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Study of 1D supersonic nozzle flow using MacCormack method

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Abstract**—***The supersonic nozzle is a convergent-divergent type nozzle in which the flow of velocity is much higher than sonic velocity. These types of nozzle have got a lot of applications as a propelling nozzle in automobile and jets. Besides that these nozzles are also used in steam turbines rockets for providing sufficient thrust to move upwards. Since the velocity inside the nozzle is very high, any abrupt change in the flow may cause serious damage to the equipment. Hence the study of these kinds of flow becomes important, so as to understand the behaviour of liquid with respect to the boundaries of the nozzle. In this project, in order to reduce the complexity of the problem, here only 1- D flow has been considered inside the nozzle. The governing equation representing the flow characteristics is been solved using MacCormack method. The problem-solving approach is taken as a time-marching approach to solving the PDE. The result of the numerical discretization process is in the form of ODE. These ODE's are solved in MATLAB to get flow result at all grid points. The final result obtained is in a two-dimensional graphical format representing characteristics like pressure, velocity, temperature, density along the x-axis.*

***Keywords—Matlab, Nozzle, numerical discretization, one-dimensional, supersonic flow*.**

I. INTRODUCTION

The project work presented here in this report is about study of fluid behaviour inside the supersonic nozzle. The flow that is here being assumed is quasi 1 Dimensional flow, that means the flow is only considered along x-axis. It includes the steady, isentropic flow through a convergent-divergent type nozzle. The inlet of the nozzle is considered to come from a reservoir which has infinite area thus having zero velocity of float at inlet. The pressure and temperature values at inlet are stagnation values. The flow is considered to expand through the nozzle is entrolically being subsonic, sonic, Supersonic in convergent, throat, divergent section respectively. The governing equations representing the prescribed flow will be represented in conservation as well as non-conservation form. These equation will be solved at all grid points considering the non-dimensional variable terms. Finally the numerical results of the conservation forms and non-conservations form will be compared.

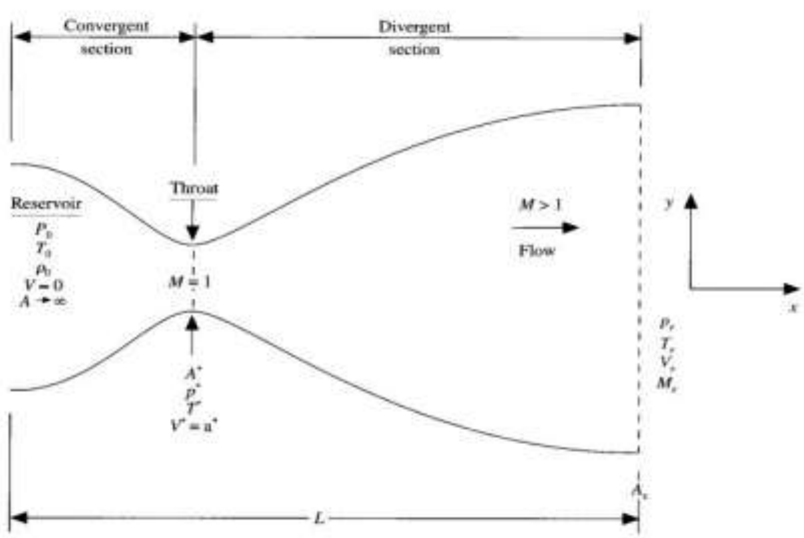

Fig1: convergent-divergent nozzle

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1.1 MacCormack method

As MacCormack method is a simple approach to solve the problems in 1D flow. It is an explicit finite-difference technique which is second order accurate in both space and time the results obtained using this method is most satisfactory and easy to understand. In this method time marching solution is used to obtain the solution at next time step using the previous time step value. Assuming that the flow field variable at a Grid Point at time are known, the value of flow field variable at the same Grid Point at next time step (t + ∆t) is calculated.

