Discussion on Weighted Similarity Measure under Intuitionistic Fuzzy Sets Environment

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***Abstract*—** *We analytically show that the findings of pattern recognition problems with weighted similarity measures under intuitionistic fuzzy sets environment that is dominated by relative weights of elements in the universe of discourse for the discrete case and the weighted function for the continuous case. In the past, researchers focus on constructing new similarity measures or developing new algorithms applying their similarity measures. Hence, previous results depended on a special weight to decide the pattern of the sample that may be required further considerations. How to select a proper weight will be an important issue for researchers in the future when deal with pattern recognition problems.*

***Keywords*— Pattern recognition, intuitionistic fuzzy sets, similarity measure.**

I. INTRODUCTION

More and more business had being developed in the real world, so it is hard for a company to get accurate market data. Therefore, fuzzy set data may be a kind of data that can be obtained more easily to explain the market and managerial situation. If the data is a traditional fuzzy set, there are some methods to be applied to solve the fuzzy problem. Recently, many similarity measures have been proposed, for examples, Hung and Lin [1], Julian et al. [2], Hung and Lin [3], Tung et al. [4], Yen et al. [5], Hung and Wang [6], Chu and Guo [7], Hung et al. [8], and Tung and Hopscotch [9], for measuring the degree of similarity between fuzzy sets, under a kind of special fuzzy sets, which is called Intuitionistic Fuzzy Sets (IFSs) that were initiated presented by Atanassov [10, 11]. Since Atanassov originated the idea of IFSs, many different similarity measures between IFSs have been proposed in the literature. Atanassov and Rangasamy[12] and Kuppannan et al. [13] provided practical applications for IFSs. The importance of suitable distance measures between IFSs takes place because they play an important role in the theoretical development and implication problem. Two existing similarity measures for IFSs were proposed by Li and Cheng [14] to indicate the dominated factor for the selection of pattern recognition problems. This paper is a detailed analysis for the similarity measure of intuitionistic fuzzy sets (IFSs) in Li and Cheng [14]. Mitchell [15] already provided an example to demonstrate that the similarity measure proposed by Li and Cheng [14] may lead to counter-intuitive result. However, there are 373 papers continuously referred to Li and Cheng [14]. Owing to the high citation, it deserves a detailed study of their paper. To be compatible with previous results, we directly study the examples of Li and Cheng [14] to show that their weighted similarity measures contained inherent problems that is their results are dependent on weights of elements in the universe of discourse for the discrete case and the weighted function for the continuous case. Hence, how to derive the weights or the weighted function should be the crucial issue in the future research. Our consideration will offer a patch work to enhance the operational development of similarity measure for pattern recognition under IFSs. Recently, there is a trend to improve published papers, for example, Hung et al. [16], Lin et al. [17], Tung [18], and Chao et al. [19]. Following this trend, we will provide improvements for Li and Cheng [14]. In this paper, based on the same numerical examples of Li and Cheng [14], we will demonstrate that the proposed measures of Li and Cheng [14] performs dependent on the relative weight in pattern recognition. Our findings presented here could arouse attention to take care of the decision of relative weights in the selection and applications of similarity measures for IFSs and vague sets in practice.

II. MATERIALS AND METHODS

Let X be a fixed set. An IFS A in X is an object having the form $A=\left\{\left\langle x,t_A\left(x\right), f_A\left(x\right)\right\rangle \middle| x\in X\right\}$ where the function $t_A : X \to \left[0,1\right]$ and $f_A : X \to \left[0,1\right]$ define the degree of membership and degree of non-membership, respectively, and for every $x \in X$, $0 \le t_A(x) + f_A(x) \le 1$. Let $IFSs(X)$ denote the set of all IFSs in X. Next, we define the order relation. $A, B \in IFSs(X)$, $A \subseteq B$ is defined as $t_A(x) \le t_B(x)$ and $f_A(x) \ge f_B(x)$ for every $x \in X$.

