Comparison of cn estimation approaches

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Abstract— *The Curve Number (CN) method used for estimating direct runoff depths from rainstorms (developed by NRCS in 1954) is based on a CN parameter (representing the hydrological properties of a catchment) and a λ parameter (representing the initial abstraction ratio Ia/S). In this paper, the CN parameter is determined for 10 small Slovak basins using the original SCS-CN method ($CN_{tabulated}$) and empirically from rainfall-runoff events for a 13-year period using four different approaches (asymptotic fitting, arithmetic, median, and a method used by Hawkins et al. [9]) for both natural and ordered P-Q pairs ($CN_{empirical}$). The $CN_{empirical}$ numbers were evaluated and consequently used to determine the direct runoff for both λ equal to 0.2 and 0.05. The results show differences among the selected approaches that indicate variances in the direct runoff computed. The large range of the empirical CN numbers indicates uncertainty. However, the NRCS does not define the uncertainty of the tabulated curve numbers either, and the original data is not available. As a result, this paper will highlight the uncertainty of tabulated CN numbers.*

***Keywords*— Curve Number method, Rainfall-runoff events, Asymptotic fitting.**

I. INTRODUCTION

The SCS-CN method is a rainfall-runoff model developed for the United States by the U.S. Soil Protection Service (USDA SCS, now Natural Resources Conservation Service (NRCS)). In this method a relationship between watershed characteristics and an antecedent rainfall meets in a Curve Number parameter. With this simple parameter a rainfall depth is transformed to a runoff depth. The tables and figures for estimating the CN parameter for the soil cover complexes of the USA are given in the NRCS publication [19]. Direct runoff can be computed using the tabulated CN based on the land´s use and condition, the hydrologic soil group, and the rainfall depth as:

$$Q = \frac{(P - \lambda S)^2}{S + (P - \lambda S)} \tag{1}$$

$$S = \frac{25400}{CN} - 254 \tag{2}$$

Where Q is the direct runoff [mm], P is storm rainfall [mm], S is the potential retention [mm], λ is the potential retention parameter, and CN is a curve number parameter [-], valid for P > Ia. For P < Ia, Q = 0.

In the original methodology the classification of antecedent moisture condition (AMC) was developed, which classifies the rainfall-runoff events into the AMC I, II and III classes. These correspond to low, medium and high soil moisture conditions depending on the total antecedent rainfall depth for the previous 5 days [23]. The uncertainties of the original concept of the antecedent conditions have been questioned, and the SCS-CN method has been analyzed by many authors [7] [8] [10] [13] [16] [21].

Also, the initial abstraction coefficient (λ) has been questioned. New methods and the availability of data with long rows of observations have led to a review of the original relation Ia = 0.2*S. Mockus (1972) concluded that the coefficient varied in the interval λmin = 0.013 to λmax = 2.1 and that the mean value of λ is approximate and can have a negative effect on the accuracy of the computed runoff. Cazier and Hawkins [2] analyzed the data of 109 small basins and determined that the most common value for the parameter λ was 0 and that the average value was 0.0006. Baltas et al. [1] analyzed the relationship Ia / S at a catchment in Greece (15.18 km^2); here the average coefficient λ was 0.014, and the design value was determined to be 0.037. Hawkins and Khojeini [11] analyzed data for 97 small basins and determined that the coefficient λ ranged from 0 to 0.0966 for the data generated and equalled zero for the observed. Jiang [15] used two methods to evaluate the coefficient λ and analyzed 307 river basins. He found that 90% of the values were less than 0.2. He therefore proposed the coefficient λ =

0.05, which corresponds better to the empirical data. For the proposed parameter, λ equalled 0.05, where $R^2 = 0.993$, for the calculated and measured real data.

The CN parameter is assumed to be constant at each watershed. In practice, however, the CN parameter differs from storm to storm, which is a result of changes in the antecedent conditions and the variability of the storm´s morphology. The Curve Numbers were empirically derived from the local rainfall-runoff data of the eastern, midwestern and southern United States. Despite the fact that it was developed based on the empirical data of the USA, the method is used in many countries all over the world. The SCS-CN method works with mean values, which leaves even more room for the degree of uncertainty. Of all these factors may enlarge the errors of an estimated runoff depth.

