Application of Physical Similarity in the Transfer of Results from a Model to a Prototype

Ing. Romana Dobáková PhD.[1], doc. Ing. Natália Jasminská PhD.[2], doc. Ing. Tomáš Brestovič PhD.[3]

Faculty of Mechanical Engineering, Department of Power Engineering, Vysokoškolská 4, 042 00 Košice, Slovakia

Abstract - *The process of heat distribution through heat networks is accompanied with significant loss. That is why distributors of heat and hot water are nowadays very interested in these issues. The drawbacks of the existing methods of heal loss calculation brought the necessity of searching new procedures of such calculation.*

The article presents a possibility of using the similarity theory and modelling in order to transpose results from an experimental heat network to any other network that will be similar to the experimental network with regard geometry, kinematics, as well as heat parameters.

Keywords - heat loss, heat network, physical similarity, simplex.

I. INTRODUCTION

The heat loss is identified using the so-called balance method in which it is necessary to know the temperature of a heat-conveying material in the beginning and at the end of the examined section, with sufficient accuracy, and the water flow rate in the supply as well as return pipelines. The measurements of the temperature differential for the heat-conveying material in short sections require the use of the state-of-the-art measuring technology which, however, cannot be permanently installed in every network, especially when the entire distribution system covers several tens of kilometres. Such methodology for the heat loss identification is appropriate mainly for very long heat distribution sections in which an error in the measurement of the temperature differential would not cause a substantial error in the subsequent calculation of the heat loss.

II. SIMILARITY THEORY AND MODELLING

The drawbacks of the balance method regarding the expression of heat loss brought the necessity of examining new methods how to express such loss.

The present calculation of heat loss was made applying the physical similarity and modelling.

Two physical phenomena are only similar if all the parameters determining them are similar. In the case of similar phenomena that depend on several physical parameters, there are probably certain relationships between similarity constants.

Similarity criteria are divided into simplexes and complexes. Simplex is the ratio of two physical parameters with identical names. Complex is the ratio of parameters with different names (dimensionless). In other words, complex is an invariant consisting of parameters bearing different names.

A physical phenomenon expressed by an equation that cannot be directly solved is described using criterial equations. The method of their application is based on the substitution of relevant dimensional parameters for similarity criteria. Mutual functional relationships between the criteria are then identified experimentally. Following an experiment, criterial equations represent the main mathematical relationship that is attributable to a group of similar phenomena. The similarity theory facilitates the deduction of a general form of a criterial equation using the method of dimensional analysis. The dimensional analysis is characterised with simplicity of input data as it facilitates the identification of the quantity and types of similarity criteria merely on the basis of the opinion on the problem, without knowing a particular form of the complete physical equation.

In a general case, it is a complete physical equation expressing the relationships between n relevant parameters V_1, V_2, ..., V_n of various dimensions, in particular the following:

$$f(V_1, V_2, \ldots V_n) = 0 \qquad (1)$$

According to Buckingham's π - theorem, equation (1) can be written as follows:

$$f(\pi_1, \pi_2, \ldots \pi_k) = 0 \tag{2}$$

$$\text{or} \quad \pi_1 = \psi(\pi_2, \ldots \pi_k) \tag{3}$$

It follows from the requirement of dimensional uniformity that parameters V_1, V_2, ..., V_n together form a group used in the equation (1):

$$\pi_1 = V_1^{x_{1i}} \cdot V_2^{x_{2i}} \ \ldots V_n^{x_{ni}} \tag{4}$$

where i = 1, 2, ...,k and π_1 is a dimensionless variable.

All independent dimensionless π - arguments that may be created from n relevant dimensional parameters V_1, V_2, ..., V_n may be found on the premise that there is a determined system of criteria based on m dimensionally independent units $z_1, z_2, \ldots, z_m$ ($m < n$) and that we know respective defining equations. Exponents x_i may be identified by solving the following equation:

$$A \cdot x_i = 0 \tag{5}$$

where A is the rectangular dimensional matrix (n x m) with the rank $h \leq m$ and $x_{1i}, x_{2i}, \ldots, x_{ni}$ are unknown exponents.