For a pattern recognition problem, if a pattern A_{i_0} satisfies $S\left(A_{i_0}, B\right) = \max_{1 \le i \le m} S\left(A_i, B\right)$, and $S\left(A_{i_0}, B\right) > \max_{1 \le i \le m, i \ne i_0} S\left(A_i, B\right)$ according to the principle of the maximum degree of similarity between IFSs, researchers can decide that the sample *B* belongs to the pattern A_{i_0} .

In the following two numerical examples, we will show that the similarity measures of Li and Cheng [14] will dependent on the relative weights of IFS so their approach may cause different results for pattern recognition problems when the relative weights are arbitrary selected.

For the discrete case with the universe of discourse $X = \{x_1, x_2, ..., x_n\}$, Li and Cheng [14] assumed the similarity measure between two IFSs A and B as

$$S_d^p\left(A, B\right) = 1 - \frac{1}{\sqrt[p]{n}} \sqrt[p]{\sum_{i=1}^{n} \left(\varphi_A\left(i\right) - \varphi_B\left(i\right)\right)^p} \tag{1}$$

Where

$$\varphi_A\left(x_i\right) = \frac{t_A\left(x_i\right) + 1 - f_A\left(x_i\right)}{2} \tag{2}$$

Which was introduced originally in Tanev [20].

2.1 Example 1 of Li and Cheng [14].

There are three patterns in IFSs of

$X = \{x_1, x_2, x_3\}$,

$A_1 = \{\langle 1, 0\rangle, \langle 0.8, 0\rangle, \langle 0.7, 0.1\rangle\}$, $A_2 = \{\langle 0.8, 0.1\rangle, \langle 1, 0\rangle, \langle 0.9, 0\rangle\}$, $A_3 = \{\langle 0.6, 0.2\rangle, \langle 0.8, 0\rangle, \langle 1, 0\rangle\}$, respectively, where $t_{A_1}\left(x_1\right) = 1$ and $f_{A_1}\left(x_1\right) = 0$ for pattern A_1. Consider a sample $B \in IFSs(X)$ which will be recognized, with $B = \{\langle 0.5, 0.3\rangle, \langle 0.6, 0.2\rangle, \langle 0.8, 0.1\rangle\}$. We referred their findings in the next table.

- **Notation**

D is the demand rate per unit of time.

h is the holding cost per unit, per unit of time.

K is the ordering cost (setup cost) per order.

p is the backorder cost per unit, per unit of time (linear backorder cost).

Q is the order quantity.

$Q - S$ is the beginning inventory level, after backlogged quantity S .

r is an auxiliary expression, with $r = \left(h/p\right)$.

S is the backlogged amount.

TC is the total cost per unit of time.

π is the backorder cost per unit (fixed backorder cost).

- **Assumptions**

1. The model is developed for only one product.

2. The inventory model is developed for an infinite planning horizon such that the goal is to minimize the first (and repeated) replenishment cycle.
3. The demand rate is constant over the entire planning horizon.
4. The shortages are allowed and fully backlogged.
5. There are two types of backorder costs: a linear backorder cost that is applied to average backorders per unit of time and a fixed cost that is applied to maximum backorder level without considering the backlogged waiting period.

TABLE 1
SUMMARY OF THEIR FINDINGS IN LI AND CHENG [14]

				$S_d^p(A_1,B)$	$S_d^p(A_2,B)$	$S_d^p(A_3,B)$
$x_1 = 1/3$	$x_2 = 1/3$	$x_3 = 1/3$	$p = 1$	0.78	0.80	0.85
$x_1 = 1/3$	$x_2 = 1/3$	$x_3 = 1/3$	$p = 2$	0.74	0.78	0.84
$x_1 = 0.5$	$x_2 = 0.3$	$x_3 = 0.2$	$p = 2$	0.696	0.779	0.853

Based on the similarity measure on the above table, Li and Cheng [14] concluded that the sample B belongs to the pattern A_3.