The dominant weakness of the method is its single parameterization of the potential retention (S), which combines several important hydrological processes, thus the tabulated CNs are based on some of the factors dedicated to a landscape´s retention of water. The CN method seems to be based on runoff generation through infiltration excess and is similar to the Horton [14] infiltration equation. It might be good to relate the curve number runoff equation to either infiltration excess or saturation excess runoff [3] [4]. Moreover, the tabulated CNs for woodlands and orchards are said to have been extrapolated from rainfall-runoff data from just one wooded plot or watershed, and in recent years other limitations such as these have become even more obvious [6] [5]. Therefore, to apply the CN method without greater difficulty, the runoff estimates for humid forested watersheds have to be better understood, and the runoff equation should be related to Hortonian runoff processes and variable saturated source areas. The accuracy of the Curve Number Method has to be questioned as the use of the current tabulated CNs results in overdesigning hydrological infrastructures by billions of dollars annually [20].

The objective of this paper is to point out the uncertainty of tabulated curve numbers and differences in the empirical CN identified.

II. STUDY AREA

The Upper Hron River basin was chosen for this study. The ten small watersheds selected were (W1) Havraník, (W2) Šaling, (W3) Brôtovo, (W4) Osrblianka, (W5) Bystrianka, (W6) Harmanec, (W7) Ramžiná, (W8) Starohorský potok, (W9) Hukava, and (W10) Jasenica. Each of the watersheds has forest cover as its dominant land use. The land use changes during periods when rainfall and runoff were measured in the selected watersheds has not been considered. Streamflow and rainfall records were obtained for a 13-year period (1989 - 2002). The major hydrological soil group of the drainage is B. The areas of the watersheds are in an interval of 9.28 to 82.97 km^2.

TABLE 1
CATCHMENTS CHARACTERISTICS

N.	River	Gauging station		Area (km^2)	Woods	Urban.	Grass	Agric	Shrub
		ID	Name		%				
W1	Havraník	6960	Zlatno	16.72	78.5	0	13.7	1.3	6.7
W2	Šaling	7029	Čierny Balog	24.98	78.4	0.3	7.1	4.9	9.4
W3	Hron, Brôtovo	7033	Čierny balog	9.28	87.8	0	2.1	0	10.1
W4	Osrblianka	7050	Osrblie	27.77	93.6	1.8	0	4.6	0
W5	Bystrianka	7058	Mýto pod Ďumbierom	22.48	50.2	0	0	0	49.8
W6	Harmanec	7120	Dolný Harmanec	23.1	92.5	0	0	0	7.5
W7	Ramžiná	7140	Staré Hory	12.29	92.5	0.6	0	2.6	4.2
W8	Starohorský potok	7145	Staré Hory	62.61	77.4	0.6	2	5.6	14.3
W9	Hukava	7183	Hriňová	9.96	86.7	0	2.9	4.4	6
W10	Jasenica	7241	Hronská Breznica	82.97	92.5	0.6	0	2.6	4.2

