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The Study of Laplace Transform and It's Apllication in Real World Problems

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Abstract *- Laplace transform is used in different area of engineering and science, Laplace transforms help to solve critical problems. Laplace transform is used in electric circuit analysis .In this work author used the Laplace transform and discuss the method for solving any differential equation by using Laplace transform since Laplace transform make it easier to solve critical engineering problems*.

Keywords- Laplace transform, differential equation, inverse of Laplace transform, wave and circuit

I. INTRODUCTION

The solution of physical problem has been a challenge to the all.so in engineering field solution of differential equation is simplified using Laplace transform. A Laplace transform is a very different function that can transform a real function of time t to one in complex plane s refer to as frequency domain. It use in various areas of physics, electrical engineering, control engineering, optics, mathematics and signal processing. But we will discuss some of these areas in a paper. The name of this transform originates from a French mathematician, Pierre-Simon Laplace, receiving the name in honor of the late great mathematician due to him using a very similar transform in his work. Study of Laplace transforms has become an essential part of any branch involving mathematics such as engineering, mathematics, physics. Even in chemistry sometimes are required to have an understanding of what a Laplace transform is .the mostly the users of this transform are engineers because its applications in circuits and many other types of systems that deal with sinusoids and exponentials. The Laplace transform is also used in cell phone cause it is type of two way receivers and in heating, ventilation and many more Along with these applications, some of its more well-known uses are in electrical circuits and in analog signal processing, which will be the subjects explored in this paper.

II. DEFINATION OF LAPLACE TRANSFORM

if f(t) is function of t then definite integral $\int_0^\infty e^{-st} f(t)dt$ If it exist will be a function of parameter and is denoted by f(s) is called Laplace transform of f(t) thus

$$L\{f(t)\} = \int_0^\infty e^{-st} f(t)dt = F(s) \tag{1}$$

This function transforms the equation from being in the time domain to being in the complex domain where s is a complex variable representing frequency denoted by the following equation, s=u+iw In the case of the equation denoting s, u and ware both real numbers with i being the complex portion. This means we are putting the differential equation into a completely different domain, as previously mentioned with u and iw being our individual coordinates respectively. This domain will be referred to as the frequency domain.

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III. SOME IMPORTANT PROPERTIES LAPLACE TRANSFORM

3.1 Linearity Property:

If $f(t)$ and $g(t)$ are any two functions of and t, are any two constant α, β then,

$$L[\alpha f(t)+\beta g(t)]=\alpha L[f(t)]+\beta L[g(t)] \quad (2)$$

3.2 Shifting Property:

If $L[f(t)]=F(s)$, (3)

Then $L[e^{at}f(t)]=F(s-a)$ (4).

3.3 Multiplication by t^n Property:

$L[f(t)]=F(s)$, Then (5)

$$L[t^n f(t)]=(-1)^n\frac{d^n}{ds^n}[F(s)]$$

3.4 Laplace Transform of Derivative:

If $L[f(t)]=F(s)$, then (6)

$$L[f^n(t)]=s^nF(s)-s^{n-1}f(0)-s^{n-2}f'(0)-s^{n-3}f''(0)-\cdots-f^{n-1}(0).$$

3.5 Laplace Transform of Bessel's function:

$L[J_0(t)]=\frac{1}{\sqrt{s^2+1}}$, where (7)

$J_0(t)=\sum_{k=0}^{\infty}(-1)^k\frac{(\frac{1}{4}t^2)^k}{(k!)^2}$ Is called Bessel's function. (8)

3.6 Inverse Laplace Transform:

$L[f(t)]=F(s)$, Then (9)

$L^{-1}[F(s)]=f(t)$ Is called inverse Laplace Transform of $F(s)$. (10)

