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The Study of Laplace Transform and Fourier Transform

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Abstract—*In this paper we study the basics of Laplace transform and Fourier transform it deals with what a Laplace transform and Fourier Transform means and its application. The history of both the transformation and properties with examples on differential equation have been discussed.*

Keywords—Fourier transform,Laplace transform, Ordinary differential equation, Partial differential equation.

Introduction

Mathematics is everywhere in the world, it is used in every field. It is in every phenomenon, every technology, every observation, every experiment and more. Theknowledge of Laplace transform and Fourier transform is required for engineers and scientist in recent year because it is essential part of mathematical background. And the Laplace transform is used to find out the solutions of ordinary differtial equation whereas Fourier transform is used to find solution of ordinary differential equations.
The method of Laplace Transform give the directly Advantage for the solution of differential equations with given boundary values without the necessity of first finding the general solution and then evaluating from it the arbitrary constants.
Moreover the ready tables of Laplace transforms reduce the problem of solving differential equations to mere algebraic manipulation. Fourier transform when applied the partial differential equation reduces the number of its independent variables by one.it is highly useful in the study of conduction of Heat, Wave propagation, communication etc.

I. HISTORY

1.1) Laplace transform

Laplace transform was formulated by Pierre-Simon Laplace [1749-1827] who was well known French mathematician astronomer whose work contributed greatly to the development mathematical astronomy and statistic. He formulated Laplace's equation, pioneered and developed the Laplace transforms, it is used in many branches of science. The laplaciandiffertial operator is named after him. He is remembered as one of the greatest science of all time and is called Newton of France.

1.2) Fourier transform

Fourier transform was formulated by Jean Baptiste Joseph Fourier [1768-1830] a great French mathematician and physicist he is best remember for pioneering Fourier series, Fourier transform He went to Egypt with napoleon bonapararte in 1798 and was made governor of lower Egypt he contributed several mathematical papers to Cairo institute founded by napoleon He was permanent secretary of French academy of sciences in 1830 he was elected to royal Swedish academy of science Key idea is that the Fourier transforms changes a function on one space into another function on a different space.
Fourier transforms are properly a subdomain of harmonic analysis, which is a very general and powerful set of mathematical ideas. The Laplace transform is similar to the transform. The Fourier transform of a function is a complex function of a real variable were as Laplace transform of a function is a complex function of a complex variable.

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II. DEFINITION

2.1) LAPLACE TRANSFORM:-

Let $f(t)$ be a function of t defined for all positive values of t. then the LAPLACE TRANSFORM of $f(t)$,denoted by L{ $f(t)$}is define by

$$L\{f(t)\} = \int_0^\infty e^{-st} f(t)dt$$

Provided that the integration exist s is the parameter which may be real or complex no.$L\{f(t)\}$ Been clearing a function of s is briefly written as $\bar{f}(s)$ i.e. $L\{f(t)\} = \bar{f}(s)$

$$\therefore\ f(t) = L^{-1}\{\bar{f}(s)\}$$

Then $f(t)$is called inverse Laplace transform of $\overline{f(s)}$

2.2) FOURIER TRANSFORM:-

If a function $f(t)$ is defined on (-∞, ∞), is piecewise continuous in each finite interval and is absolutely in (-∞, ∞) then the integral $\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(t)e^{-ist}\,dt$ is called the Fourier Transform f (t) and is denoted by F { $f(t)$} or $F(s)$ thus

$$F(s) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(t)e^{-ist}\,dt$$

III. PROPERTIES

3.1) LAPLACE TRANSFORM:-

1 .LinearityProperty:

If$f(t)$and$g(t)$areanytwofunctions of t and α, β are any two constantthen,

$$L[\alpha\, f(t) + \beta g(t)] = \alpha L[f(t)] + \beta L[g(t)]$$

2. Shifting Property:

If $L[f(t)] = F(s)$, then$L[e^{at} f(t)] = F(s-a)$.

3. Multiplication byt^nProperty:

$L[f(t)] = F(s)$, Then

$$L[t^n f(t)] = (-1)^n \frac{d^n}{ds^n}[F(s)]$$

4. Laplace Transform of Derivative:

If$L[f(t)] = F(s)$,then

$$L[f^n(t)] = s^nF(s) - s^{n-1}f(0) - s^{n-2}f'(0) - s^{n-3}f''(0) - \cdots - f^{n-1}(0).$$

5. Laplace Transform of Bessel's function:

$L[J_0(t)] = \frac{1}{\sqrt{s^2+1}}$,where

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$J_0(t) = \sum_{k=0}^{\infty}(-1)^k \frac{(\frac{1}{4}t^2)^k}{(k!)^2}$ Is called Bessel'sfunction.