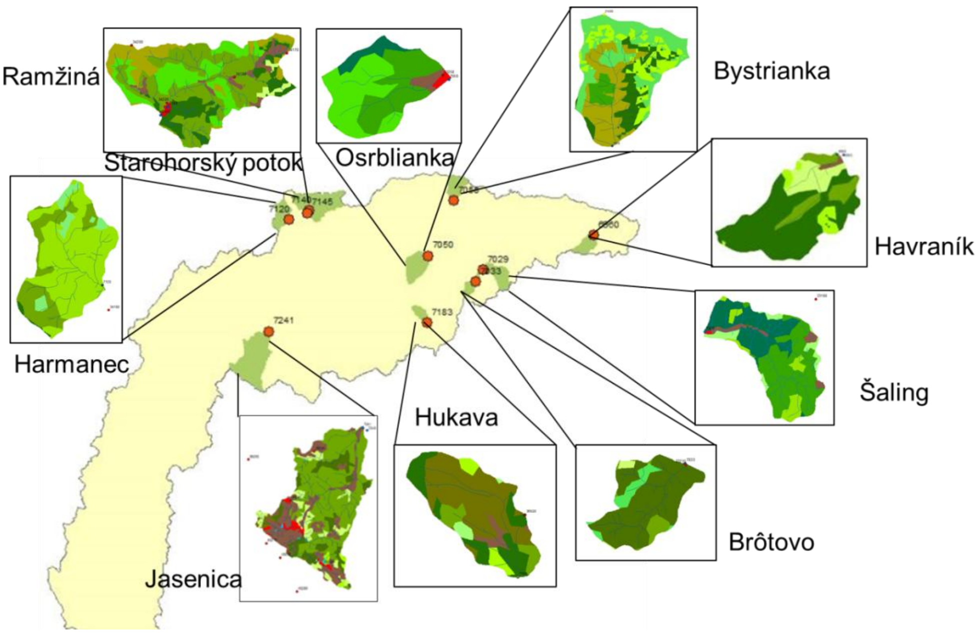


FIG 1 – LOCATION AND LAND USE OF THE CATCHMENTS ANALYZED.

III. METHODOLOGY

The rainfall and runoff depths measured for each event were used to determine the curve number ($CN_{EVENT,\,i}$) for each year of record at all ten watersheds. The representative watershed curve numbers (CN_{REP}) for each of the watersheds were selected from a set of CN_{EVENT} determined using the (1) arithmetic mean, (2) median, (3) Hawkins [9], and (4) asymptotic value procedure. To access the accuracy of the tabulated curve numbers (CN_{TAB}), these four calibrated values were compared to the CN_{TAB} , which is based on the specific hydrological soil group (Group A, B, C, or D), cover-complex (land use, treatment, or practice), and hydrological condition (poor, fair, or good) of each watershed. The detailed procedures for each technique follow.

3.1 CN_{EVENT} determination

Eq. (1) is solved via the quadratic formula for S and via equation (2), a CN_{EVENT}. Thus, any P-Q pair (0 < Q < P) has its own $CN_{EVENT,\,i}$, and this data-derived value will not be constant, as the original methodology assumed. The $CN_{EVENT,}$ is computed as:

$$CN_{EVENT,\ i} = \frac{25400}{5\left(P + 2Q - \sqrt{4Q^2 + 5PQ}\right) + 254}\,(mm) \qquad (3)$$

Where Q is the measured runoff in [mm] and P is the measured rainfall depth in [mm] for *i* number of events.

A set of CN_{EVENT} for a particular watershed is averaged to determine the CN_{REP}.
A median of the $CN_{EVENT,\,i}$ for a particular watershed is computed to determine the CN_{REP}. The median was determined for the original curve number tables using the Graphic Method. The direct runoff was plotted versus the rainfall volume to determine the curve, which divides the plotted points into two equal groups. The curve number for that curve is the median curve number.

The CN_{EVENT} for a watershed is averaged for only those events to which it applies:

$$P_i/S_m \geq 0{,}46 \qquad (4)$$

$$S_m = \sum S_i/n \qquad (5)$$

Where:

P_i is rainfall depth of the ordered event (from low to higher),

S_m is the mean value of the initial abstraction for n events.

When $CN_{EVENT,\,i}$ is derived from actual storm data (section 3.1), a secondary relationship almost always occurs between a set of $CN_{EVENT,}$ and the storm rainfall depths ($P_{,\,i}$). The curve number varies with the rainfall events that occur at different frequencies, as it is generally a function of the design return interval or frequency. This is a fundamental problem of the Curve Number Method, as it does not take the different frequencies into account. This asymptotic method corrects this problem.

The calculated $CN_{EVENT,i}$ mostly approaches a constant value with increasing rainfall. However, three types of $CN_{EVENT,i}$-responses to the frequency of the occurrence of a rainfall have been observed: standard, complacent, and violent responses. The most common is the standard behavior that occurs when the ratio of the rainfall and runoff becomes constant for the increasing rainfall, and the $CN_{EVENT,\,i}$ approaches a near-constant, minimum value (CN_{min}). The complacent behavior, in which the observed $CN_{EVENT,\,i}$ declines steadily with an increasing rainfall depth, shows no apparent tendency to achieve a stable value. An asymptotic curve number (CN_{min}) cannot be defined, and different curve numbers must be defined for different design return intervals. The response is violent, when the set of $CN_{EVENT,\,i}$ suddenly rises and asymptotically approaches an apparent constant maximum value; the watershed generates more runoff per millimeter of rainfall as the rainfall event increases [22]. Table 2 shows the overview of the models used in this analysis.