For the continuous case, Li and Cheng [14] assumed the similarity measure of two IFSs A and B as

$$S(A,B) = 1 - \int_a^b w(x)[\varphi_A(x) - \varphi_B(x)]dx \tag{3}$$

where $w(x)$ is the weight function. For the continuous case, they applied the uniform distribution with weight $\omega(t):[a,b] \to [0,1]$, $\int_a^b \omega(t)dt = 1$ so that they used $\omega(t) = 1/(b-a)$, for $a \le t \le b$.

2.2 Example 2 of Li and Cheng [14].

They assumed that two patterns, A_1 and A_2 with a sample, B are represented by IFSs, where

$$t_{A_1}(x) = \begin{cases} 0.8(x-1), & 1 \le x < 2, \\ 4(5-x)/15, & 2 \le x \le 5, \end{cases} \tag{4}$$

$$f_{A_1}(x) = \begin{cases} 1.9 - 0.9x, & 1 \le x < 2, \\ 0.3x - 0.5 & 2 \le x \le 5, \end{cases} \tag{5}$$

$$t_{A_2}(x) = \begin{cases} 0.2(x-1), & 1 \le x < 4, \\ 0.6(5-x), & 4 \le x \le 5 \end{cases} \tag{6}$$

$$f_{A_2}(x) = \begin{cases} 1.3 - 0.3x, & 1 \le x < 4, \\ 0.9x - 3.5 & 4 \le x \le 5, \end{cases} \tag{7}$$

$$t_B(x) = \begin{cases} 0.3(x-1), & 1 \le x < 3, \\ 0.3(5-x), & 3 \le x \le 5 \end{cases} \tag{8}$$

$$f_B(x) = \begin{cases} 1.4-0.4x, & 1 \le x < 3, \\ 0.4x-1, & 3 \le x \le 5. \end{cases} \tag{9}$$

We recall their findings of $S(A_1, B) = 0.85$ and $S(A_2, B) = 0.86$ such that Li and Cheng [14] claimed that sample B should belong to the pattern A_2.

III. RESULTS

3.1 Our discussion for Example 1 of [14]

Based on equation (2), we derive that $\varphi_{A_1} = (1.0, 0.9, 0.8)$, $\varphi_{A_2} = (0.85, 1.0, 0.95)$, $\varphi_{A_3} = (0.7, 0.9, 1.0)$ and $\varphi_B = (0.6, 0.7, 0.85)$. For the discrete case, we assume that the weight is $(\omega_1, \omega_2, \omega_3)$ under the condition $p = 1$, and then we recall the 1-norm distance, say $d(A_i, B)$ with $d(A_i, B) = 1 - S_d^1(A_i, B)$. Hence, we know that to have the maximum similarity is equivalent to have the minimum distance. We find that

$$d(A_1, B) = 0.4\omega_1 + 0.2\omega_2 + 0.05\omega_3, \tag{10}$$

$$d(A_2, B) = 0.25\omega_1 + 0.3\omega_2 + 0.1\omega_3, \tag{11}$$

and

$$d(A_3, B) = 0.1\omega_1 + 0.2\omega_2 + 0.15\omega_3. \tag{12}$$

From the above derivation, if we select that $\omega_1 = 0.1$, $\omega_2 = 0.1$ and $\omega_3 = 0.8$, then it yields that

$$d(A_1, B) = 0.1 < d(A_2, B) = 0.135 < d(A_3, B) = 0.15 \tag{13}$$

then the sample B belongs to the pattern A_1.

On the other hand, if $\omega_1 = 0.3$, $\omega_2 = 0.1$ and $\omega_3 = 0.6$, then it yields that

$$d(A_3, B) = 0.14 < d(A_2, B) = 0.165 < d(A_1, B) = 0.17 \tag{14}$$

then the sample B belongs to the pattern A_3.

In the following, we will develop a theoretical result to show that sample B cannot be classified to the pattern A_2.

Lemma 1.

