boldsymbol{R}_{i4}\times\boldsymbol{R}_{i5})^{\mathrm{T}}}{\boldsymbol{R}_{i3}\cdot(\boldsymbol{R}_{i4}\times\boldsymbol{R}_{i5})}=\infty \tag{24}$$

$$\boldsymbol{J}_{vij}=[\boldsymbol{\delta}_{ij}^{\mathrm{T}}\quad(\hat{\boldsymbol{e}}_i\,\boldsymbol{\delta}_{ij})^{\mathrm{T}}]_{1\times6}+(\hat{\boldsymbol{e}}_{ij}\,\boldsymbol{\delta}_{ij})^{\mathrm{T}}(\boldsymbol{E}_{3\times3}-\boldsymbol{D}_3)\frac{\boldsymbol{D}_2}{D_0}[\hat{\boldsymbol{\delta}}_i^2\quad-\hat{\boldsymbol{\delta}}_i^2\,\hat{\boldsymbol{e}}_i]+(\hat{\boldsymbol{e}}_{ij}\,\boldsymbol{\delta}_{ij})^{\mathrm{T}}[\boldsymbol{0}_{3\times3}\quad\boldsymbol{D}_3]=\infty\ |\boldsymbol{J}|\to\infty$$

It is known from (i.e., (24)) that when the planer limbs Q_i (i=1,2) is in the same plane with the moving platform m($\boldsymbol{R}_{i3}$||$\boldsymbol{R}_{i4}$)(see Fig. 3).A local singularity of the 5-DoF PM with 2*Qi*occurs.

When $\boldsymbol{r}_i$|$\boldsymbol{G}_i$ are satisfied, it leads to:

$$\boldsymbol{r}_i\perp\boldsymbol{D}_1\ D_0=\boldsymbol{r}_i\cdot\boldsymbol{D}_1=0$$

$$\boldsymbol{J}_{vij}\to\infty\ |\boldsymbol{J}|\to\infty \tag{25}$$

But for the 5-DoF PM with 2*Qi*,Due to the restriction of the linear active rods length, it is impossible to reach the position.

In the *SPR* type active leg, when $\boldsymbol{\delta}_i$‖ $\boldsymbol{e}_i$ are satisfied, it leads to:

$$\boldsymbol{e}_3\times\boldsymbol{\delta}_3=0$$

$$J_{r3}=[\delta_3^T\quad e_3\times\delta_3]_{1\times6}\to0,\ |\boldsymbol{J}|\to0 \tag{26}$$

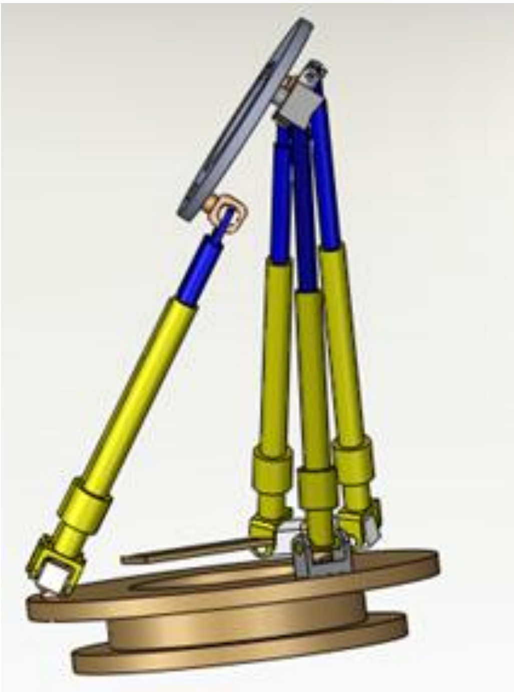


FIGURE 3. THE LOCAL SINGULARITIES OF THE PROPOSED 5-DOF PM

VII. CONCLUSION

1. A novel 5-DOF parallel manipulator (PM) with 2 planar limbs is proposed and its structure characteristics and merits are analyzed. The formulae for solving its kinetostatics are derived.
2. When given the input displacement, velocity, acceleration of the proposed PM, its output displacement, velocity, acceleration can be solved by using derived formulae. The analytic solutions of coordinated kinematics for the proposed parallel manipulator are verified by its simulation solutions.
3. The proposed PM has higher rigidity, and more room for arranging multi-finger mechanisms without interference among active legs. Each of active legs is only the subjected to a linear force along active leg, the active leg and has a large capability of load bearing.
4. The proposed PM has potential applications for of forging operator, manufacturing and fixture of parallel machine tool, assembly cells, CT-guided surgery, health recover and training of human neck or waist, and micro–Nano operation of bio-medicine, and rescue missions, industry pipe inspection. Theoretical formulae and results provide foundation for its structure optimization, control, manufacturing and applications.

ACKNOWLEDGEMENTS

The authors would like to acknowledge （1） National Natural Science Foundation of China(Grant No.51175447),（2）Project supported by Natural Science Foundation of Hebei(Grant No. E2016203379).

REFERENCES

[1] Huang Z, Kong L F, and Fang Y F, "Theory on Parallel Robotics and Control," Machinery Industry, Beijing, China, 1998.

[2] Gallardo-Alvarado J, "Kinematics of a Hybrid Manipulator by Means of Screw Theory,"Multibody Syst. Dyn., 14 (3–4), 2005, pp. 345–366.

[3] XIE F G, Li T M and Liu X J, "Type Synthesis of 4-DOF Parallel Kinematic Mechanisms Based on Grassmann Line Geometry and Atlas Method,"Chinese Journal of Mechanical Engineering. 26 (6), 2013, pp. 1073–1081.

[4] HE J and GAO F, "Type Synthesis for 4-DOF Parallel Press Mechanism Using GF Set Theory, "Chinese Journal of Mechanical Engineering.2015, 26 (6), pp. 851–859.

[5] Wu Y, Gosselin CM, "Synthesis of reactionless spatial 3-DoF and 6-DoF mechanisms without separate counter rotations,"Int. J. Rob. Res.23, 2004, pp.625–642.

[6] Lu Y, and Li XP, "Kinetostatics Analysis of a Novel 6-DOF Parallel Manipulator with Three Planar Limbs and Equipped with Three Fingers," Chinese Journal of Mechanical Engineering. 27 (5), 2014, pp. 919–926.

[7] Merlet, J. P, Jacobian, Manipulability, Condition Number, and Accuracy of Parallel Robots, ASME J. Mech. Des., 2006,128 (1), pp. 199–206.

[8] Canfield, S. L, Soper, R. R, and Reinholtz, C. F, "Velocity Analysis of Parallel Manipulators by Truss Transformations," Mech. Mach. Theory, 1999, 34 (3), pp. 345–357.

[9] Zhou K, Zhao J -S. Tan Z. -Y and Mao, D-Z, "The Kinematics Study of a Class of Spatial Parallel Mechanism With Fewer Degrees of Freedom,"Int. J. Adv. Manuf. Technol.,2005,25 (9–10), pp. 972–978.

[10] Lu Y and Hu B, "Unification and Simplification of Velocity/Acceleration of Limited-DOF Parallel Manipulators with Linear Active Legs," Mech. Mach. Theory, 2008,43 (9), pp. 1112–1128.

[11] MERLET J P. Parallel robots[M]. 2nd ed. Netherlands: Springer, 2006. pp. 97–105