3.7 Inverse Laplace Transform by Convolution Theorem:

If $L^{-1}[\phi_1(s)]=f_1(t);\ L^{-1}[\phi_2(s)]=f_2(t)$ then

$$L^{-1}[\phi_1(s).\phi_2(s)]=\int_0^t f_1(u).f_2(t-u)\,du \quad \textbf{(11)}$$

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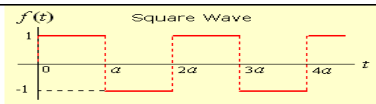
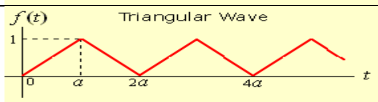
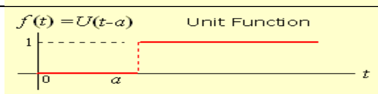
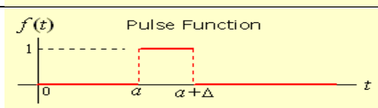
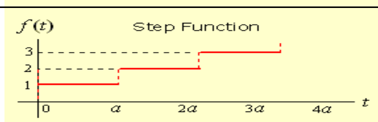
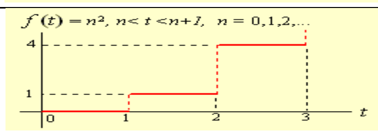
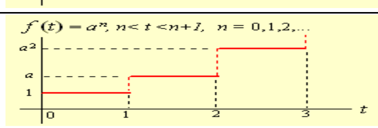
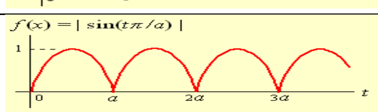
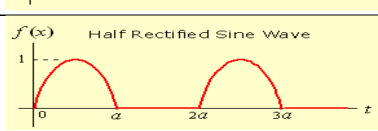
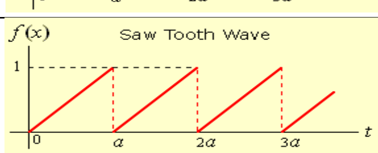
IV. APPLI CATION OF LAPLACE TRANSFORM:

- Laplace Transforms of Common Wave Forms
- Analysis of electronic circuits:
- System modeling:
- Digital signal processing:
- Nuclear Physics:
- Process Control:

4.1 Laplace Transforms of Common Wave Forms

$f(t)$	$F(s)$
	$\frac{1}{s}\tanh\left(\frac{as}{2}\right)$
	$\frac{1}{as^2}\tanh\left(\frac{as}{2}\right)$
	$\frac{e^{-as}}{s}$
	$\frac{e^{-as}\left(1-e^{-s\Delta}\right)}{s}$
	$\frac{1}{s\left(1-e^{-as}\right)}$
	$\frac{e^{-s}+e^{-2s}}{s\left(1-e^{-s}\right)^2}$
	$\frac{1-e^{-s}}{s\left(1-ae^{-s}\right)}$
	$\frac{\pi a}{a^2s^2+\pi^2}\coth\left(\frac{as}{2}\right)$
	$\frac{\pi a}{\left(a^2s^2+\pi^2\right)\left(1-e^{-as}\right)}$
	$\frac{1}{as^2}-\frac{e^{-as}}{s\left(1-e^{-as}\right)}$

4.2 Analog Signal Processing

Laplace transforms is very important part of modern science and in different branches so it is popular among the people who are dealing with these subject. In the late 1900s, the transform has fixed itself as a very necessary component for mathematics,

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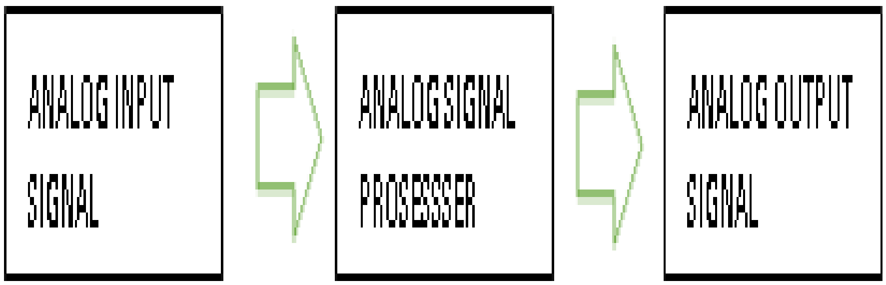
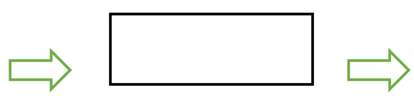
engineering, physics, and other sciences they are familiar with and understand how to use it. The Laplace transform may be famous for analyzing circuits, but it is a very different transform that any mathematician should have knowledge of due to its versatility. The way of analog signals are given following diagram:



The system's input, processor and output are continuous time functions. For the input and output, the labels are x(t) and y(t) respectively. Also, for the purposes of this example we will label the analog signal processor in the middle by h(t).Now Laplace transform is actually used to figure out how the system behaves depending on what input is applied to it, and from there we can discover quite a few things about the system. This means we are trying to find out what the values of y(t) are when we plug in x(t) to the system. We can take the Laplace transform of this to get it into the complex s domain. By taking the Laplace transform, we get X(s) and Y(s}, replacing our previous functions x(t) and y(t), along with getting the transfer function, H(s). Note that H(s) is the analog signal processor from the previous diagram and that the equation that will be mentioned below applies to many more fields than just analog signal processing. Reason we include it is because we take the Laplace transform of the processor as well so to get an accurate equation. It is also the processor that X(s) goes through to give the output Y{s). This relationship can be seen in the following diagram, replacing the previous diagram with another one where the variables are now in the complex plane:

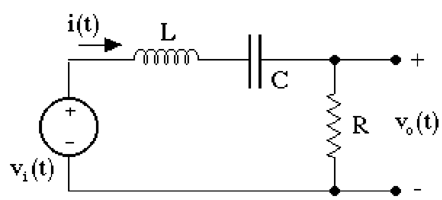
X(S) Y(S)



X(S) Y(S) With this new system in the s plane, we can now figure out what the value of the transfer function, H(s) We do this by first writing the equation in the form we know we can write it as, by recognizing that to get Y(s) we have to multiply the other two together(s) = H(s)X(s)

4.3 Circuit Analysis:

RLC Circuits: - An Example of the Application of Laplace Transforms



Kirchhoff's voltage law

$$v_i(t) = i(t)\,R + L\frac{d\,i(t)}{dt} + \frac{1}{C}\int_{0_-}^{t} i(t')\,dt' \tag{12}$$

Kirchhoff's voltage law transformed

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$$\begin{aligned} V_1(s) &= R\,I(s) + s\,L\,I(s) - L\,i(0_-) + \frac{1}{s\,C}\,I(s) \\ &= \left[R + s\,L + \frac{1}{s\,C}\right] I(s) - L\,i(0_-) \\ &= Z(s)\,I(s) - L\,i(0_-) \end{aligned} \quad (13)$$

Where $Z(s) = R + s\,L + \frac{1}{s\,C}$ (14)

Example 1 - A series RLC circuit excited by a unit impulse function - that is to say

$$V_1(s) = 1 \quad (15)$$

(The unit impulse always gives the "natural response" of any circuit. Natural as compared to "forced" response.)

$$I(s) = \frac{1}{Z(s)} = \frac{1}{R + s\,L + \frac{1}{s\,C}} = \frac{1}{L}\left\{\frac{s}{s^2 + s\,\frac{R}{L} + \frac{1}{LC}}\right\} \quad (16)$$

This is to be compared to the transform pair

$$\begin{aligned} f(t) &= \sqrt{c^2 + d^2}\,\exp(-a\,t)\cos\left(\omega t - \tan^{-1}\frac{d}{c}\right) \\ F(s) &= \frac{c(s+a) + d\,\omega}{(s+a)^2 + \omega^2} \end{aligned} \quad (17)$$

Therefore, if

$$\begin{aligned} & a = \frac{R}{2\,L} \;\Rightarrow\; c = 1 \;\Rightarrow\; d = -\frac{\omega}{a} = -\frac{2\,\omega\,L}{R} \\ & \omega^2 = \frac{1}{LC} - a^2 = \frac{1}{LC} - \left(\frac{R}{2\,L}\right)^2 \\ & \sqrt{c^2 + d^2} = \frac{\sqrt{a^2 + \omega^2}}{a} = \frac{2\,L}{R}\sqrt{\frac{1}{LC}} \\ & \tan^{-1}\frac{d}{c} = -\tan^{-1}\left(\frac{2\,\omega\,L}{R}\right) \end{aligned} \quad (18)$$

$$\boxed{v_2(t) = i(t)\,R = 2\sqrt{\frac{1}{LC}}\exp\left(-\frac{R}{2\,L}\,t\right)\cos\left\{\omega\,t + \tan^{-1}\left(\frac{2\,\omega\,L}{R}\right)\right\}}$$