$S(A_2, B) = \max\{S(A_i, B) : i = 1, 2, 3\}$ if and only if $w_1 = 0.25$, $w_2 = 0$ and $w_3 = 0.75$.

(Proof of Lemma 1) We assume $\omega_1 = x$, $\omega_2 = y$ and $\omega_3 = 1 - x - y$ to simplify the expression. It yields that $d(A_2, B) \le d(A_1, B)$ if and only if

$$0.2x \ge 0.05y + 0.05. \tag{15}$$

On the other hand, we know that $d(A_2,B)\le d(A_3,B)$ if and only if

$$0.2x+0.15y\le 0.05. \tag{16}$$

If we want $d(A_2,B)$ is the smallest distance, then from equations (15) and (16), it follows that

$$0.2x\ge 0.25y+0.05\ge 0.2x+0.2y. \tag{17}$$

By equation (17), it implies that $y=0$, $x=0.25$ and

$$d(A_2,B)=d(A_1,B)=d(A_3,B). \tag{18}$$

Therefore, under the usual condition, three weights, w_1, w_2 and w_3 are all positive so we cannot construct an example in which the sample B belongs to the pattern A_2 such that we derive the next theorem.

Theorem 1.

Under the conditions $w_i>0$, for $i=1,2,3$ and $w_1+w_2+w_3=1$ then sample *B* cannot be decided belonging to pattern A_2.

Based on above discussion, different relative weights will imply different patterns for the given sample. Our work demonstrates that the weights of IFSs are the most essential factor to decide the sample B belongs to which pattern.

3.2 Our discussion for Example 2 of [14]

According to equation (2), we follow Li and Cheng [14] approach to transfer three IFSs into ordinary fuzzy sets as

$$\varphi_{A_1}(x)=\begin{cases}0.85(x-1), & 1\le x<2,\\ 17(5-x)/60, & 2\le x\le 5,\end{cases} \tag{19}$$

$$\varphi_{A_2}(x)=\begin{cases}0.25(x-1), & 1\le x<4,\\ 3(5-x)/4, & 4\le x\le 5,\end{cases} \tag{20}$$

and

$$\varphi_{B}(x)=\begin{cases}0.35(x-1), & 1\le x<3,\\ 0.35(5-x), & 3\le x\le 5.\end{cases} \tag{21}$$

We will show that different weight function $\omega(x)$ will influence the results for the pattern selection. We assume that

$$\omega(x)=\begin{cases}\lambda_1, & 1\le x<3,\\ \lambda_2, & 3\le x\le 5,\end{cases} \tag{22}$$

under the condition that $0\le\lambda_1$, $0\le\lambda_2$, and $\lambda_1+\lambda_2=0.5$.

Remark. The restriction of $\lambda_1+\lambda_2=0.5$ is our special treatment such that the uniform weighted function, where $\omega(x)=0.25$, for $1\le x\le 5$, proposed by Li and Cheng [14] contains in our design of Equation (22).

We will develop next two lemmas to help us to decide the similarity measure for the continuous case.

Lemma 2. We show that

$$S(A_1,B)=1-\lambda_1 a_1-\lambda_2 b_1, \tag{23}$$

where $a_1=\frac{\varphi_{A_1}(2)}{2}+\left(\frac{\varphi_{A_1}(2)+\varphi_{A_1}(3)}{2}\right)+\varphi_B(3)\ -2\left[\left(\varphi_{A_1}(3)+\varphi_{A_1}(c_1)\right)\frac{3-c_1}{2}+\varphi_{A_1}(c_1)\frac{c_1-1}{2}\right]$,

$$b_1=\left(\varphi_B(3)-\varphi_{A_1}(3)\right)\left(\frac{5-3}{2}\right)$$

with c_1 is the x-coordinate of the intersection of $\varphi_{A_1}(x)$ and $\varphi_B(x)$ for $1<x<5$.