TABLE 2
MODEL DESCRIPTIONS WITH VALUES OF ALFA

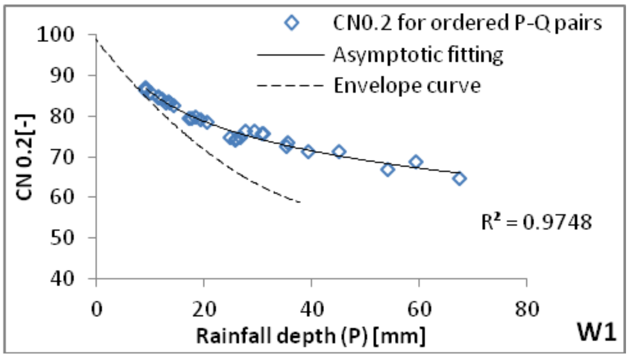
Model	Method	λ	P:Q pairs
M1	Arithmetic mean	0.05	Ordered
M2	Arithmetic mean	0.2	Ordered
M3	Arithmetic mean	0.05	Natural
M4	Arithmetic mean	0.2	Natural
M5	Median	0.05	Ordered
M6	Median	0.2	Ordered
M7	Median	0.05	Natural
M8	Median	0.2	Natural
M9	Hawkins (1985)	0.05	Ordered
M10	Hawkins (1985)	0.2	Ordered
M11	Hawkins (1985)	0.05	Natural
M12	Hawkins (1985)	0.2	Natural
M13	Asymptotic fit	0.05	Ordered
M14	Asymptotic fit	0.2	Ordered
M15	Tabulated CN	0.05	-
M16	Tabulated CN	0.2	-

IV. RESULTS

For each watershed was computed four curve numbers, which are based on the arithmetic mean, median, Hawkins [9] method and tabulated CNs.The arithmetic mean and median of CN value for each watershed are quite equivalent .Except for the asymptotic method, where only ranked rainfall and runoff depths were used, both ranked and unranked P-Q pairs were used to compute the calibrated curve numbers. All the procedures were performed using initial abstraction coefficients equal to 0.2 and 0.05. Watershed CNs using asymptotic procedures could not be determined for watersheds with complacent responses. Unfortunately, most of the selected watersheds showed complacent behavior. Fig. 4 and 5 are examples of such complacent watersheds, where the rainfall depth and Curve Number have a linear relationship. There is a great difference among the watershed Curve numbers. CNs with λ equal to 0.05 have lower values than CNs with λ equal to 0.2.

TABLE 3
CURVE NUMBER USING ASYMPTOTIC PROCEDURE FOR ANALYSES CATCHMENTS

	λ = 0.2	λ = 0.05
W1	64	47
W2	-	-
W3	-	42
W4	55	-
W10	-	59



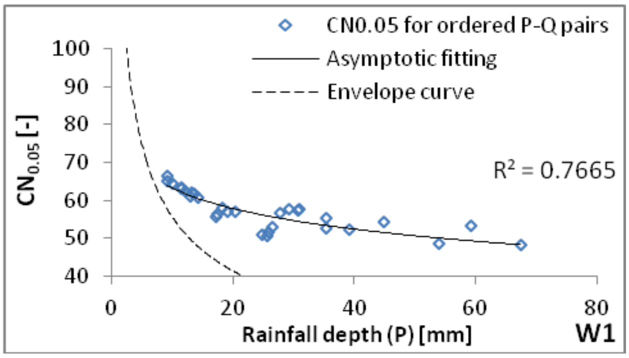
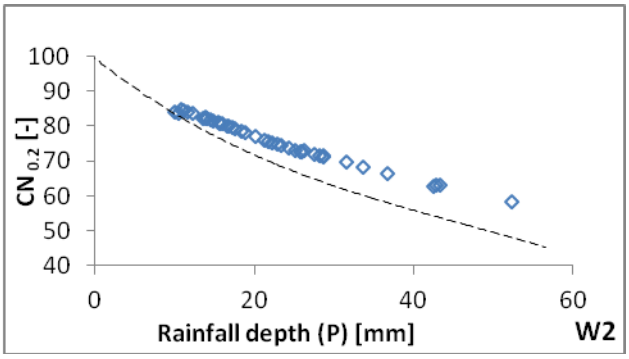