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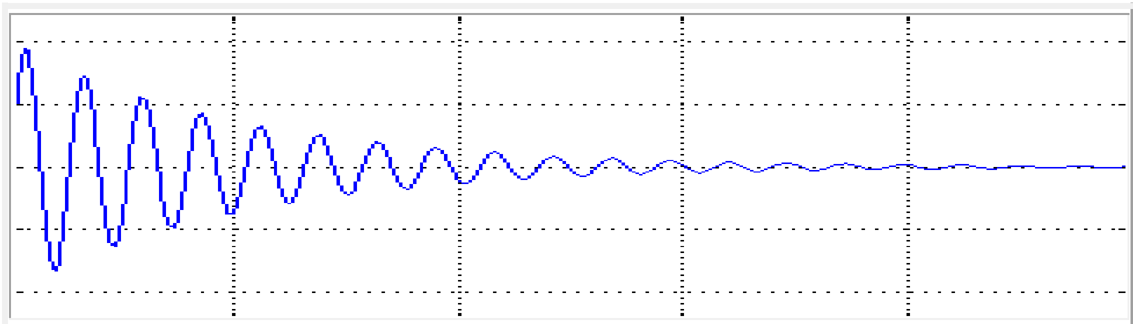
Where $$\omega = \sqrt{\left[\frac{1}{LC} - \left(\frac{R}{2L}\right)^2\right]} \quad (19)$$



Comparison - A series RC circuit excited by a unit impulse function - that is to say

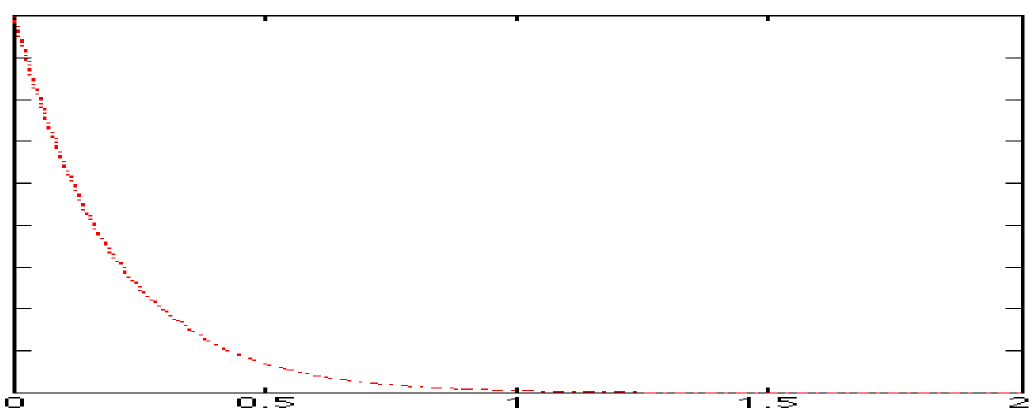
$$V_i(s) = 1 \quad (20)$$

In general

$$\frac{V_o(s)}{V_i(s)} = \frac{R\,I(s)}{Z(s)\,I(s)} = \frac{s\,R\,C}{1 + s\,R\,C} \quad (21)$$

And in particular

$$V_o(s) = \frac{s\,R\,C}{1 + s\,R\,C} = s\,\frac{1}{s + \frac{1}{R\,C}} \quad (22)$$



V. CONCLUSION

The paper presented the application of Laplace transform in different areas of engineering. And also Laplace transform is a very effective mathematical tool to simplify very critical problem in the area of stability and control .in this paper we used some examples of electric circuit and get output easily by using Laplace transform. The waves can be easily form by Laplace transform.

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