(Proof of Lemma 2) We know that $\frac{\varphi_{A_1}(2)}{2}$ is the area of the triangle from φ_{A_1} and the area of the trapezoid is $\frac{\varphi_{A_1}(2)+\varphi_{A_1}(3)}{2}$ from φ_{A_1}. Moreover, $\varphi_B(3)$ is the area of the triangle from φ_B, $[\varphi_{A_1}(3)+\varphi_{A_1}(c_1)]\frac{3-c_1}{2}$ is the area of the area of a trapezoid from φ_{A_1}, and $\varphi_{A_1}(c_1)\frac{c_1-1}{2}$ is an area of a triangle by φ_{A_1}. Hence, the integration of $\lambda_1\int_1^3[\varphi_{A_1}(x)-\varphi_B(x)]dx$ can be simplify as $\lambda_1 a_1$.

Consequently, we can derive the same result for the similarity measure between pattern A_2 and sample B.

Lemma 3. We show that

$$S(A_2,B)=1-\lambda_1 a_2-\lambda_2 b_2, \tag{24}$$

where $a_2=\left(\varphi_B(3)-\varphi_{A_2}(3)\right)\left(\frac{3-1}{2}\right)$

$$b_2=\frac{\varphi_{A_2}(4)}{2}(5-4)+\left(\frac{\varphi_{A_2}(4)+\varphi_{A_2}(3)}{2}\right)+\varphi_B(3)\ -2\left[\left(\varphi_{A_2}(3)+\varphi_{A_2}(c_2)\right)\frac{c_2-3}{2}+\varphi_{A_2}(c_2)\frac{5-c_2}{2}\right]$$

with c_2 is the x-coordinate of the intersection of $\varphi_{A_2}(x)$ and $\varphi_B(x)$, for $1<x<5$.

If we consider the same data as the second example of Li and Cheng [14], based on the assumptions in Lemma 2 for a_1 and b_1, we derive that $a_1=0.461404$ and $b_1=0.133333$. By the same argument, based on the assumptions in Lemma 3 for a_2 and b_2, we derive that $a_2=0.2$ and $b_2=0.366667$. Hence, we know that

$$S(A_1,B)=1-0.461404\lambda_1-0.133333\lambda_2, \tag{25}$$

and

$$S(A_2,B)=1-0.2\lambda_1-0.366667\lambda_2. \tag{26}$$

We list some possible combination of different λ_1 and λ_2, under the restriction $\lambda_1+\lambda_2=0.5$ in the next table.

TABLE 2
NUMERICAL EXAMPLE OF [14] UNDER CONTINUOUS CASE

	$\lambda_1 = 0.1$	$\lambda_1 = 0.2$	$\lambda_1 = 0.25$	$\lambda_1 = 0.3$	$\lambda_1 = 0.4$
$S(A_1, B)$	0.900526	0.867719	0.851316	0.834912	0.802105
$S(A_2, B)$	0.833333	0.85	0.858333	0.866667	0.883333

First, we observe that the findings of Li and Cheng [14] with $S(A_1,B)=0.85$ and $S(A_2,B)=0.86$ that is consistent with our derivations in Table 2, where our results are $S(A_1,B)=0.851316$ and $S(A_2,B)=0.858333$, after we round off the second decimal place, then the findings of Li and Cheng [14] are derived.

Motivated by the numerical examples for similarity measure of the table 2, we will derive a theoretical result to clearly point out that the pattern recognition is strongly influence by the weighted function $\omega(t)$.

Lemma 4. There is a unique point, say λ_0 such that

(1) if $0 \le \lambda_1 \le \lambda_0$, then $S(A_1,B) \ge S(A_2,B)$, and

(2) if $\lambda_0 \le \lambda_1 \le 0.5$, then $S(A_1,B) \le S(A_2,B)$.