III. APPLICATION OF THE SIMILARITY THEORY TO A PARTICULAR HEAT NETWORK

A mathematical model used for the calculation of total heat losses is based on the dimensional analysis and the mathematical interpretation thereof is affected by a correct choice of relevant parameters which are assumed to have a significant influence on the given phenomenon. Using the selected relevant parameters affecting heat losses in heat networks, one simplex (π_1) and one complex (π_2) were compiled as follows:

$$\pi_1 = \frac{T_\mathrm{i}}{T_\mathrm{e}} \tag{6}$$

$$\pi_2 = \frac{P}{T_\mathrm{i} \cdot \lambda_{\mathrm{ins}} \cdot l} \tag{7}$$

Two devices in which physically similar phenomena are taking place are differentiated by names. One is referred to as the Model (M) and the other as the Prototype (P).

In order to derive the characteristics of the Prototype from the characteristics of the Model, the underlying problems must be of the same nature and the corresponding similarity criteria (dimensionally independent dimensionless arguments for which the following applies: $\pi_{(M)} = \pi_{(P)}$) of the Prototype and of the Model must be of identical magnitudes. Conclusions made on the basis of conformity of criteria are referred to as model laws and their number corresponds to the number of dimensionless arguments:

$$c_1 \cdot c_2 \cdot c_3 \ldots\ldots\ldots c_n = 1 \tag{8}$$

where c_1 to c_n are referred to as *similarity constants*.

The similarity constant is the ratio of parameters with identical dimensions that expresses their proportionality in corresponding points within similar systems. It can be mathematically formulated as the following ratio:

$$c_i = \frac{V_{i(\mathrm{M})}}{V_{i(\mathrm{P})}} \tag{9}$$

where V_i is the relevant parameter.

$$c_l = \frac{l_{(\mathrm{M})}}{l_{(\mathrm{P})}} = \text{const.} \tag{10}$$

where c_l is referred to as the similarity constant for the change in length. It can be an arbitrary positive number. If $c_l = 1$, the Prototype and the Model are identical in size.

The network that was subjected to the analysis described in the article is regarded, within the following considerations, as the Model (M). The results may be transferred from the Model to any heat network that is regarded as the Prototype (P) using the similarity indicators.

The heat network characteristics are as follows: the network is lead underground; its nominal diameter is DN125; external diameter of the pipeline d_2 is 133 mm; pipeline wall thickness s is 3.6 mm; insulation thickness s_{ins} is 33.5 mm; length of the examined network L is 78 m; mean value of the coefficient of thermal conductivity of the soil λ_z is 1.35 W·m^{-1}·K^{-1}; mean value of the coefficient of heat transfer from the ground surface to the external environment α_0 is 3 W·m^{-2}·K^{-1}; depth of the pipeline bed H is 1.06 m. The temperature of water supplied to the supply pipeline ranged from 55 to 70 ºC. The temperature of water coming out of the return pipeline ranged from 50 to 60 ºC. The calculated temperature of the environment t_e for the given location was -15 ºC (258.73 K).

When the physical similarity is applied to the Model and the Prototype, two phenomena are only similar if the following conditions apply:

$$\pi_{1(\mathrm{M})} = \pi_{1(\mathrm{P})} \tag{11}$$

$$\pi_{5(\mathrm{M})} = \pi_{5(\mathrm{P})} \tag{12}$$

When we break down dimensionless arguments for the Model and for the Prototype according to formulas (11) and (12), using relevant parameters, we will get the following forms:

$$\frac{T_{\mathrm{i(M)}}}{T_{\mathrm{e(M)}}} = \frac{T_{\mathrm{i(P)}}}{T_{\mathrm{e(P)}}} \tag{13}$$

$$\frac{P_{\mathrm{AS(M)}}}{T_{\mathrm{i(M)}} \cdot \lambda_{\mathrm{ins(M)}} \cdot l_{\mathrm{(M)}}} = \frac{P_{\mathrm{AS(P)}}}{T_{\mathrm{i(P)}} \cdot \lambda_{\mathrm{ins(P)}} \cdot l_{\mathrm{(P)}}} \tag{14}$$