FIG 2 – ASYMPTOTIC FITTING FOR λ = 0.2 (W1)

FIG 3 – ASYMPTOTIC FITTING FOR λ = 0.05 (W1)

Asymptotic CN example for watershed W1 (Havraník) fitted to a ranked rainfall and runoff series for standard behavior. The envelope curve is a function $CN_{o,\ 0.2}$ = 2540/(25.4+P/2), and $CN_{o,\ 0.05}$ =1270/(12.7+P) defines a threshold below which no runoff occurs.



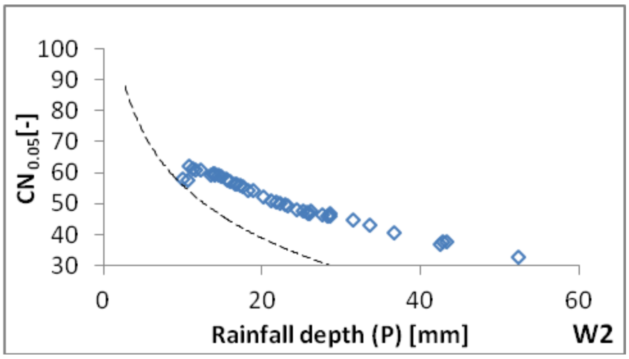
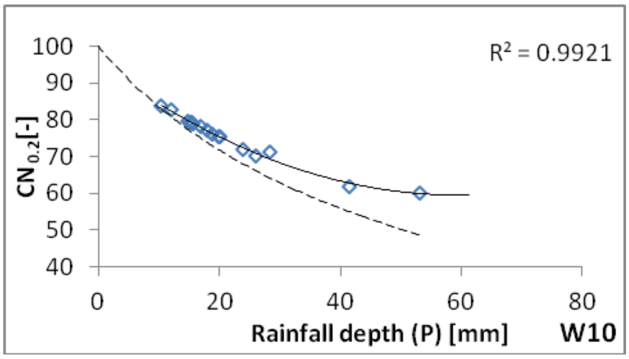


FIG 4 – ASYMPTOTIC FITTING FOR λ = 0.2 (W2)

FIG 5 – ASYMPTOTIC FITTING FOR λ = 0.05 (W2)



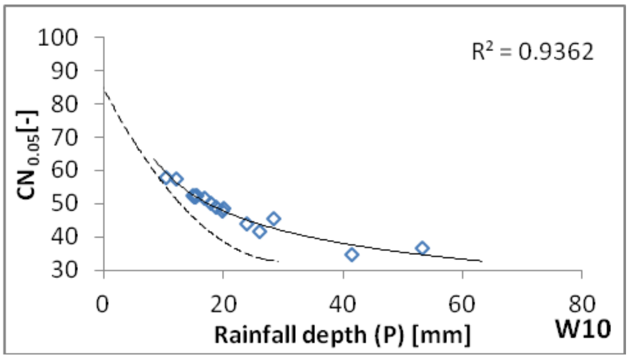


FIG 6 – ASYMPTOTIC FITTING FOR λ = 0.2 (W10)

FIG 7 – ASYMPTOTIC FITTING FOR λ = 0.05 (W10)

The watershed W2 (Šaling) data were not sufficient to determine whether the asymptotic fit is standard or complacent and to determine the asymptotic curve numbers. See Fig. 4 and 5, for example. This occurred at most of the watersheds. Only in a few cases (Table 6), was it possible to determine the asymptotic CN (Fig 6 and 7). The asymptotic curve numbers of all the watersheds were smaller than the curve numbers based on the other procedures. In some cases only the asymptotic CN for λ = 0.2 (M14) or 0.05 (M13) could be determined. As Fig 8 shows, the variability in the computed runoff using different procedures is great. Thus it is essential to choose the best method to determine the representative watershed Curve Number.