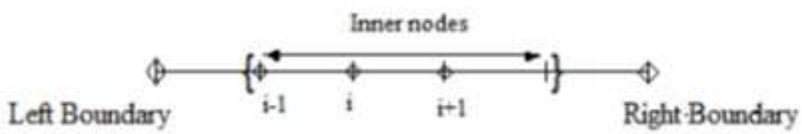

Fig 1.1: 1-D grid representation

Let us consider f as a function which can represent temperature, velocity or density.

1.1.1 Predictor step:

In this we calculate $(\frac{\partial f}{\partial t})^t$

It is predicator step, let us represent it a $(\frac{\partial f}{\partial tp})^t$

Finding solution update

$f^{t+\Delta t} = f_{initial} + (\frac{\partial f}{\partial tp})^t \Delta t$

$f^{t+\Delta t}$ =updated solution

1.1.2 Corrector step:

Consider $f^{t+\Delta t}$ values and find $(\frac{\partial f}{\partial tc})^{t+\Delta t}$

1.1.3 Average step:

This step is about finding arithmetic mean of predicator step and corrector step.

$(\frac{\partial f}{\partial t})_{avg}=0.5((\frac{\partial f}{\partial tp})^t + (\frac{\partial f}{\partial tc})^{t+\Delta t})$

This allows us to obtain the final “corrected” value of function at time (t + ∆t)

$f^{t+\Delta t} = f^t + (\frac{\partial f}{\partial t})_{avg} \Delta t$

II. PROPOSED METHODOLOGY

2.1 Governing Equation

2.1.1 Conservative forms of governing equation

The conservative forms of governing equation are:

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(1) Continuity equation:

$$\frac{\partial(\rho'.A')}{\partial t'}+\frac{\partial(\rho'.A'.V')}{\partial x'}=0$$

(2) Momentum equation:

$$\frac{\partial(\rho'.A'.V')}{\partial t'}+\frac{\partial\left(\rho'.A'.V'^2+\left(\frac{1}{\gamma}\right)\rho'.A'\right)}{\partial x'}=\left(\frac{1}{\gamma}\right).p'.\frac{\partial A'}{\partial x'}$$

(3) Energy equation:

$$\frac{\partial\left(\rho'.\left(\frac{e'}{\gamma-1}+\frac{\gamma}{2}.V'^2\right).A'\right)}{\partial t'}+\frac{\partial\left(\rho'.\left(\frac{e'}{\gamma-1}+\frac{\gamma}{2}.V'^2\right).V'.A'+p'.A'.V'\right)}{\partial x'}=0$$

2.1.2 Non-conservative forms of governing equation

The non-conservative forms of governing equation are:

1) Continuity equation:

$$\frac{\partial\rho'}{\partial t'}=-\rho'.\frac{\delta V'}{\delta x'}-\rho'.V'\frac{\partial(lnA')}{\partial x'}-V'.\frac{\delta\rho'}{\delta x'}$$

2) Momentum equation:

$$\frac{\partial V'}{\partial t'}=-V'.\frac{\partial V'}{\partial x'}-\left(\frac{1}{\gamma}\right)\left[\frac{\partial T'}{\partial x'}+\left(\frac{T'}{\rho'}\right).\frac{\delta\rho'}{\delta x'}\right]$$

3) Energy equation:

$$\frac{\partial T'}{\partial t'}=-V'.\frac{\partial T'}{\partial x'}-(\gamma-1).T'\left[\frac{\delta V'}{\delta x'}+V'.\frac{\partial(\ln(A'))}{\partial x'}\right]$$

III. RESULT

3.1 Conservative form

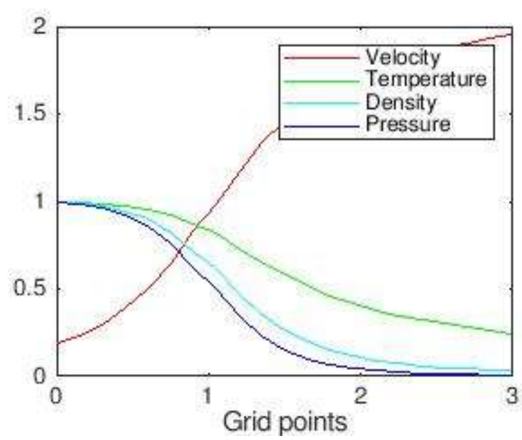

Fig 3: Showing variation of Velocity, Temperature, Density and Pressure of Conservative form

1) The velocity profile clearly shows there is subsonic velocity at the inlet of nozzle. Thus, even if the reservoir at inlet of nozzle has zero velocity, the grid point has still some velocity. And it increases from inlet to being supersonic at outlet.