6. Inverse LaplaceTransform:

$L[f(t)] = F(s)$,Then

$L^{-1}[F(s)] = f(t)$ Is called inverse Laplace Transform of $F(s)$

7. Inverse Laplace Transform by Convolution Theorem:

if

$$L^{-1}[\phi_1(s)] = f_1(t);\quad L^{-1}[\phi_2(s)] = f_2(t)$$

Then,

$$L^{-1}[\phi_1(s).\phi_2(s)] = \int_0^t f_1(u).f_2(t-u)\,du$$

3.2) FOURIER TRANSFORM:-

1. Linear Property:

If $F(s)$ and $G(s)$are Fourier Transforms of $f(t)$ and $g(t)$respectively

ThenF[a f(t) + b g(t)] = a F(s) + b G(s)

2. Change of scale property:

If F(s) is complex Fourier Transforms of $f(t)$Then $\{f(at)\} = \frac{1}{a}F\left(\frac{s}{a}\right)$, a≠0

3. Shifting Property:

If F(s) is complex Fourier Transforms of f(t)

Then $F\{f(t-a)\} = e^{isa}F(s)$

4. Modulation Theorem:

If F(s) is complex Fourier Transforms of f(t)

Then $F\{f(t)cosat\} = \frac{1}{2}$ (F(s+a)+ F(s-a))

IV. EXAMPLES

1) **solve $(D^3 - 3D^2 + 3D - 1)y = t^2e^t$ given that y (0) =1, y' (0) =0, y" (0) =-2.**
   Sol:-
   Taking the Laplace transform on both side, we get

$$[s^3\bar{y} - s^2y(0) - sy'(0)] - 3[s^2\bar{y} - sy(0) - y'(0)] + 3[s\bar{y} - y(0)] - \bar{y} = \frac{2}{(s-1)^3}$$

Using the given conditions, it reduces to

$$\bar{y} = \frac{s^2 - 3s + 1}{(s-1)^3} + \frac{2}{(s-1)^6} = \frac{(s-1)^2 - (s-1) - 1}{(s-1)^3} + \frac{2}{(s-1)^6}$$

$$= \frac{1}{(s-1)} - \frac{1}{(s-1)^2} - \frac{1}{(s-1)^3} + \frac{2}{(s-1)^6}$$

On inversion, we obtain

$$Y(t) = L^{-1}\left\{\frac{1}{(s-1)}\right\} - L^{-1}\left\{\frac{1}{(s-1)^2}\right\} - L^{-1}\left\{\frac{1}{(s-1)^3}\right\} + L^{-1}\left\{\frac{2}{(s-1)^6}\right\}$$

$$= e^t\left(1 - t - \frac{1}{2}t^2 + \frac{1}{60}t^5\right)$$

2) **An impulsive voltage E$\delta(t)$ is applied to a circuit consisting of L, R, C in series with zero initial conditions . If I be the current at any subsequent time t find the limit of I as $t \to 0$**

Sol:-

The equation of the circuit governing the current I is

$L\frac{di}{dt} + Ri + \frac{1}{C}\int_0^t i\,dt = E\,E\delta(t)$ where i=0, when t=0

Taking Laplace transform of both sides we get

$$L[s\bar{\imath} - i(0)] + R\bar{\imath} + \frac{1}{C}\frac{1}{s}\bar{\imath} = E$$

Where R\L =2a and 1\CL=a^2+b^2

$$\bar{\imath} = \frac{E}{L}\frac{(s+a) - a}{(s+a)^2 + b^2}$$

$$= \frac{E}{L}\left\{\frac{(s+a)}{(s+a)^2 + b^2} - a\frac{1}{(s+a)^2 + b^2}\right\}$$

On inversion we get

$$= \frac{E}{L}\left\{e^{-at}\cos bt - \frac{a}{b}e^{-at}\sin bt\right\}$$

Taking limits as $t \to 0$, $i \to \frac{E}{L}$

Although the current i=0 initially yet a large current will develop instantaneously due to impulsive voltage applied cat t=0 .In fact we have determinant the limits of this current which is$\frac{E}{L}$