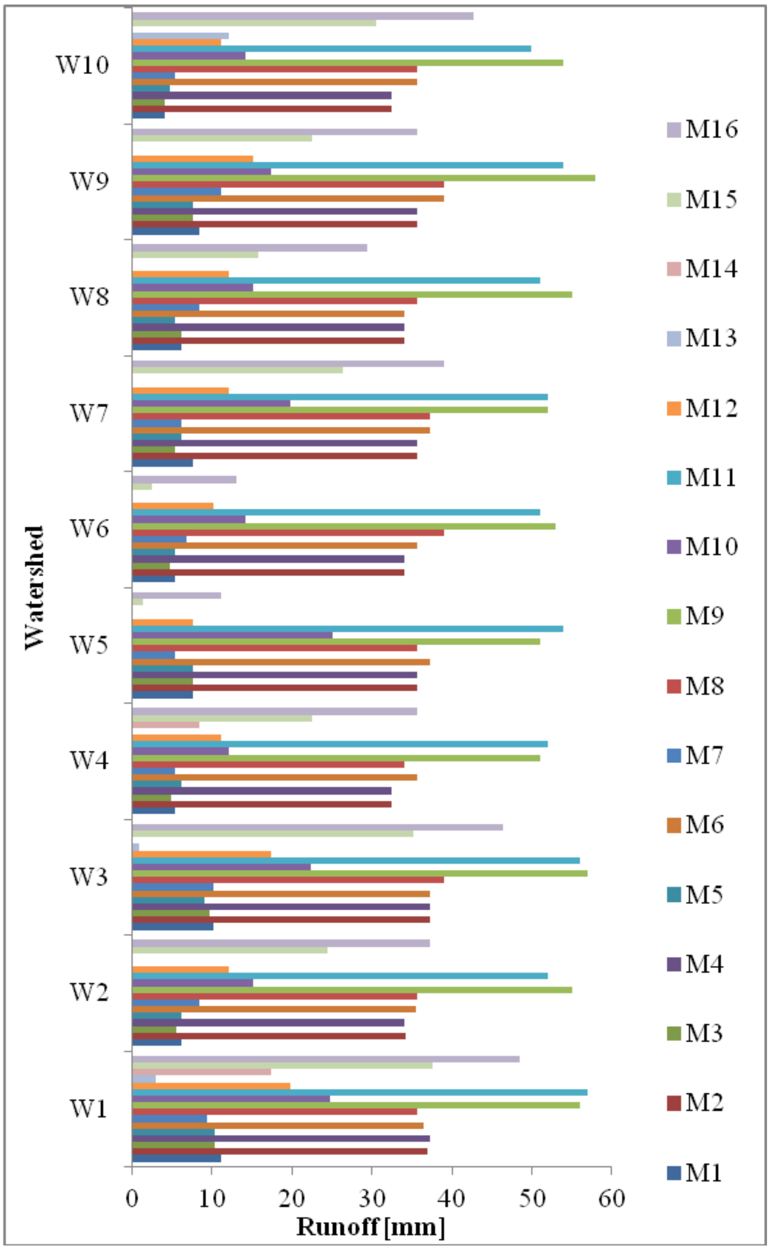


FIG 8 – RUNOFF VALUES ESTIMATED USING DIFFERENT CN VALUES AT ANALYSED STATIONS

V. CONCLUSION

The SCS-CN method is designed for estimation of direct runoff from watersheds. Many uncertainties are connected with this method.

CN value belongs to most discussed parameters. This parameter is possible to compute by various concepts.

The SCS-CN method works with mean values, which leaves even more room for uncertainty. All of these may enlarge the errors of the estimated runoff depth. With different approaches, different empirical CN values are computed; and therefore, various runoff for the same watershed can be determined.

The relative accuracy of the methods used for determining the watershed CN was performed. The best fit for the observed runoff was achieved with the use of asymptotic fitting. However, this was not applicable to all the watersheds. The median procedure for both the natural and ordered P-Q pairs with $\lambda = 0.05$ was the second best option. The worst fit was with the tabulated Curve Numbers with $\lambda = 0.05$.

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