The following applies to the constants of similarity of changes in individual physical parameters that affect heat losses in the Prototype and in the Model:

$$\frac{T_{\mathrm{i\,(M)}}}{T_{\mathrm{i\,(P)}}} = c_{T_\mathrm{i}} \tag{15}$$

$$\frac{T_{\mathrm{e\,(M)}}}{T_{\mathrm{e\,(P)}}} = c_{T_\mathrm{e}} \tag{16}$$

$$\frac{P_{\mathrm{(M)}}}{P_{\mathrm{(P)}}} = c_P \tag{17}$$

$$\frac{\lambda_{\mathrm{ins\,(M)}}}{\lambda_{\mathrm{ins\,(P)}}} = c_{\lambda \mathrm{ins}} \tag{18}$$

$$\frac{l_{\mathrm{(M)}}}{l_{\mathrm{(P)}}} = c_l \tag{19}$$

After the above listed similarity constants are substituted into formulas (11) and (12), we will get two model laws, also referred to as similarity indicators (20 and 21). Their quantity always corresponds to the quantity of dimensionless arguments.

Model laws are as follows:

$$1 = \frac{c_{T_\mathrm{i}}}{c_{T_\mathrm{e}}} \tag{20}$$

$$1 = \frac{c_P}{c_{T_i} . c_{\lambda_{ins}} . c_l} \tag{21}$$

Let us consider the case that the real Prototype differs from the Model, for example, in the quality of the used insulation, i.e., in its thermal conductivity coefficient λ_{ins}. The constants of proportionality of relevant parameters are determined by formulas (14) to (19). Given the requirement of five unknown parameters contained in the model laws and two dimensionless arguments, three constants of proportionality of relevant parameters may be chosen and two will then be calculated from the model laws.

Let the chosen similarity constants be as follows:

$$c_{T_i} = 1$$

$$c_{T_e} = 1$$

$$c_l = 1.$$

This means that the temperatures of the supplied water in the Prototype and in the Model should be identical, the temperatures of the environment around the Prototype and around the Model should be identical, and also the total lengths of the examined distribution system in the Prototype and in the Model should be identical.

The required constant of proportionality of changes in the thermal conductivity coefficient for the insulation can be calculated using the formula 15:

$$\frac{\lambda_{ins\,(M)}}{\lambda_{ins\,(P)}} = c_{\lambda_{ins}} \tag{22}$$

where $\lambda_{ins\,(M)} = 0.04$ W.m^{-1}.K^{-1} and represents the thermal conductivity coefficient for the insulation in the Model.

Let us examine the development of heat loss values for the Prototype if the thermal conductivity coefficients for the insulation in the Prototype $\lambda_{ins(P)}$ are at various levels (better as well as worse than the Model), determined by the following values:

$\lambda_{ins(P)} = 0.02$ W·m^{-1}·K^{-1},

$\lambda_{ins(P)} = 0.05$ W·m^{-1}.·K^{-1},

$\lambda_{ins(P)} = 0.08$ W·m^{-1}·K^{-1},

$\lambda_{ins(P)} = 0.09$ W·m^{-1}·K^{-1}.

The similarity constant for changes in the total heat loss will be calculated using the formula 19:

$$c_P = c_{T_i} . c_{\lambda_{ins}} . c_l \tag{23}$$

Following the substitution of particular values of similarity constants for relevant parameters for all the chosen values of the thermal conductivity coefficient in the Prototype into the formula (23), the similarity constant for heat losses, for example for $\lambda_{ins\,(P)} = 0.02$ 0.02 W·m^{-1}.K^{-1}, will have the following value:

$$c_P = c_{T_i} \cdot c_{\lambda_{iz}} \cdot c_l = 1 \cdot 2 \cdot 1 = 2$$

because if $\lambda_{ins(P)} =$ 0.02 W·m^{-1}·K^{-1}, then $c_{\lambda_{ins}} = \frac{\lambda_{(M)}}{\lambda_{(P)}} = \frac{0{,}04}{0{,}02} = 2.$