(Proof of Lemma 4)

Motivated by $a_1 = 0.461404 > a_2 = 0.2$ and $b_1 = 0.133333 < b_2 = 0.366667$, we will abstractly express equations (25) and (26) as follows:

$$S(A_1,B) = 1 - \alpha_1\lambda_1 - \beta_1\lambda_2, \tag{27}$$

and

$$S(A_2,B) = 1 - \alpha_2\lambda_1 - \beta_2\lambda_2, \tag{28}$$

where

$$\alpha_1 = \frac{\varphi_{A_1}(2)}{2} + \left(\frac{\varphi_{A_1}(2)+\varphi_{A_1}(3)}{2}\right) + \varphi_B(3) - 2\left[\left(\varphi_{A_1}(3)+\varphi_{A_1}(c_1)\right)\frac{3-c_1}{2} + \varphi_{A_1}(c_1)\frac{c_1-1}{2}\right], \tag{29}$$

$$\beta_1 = \left(\varphi_B(3) - \varphi_{A_1}(3)\right)\left(\frac{5-3}{2}\right), \tag{30}$$

$$\alpha_2 = \left(\varphi_B(3) - \varphi_{A_2}(3)\right)\left(\frac{3-1}{2}\right) \tag{31}$$

and

$$\beta_2 = \frac{\varphi_{A_2}(4)}{2} + \left(\frac{\varphi_{A_2}(4)+\varphi_{A_2}(3)}{2}\right) + \varphi_B(3) - 2\left[\left(\varphi_{A_2}(3)+\varphi_{A_2}(c_2)\right)\frac{c_2-3}{2} + \varphi_{A_2}(c_2)\frac{5-c_2}{2}\right], \tag{32}$$

under the condition of $\alpha_1 > \alpha_2$ and $\beta_2 > \beta_1$.

We derived that $S(A_1,B) \ge S(A_2,B)$ is equivalent to

$$0.5(\beta_2 - \beta_1) \geq [(\beta_2 - \beta_1) + (\alpha_1 - \alpha_2)]\lambda_1. \tag{33}$$

Hence, we knew that there is a point, say λ_0, with

$$\lambda_0 = 0.5(\beta_2 - \beta_1) / [(\beta_2 - \beta_1) + (\alpha_1 - \alpha_2)], \tag{34}$$

and then if $0 \leq \lambda_1 \leq \lambda_0$, then $S(A_1, B) \geq S(A_2, B)$, and if $\lambda_0 \leq \lambda_1 \leq 0.5$, then $S(A_1, B) \leq S(A_2, B)$.

For completeness, for the Example 2 of Li and Cheng [14], we find the value of λ_0 as $\lambda_0 = 0.235816$. From our previous discussion, it points out that the selection of pattern for the given sample is dependent on the weight function $\omega(x)$. Li and Cheng [14] asserted that the sample *B* always belonging to pattern A_2 is questionable.

Theorem 2.

If the weighted function $\omega(x)$ is expressed as equation (22), then

(a) If $0 \leq \lambda_1 < 0.235816$, then sample B belongs to pattern A_1;

(b) If $0.235816 < \lambda_1 \leq 0.5$, then sample B belongs to pattern A_2

IV. CONCLUSION

A similarity measure is a useful tool for determining the similarity of two objects. Based on the same numerical examples of Li and Cheng [14], we demonstrated that their proposed similarity measures are dominated by the relative weight of the domain for IFS in pattern recognition problems. In the past, researchers focus on developing new similarities to replace previous established similarity measures, moreover, Yen et al. [5], Hung et al. [8], Chu et al. [21] and Chou [22] constructed algorithms that is related to the size of universe of discourse for the discrete cases to repeatedly applied their proposed similarity measures. However, Yen et al. [4], Hung et al. [8], Chu et al. [21] and Chou [22] did not pay attention to how to decide relative weights for elements in the universe of discourse. Based on our discussion, we show that applying the same similarity measure with different relative weights will result in different finding for pattern recognition problems. Consequently, we point out their proposed measures to analyze the behavior of decision making that should be put more attention to the relative weight of the domain for an IFS.

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