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3) **Find the Fourier transform of :**

$$\boldsymbol{f(x)=\begin{cases}1-x^2, & |x|\le 1\\ 0, & |x|>1\end{cases}}$$

Sol:-

$F\{f(x)\}=\int_{-\infty}^{\infty} f(x)e^{isx}\,dx=F(s)$, say

$$=\int_{-\infty}^{-1}(0)\,e^{isx}\,dx+\int_{-1}^{1}(1-x^2)\,e^{isx}\,dx+\int_{1}^{\infty}(0)\,e^{isx}\,dx$$

$$=\left|(1-x^2)\frac{e^{isx}}{is}-(2x)\frac{e^{isx}}{is^2}+(-2)\frac{e^{isx}}{is^3}\right|_{-1}^{1}$$

$$=2\left(\frac{e^{is}+e^{-is}}{-s^2}\right)-2\left(\frac{e^{is}-e^{-is}}{-is^3}\right)$$

$$=-\frac{4}{s^3}(scos\ s-\sin s)$$

Now by inversion formulae we have, $f(x)=\frac{1}{2\pi}\int_{-\infty}^{\infty}F(s)e^{-isx}\,ds$

$$=-\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{4}{s^3}(scos\ s-\sin s)e^{-isx}\,ds=\begin{cases}1-x^2, & |x|\le 1\\ 0, & |x|>1\end{cases}$$

4) **Find the Fourier transform of**

$$\boldsymbol{f(x)=\begin{cases}k, & |x|<a\\ 0, & |x|>a\end{cases}}$$

Sol:-

$$F(s)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}f(x)e^{-isx}\,dt$$

$$F(s)=\frac{1}{\sqrt{2\pi}}\int_{-a}^{a}ke^{-isx}\,dt$$

$$=\frac{\boldsymbol{k}}{\sqrt{2\pi}}\left[\frac{e^{-isx}}{\boldsymbol{is}}\right]_{-\boldsymbol{a}}^{\boldsymbol{a}}$$

$$=\frac{\boldsymbol{k.2}}{\sqrt{2\pi}.s}\left[\frac{e^{isa}-e^{-isa}}{\boldsymbol{2i}}\right]$$

$$=\frac{k}{s}\sqrt{\frac{2}{\pi}}\,sinsa$$

We show below the graph of f(x) and F(s) at k=1 and a=3

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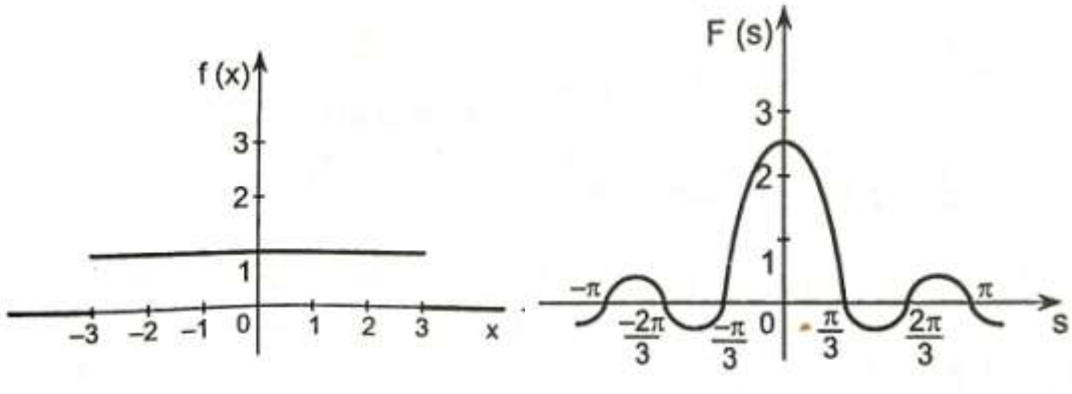

(4.1 a)(4.1 b)

V.CONCLUSION

The Laplace transform and Fourier transform has wide use in many fields especially in engineering it used to solve the differential equation and the properties of Laplace transform and Fourier transform is very helpful for finding different way of solving differential equations to mere the algebraic manipulation.In general, the Laplace transform is used when the functions are defined on the half spacei.e $t \geq 0$, whereas the Fourier transform is used when the functions defined on $(-\infty, \infty)$. Laplace Transform does a real transformation on complex data but Fourier Transform does a complex transformation on real data.

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