It follows from the above mentioned that if the quality of the insulation was twice as high as for the network that was subjected to the experimental examination described in the article, the heat loss values for the described Prototype would be twice as low. Such conclusion also follows from the measure of changes in the power dissipation $\frac{P_{(M)}}{P_{(P)}} = c_P$ in which the power dissipation in the Model $P_{(M)}$ is known and was determined for the following conditions:

$T_{i(M)} = 343.15$ K, $T_{e(M)} = 258.15$ K,

$l_{(M)} = 1$ m, $\lambda_{ins(M)} = 0.04$ W·m^{-1}·K^{-1}

These conditions correspond to a specific value of the total power dissipation of the network that was determined from the constructed Model. Its numerical value is $P_{(M)} = 40$ W.

The value of the power dissipation in the Prototype, identified using the formula 14, is 20 W. It represents a half of the value of the power dissipation in the examined Model.

$$P_{(P)} = \frac{P_{(M)}}{c_P} = \frac{40}{2} = 20 \text{ W} .$$

Table 1 contains the values of the total heat loss $P_{(P)}$ for a new Prototype that differs from the Model in the quality of insulation. In one case, the insulation was of a higher quality than in the Model ($\lambda_{iz} = 0.02$ W·m^{-1}·K^{-1}); in three other cases, it was worse than in the Model ($\lambda_{ins} = 0.05$; 0.08; and 0.09). For the purpose of comparison, the table also contains the values of power dissipation in the Model that correspond to the thermal conductivity coefficient for the insulation λ_{ins} of 0.04 W·m^{-1}·K^{-1}, i.e., the original network subjected to the experiment.

TABLE 1
HEAT LOSS IN THE NEW PROTOTYPE

$\lambda_{ins\,(P)}$	$c_{\lambda_{ins}}$	c_P	$P_{(P)} = \frac{P_{(M)}}{c_P}$
0.02	2	2	20
0.04	1	1	40
0.05	0.8	0.8	50
0.08	0.5	0.5	79
0.09	0.04	0.04	89

Fig. 1 presents the relationship between the heat loss in the Prototype and the quality of insulation, i.e. the value of its thermal conductivity coefficient λ_{ins}. The Fig. 1 indicates that with the improving quality of the used insulation (lower thermal conductivity coefficient λ_{ins}) the heat losses in the network decrease.

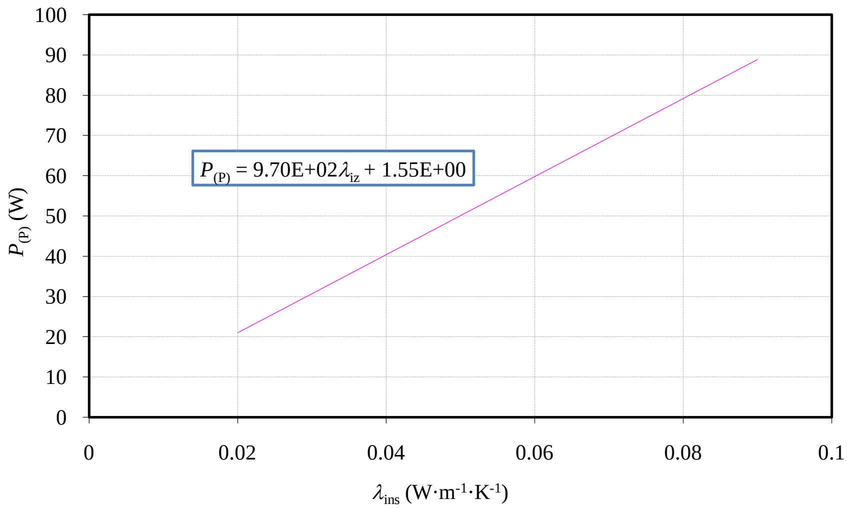

FIGURE 1: Relationship between the heat loss in the Prototype and the quality of insulation

IV. CONCLUSION

In conclusion, it is necessary to note that identification of heat losses through the examination of a physical model will find its practical applications. Introducing this procedure into real practice would not result in increased costs insured to heat manufacturers or distributors; moreover, it would facilitate clear identification of the values of total or specific heat loss in the network.

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