2) The profile of temperature, pressure and density shows gradual decrement from there stagnation value at inlet,

3.2 Non-Conservative form

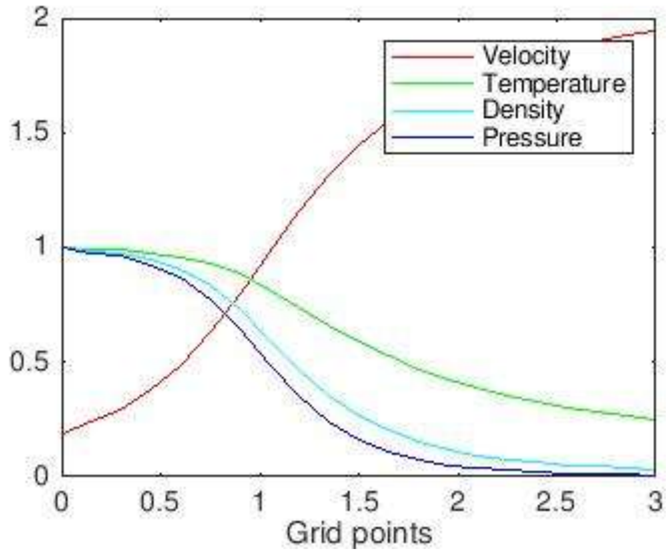

Fig4: Showing variation of Velocity, Temperature, Density and Pressure of Conservative form

1) The results obtained in non-conservation part are quite similar for first few time steps.

2) The velocity profile can be explained by method of characteristics.

3) The profile of temperature, pressure and density shows similar variation as shown in conservative part.

3.3 Mass flow rate of conservative and non-conservative form

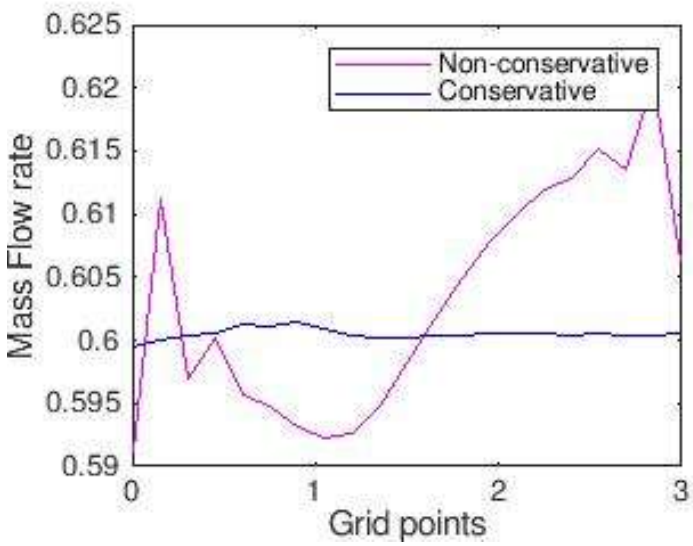

Fig 5: Showing variation of mass flow rate

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1) As shown in figure the conservative form gives the mass flow rate variation close to constant and more accurate value at the steady state.

2) But in the case of non-conservative part it shows some oscillation at inlet and outlet of the nozzle and result is less satisfactory as compared to conservative form.

IV. CONCLUSION

This study describe the theoretical behavior of fluid when the flow is conducted through supersonic nozzle. The result shows that the properties of fluid for first few time step are quite similar in both conservative and non-conservative form. This explains that the MacCormack method in comparison to other methods is the most suitable and feasible method to solve quasi 1D flow through supersonic nozzle. The result obtained in this study can be further be used for study of 2-dimensional flow in supersonic nozzle at any grid point can be studied and analyzed using Maccormack method.

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